THE

ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

VOLUME XXXIII

JANUARY—JUNE, 1911

The University of Chicago Press
CHICAGO, ILLINOIS

Published
January, March, April, May, June, 1911

Composed and Printed By
The University of Chicago Press
Chicago, Illinois, U.S.A.

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THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXXIII JANUARY 1911 NUMBER 1

ON THE ANGULAR SPEED OF ROTATION OF A LONG-ENDURING PROMINENCE

By J. EVERSHED

The prominence described by Dr. Slocum in the *Astrophysical Journal* for September 1910 (**32**, 125), and figured in Plate XII, was photographed with the Kodaikánal spectroheliograph at each successive appearance on the sun's limb. Our photographs are very similar to those taken at the Yerkes Observatory, but, contrary to Dr. Slocum's experience, our K_2 flocculi plates show the prominence also as an absorption marking on the disk of the sun at three successive meridian transits. On the three days following March 22 the prominence is such a conspicuous and remarkable object on the disk that it is difficult to understand Dr. Slocum's failure to photograph it.

Taking advantage of the exceptional opportunity afforded by the disk photographs for determining the speed of angular rotation of the prominence, I have made a series of measures of a well-defined portion of the absorption marking, and the results of these measures, together with a spectrographic determination of rotation speed, are, I think, of sufficient interest to give in some detail.

As the spring months at Kodaikánal are the most favorable for solar work, our series of photographs is very complete, and they show that the prominence endured in a more or less compact form for at least 82 days, and the region of longitude in which it

was situated became very active again a month later on May 24 and 25.

I give in the following table the dates when the prominence had attained its greatest apparent development on either limb, and the approximate limits of latitude covered.

Date	Limb	Latitude	Height
1910 Feb. 5......	West	$+2°$ to $-14°$	55″
Feb. 19......	East	0 to -22	35
March 5......	West	-6 to -20	95
March 18......	East	$+15$ to -20	100
April 1......	West	-9 to -23	40
April 14......	East	$+5$ to -25	50
April 28......	West	-7 to -23	80

Excepting during the early stages of development in February the prominence was visible for three successive days at each limb, and the dates given are the second day of visibility in each case, when the maximum apparent height was reached. The prominence was very variable in extent, and it is not easy to assign exact limits in latitude, on account of small outlying prominences apparently connected with the main mass by faint streamers. When on the eastern limb, it was always more extended northward than when on the west limb.

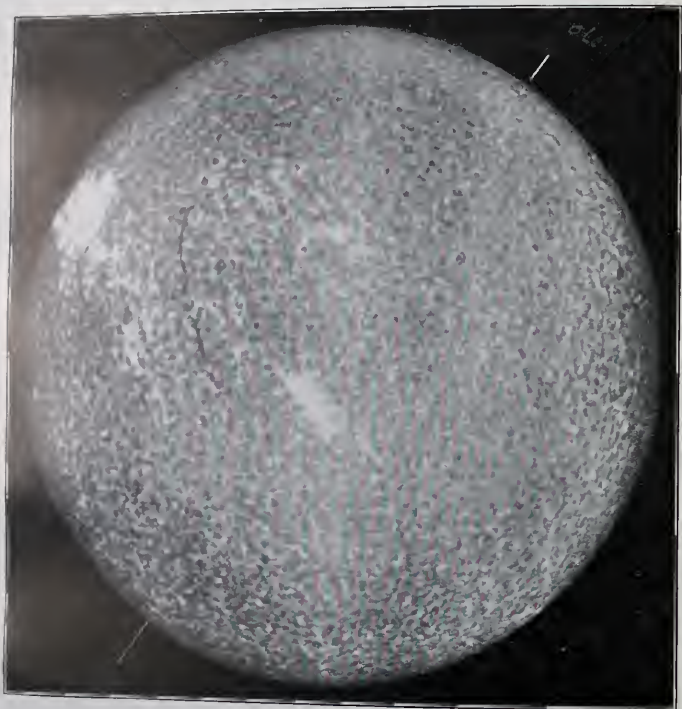
The general aspect of this "quiet" type of prominence and the rapid changes constantly in progress in the smaller details suggest a continuous renewal of the glowing gas from numerous orifices situated within the photosphere. If this be so, it is evident that in deducing values of angular rotation from successive apparitions on opposite limbs we really obtain the rotation speed of these orifices, not that of the chromosphere. But the chromospheric layer, according to the researches of Adams, has an angular rotation speed at the equator of about $6\frac{1}{2}$ per cent greater than that of the photosphere as deduced from spots and faculae: consequently the prominence gases projected into the chromosphere will constantly tend to drift westward over the photosphere, like smoke driven by the wind.

The velocities of angular rotation that I have obtained by different methods for the prominence under discussion appear to show distinctly that this westward drifting really takes place, while

PLATE I

North Pole

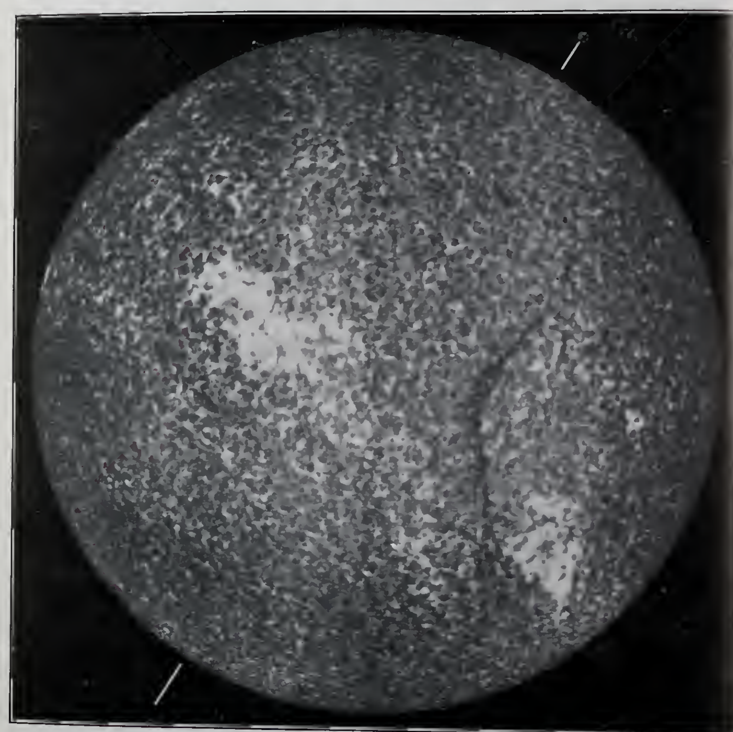
W



February 28, 1910

PLATE II

North Pole



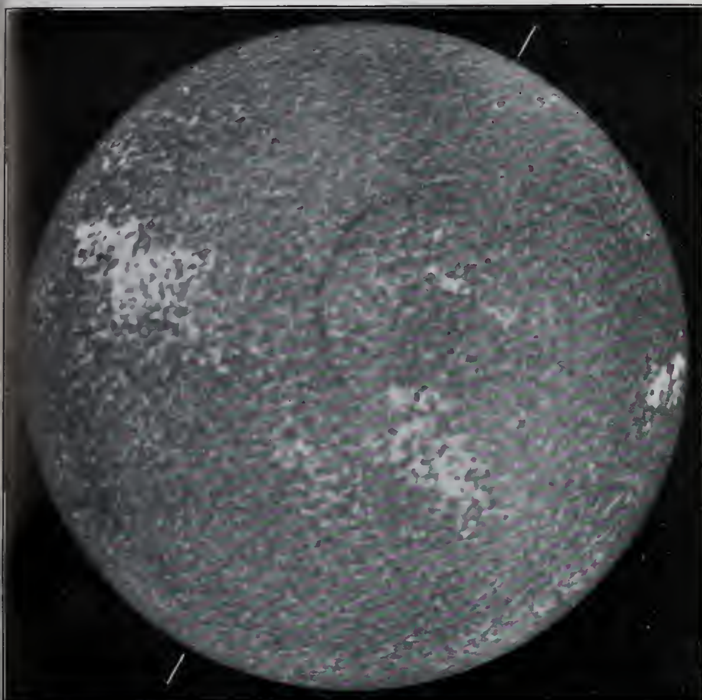
March 23, 1910

SPECTROHELIOGRAMS SHOWING DARK FLOCCULUS

Taken with calcium line K_2. Kodaikánal Observatory

PLATE III

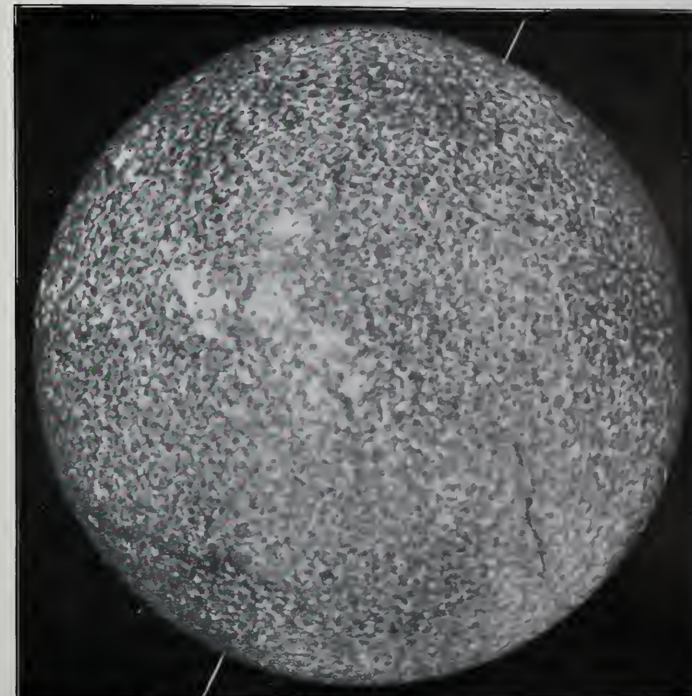
North Pole



March 25, 1910

PLATE IV

North Pole



E

April 18, 1910

SPECTROHELIOGRAMS SHOWING DARK FLOCCULUS

Taken with calcium line K_2. Kodaikánal Observatory

the locality of origin of the prominence rotates with the normal speed of the photosphere.

If we take the interval of 82 days between February 5 and April 28 to represent three synodical revolutions of the prominence, we get a rotation speed of 13°.2 per diem, or a sidereal speed, $\xi = 14°.2$ per diem. This corresponds closely with the angular speed of spots in latitude 10° to 15°. The mean latitude of the prominence at its western apparitions is $-13°$.

The estimate of the time when a prominence is at the limb or 90° from the central meridian is very uncertain, however, and the whole interval may well be subject to an error of ± 1 day, so that the above value may be over 1 per cent in error.

A much more accurate estimate is obtained by measuring the absorption marking due to the prominence projected on the disk, and deducing the times of passing the central meridian at the successive transits. The marking was photographed on February 25, 26, 27, and 28; on March 22, 23, 24, and 25; and again less distinctly every day from April 16 to 26 inclusive. In the two earlier apparitions it is shown as a bow-shaped dark streak, crossing the equator (see Plates I and II), the center of the bow lying on the equator. During February the bow was narrower than in March, and more sharply bent at the center, forming an obtuse angle; this definite point, which was in latitude $+0°.4$, was measured on the successive days, giving a good value of the daily motion, as well as the time of meridian passage. The three plates of March were similarly measured, but in this case a point had to be chosen 8° north of the equator, where the marking contracted to a narrow line, nearly perpendicular to the equator. In April only the southern arm of the bow can be seen, highly inclined to the equator, and accurate measures of the daily motion are not possible; but from measures of the western end of the marking in the plates of April 20, 22, and 23 I have estimated the time of meridian passage of a point in latitude $-3°$.

The times of meridian transit deduced from these measures are

1910 February	25, 9^h	12^m	Greenwich Civil Time
March	24, 7	20	Greenwich Civil Time
April	20, 3		Greenwich Civil Time

The first interval of 26.92 days, representing a complete synodical revolution, is equal to a mean daily sidereal motion, $\xi=14\overset{\circ}{.}37$; and the second interval of 26.84 days is equal to a mean motion of $14\overset{\circ}{.}40$. The first interval is not likely to be in error by an amount exceeding one part in 400, but the second is less trustworthy. These values may be taken to represent the equatorial velocity, notwithstanding the fact that the measures for March refer to a point in latitude $+8°$.

It is assumed, of course, that the marking photographed in February is identical with those obtained in March and April; this is probably true only in the same sense that the prominence photographed on the west limb on February 5 is identical with those photographed on later dates; that is to say, the angular velocity obtained is the velocity of the origin of the marking, not that of the absorbing gas itself. It is in quite remarkable agreement with the equatorial speed of the photosphere as determined from spots.

The measures of angular motion from day to day give quite a different result, as the following determinations of longitude measured from the central meridian clearly indicate.

DAILY MOTION IN LONGITUDE OF ABSORPTION MARKING SITUATED IN LATITUDE $+0\overset{\circ}{.}4$

Date	G.C.T.	Longitude	Motion in 24 Hours	ξ
1910 Feb. 25	$2^h 40^m$	$3\overset{\circ}{.}8$ East		
Feb. 26	2 42	10.2 West	$14\overset{\circ}{.}00$	$15\overset{\circ}{.}00$
Feb. 27	2 45	24.4 West	14.12	15.12
Feb. 28	3 01	38.7 West	14.14	15.14

Mean daily sidereal motion $= 15\overset{\circ}{.}09$

DAILY MOTION IN LONGITUDE OF ABSORPTION MARKING SITUATED IN LATITUDE $+8°$

Date	G.C.T.	Longitude	Motion in 24 Hours	ξ
1910 March 23	$2^h 31^m$	$18\overset{\circ}{.}3$ East		
March 24	2 42	2.6 East	$15\overset{\circ}{.}3$	$16\overset{\circ}{.}3$
March 25	2 21	11.7 West	14.8	15.8

Mean daily sidereal motion $= 16\overset{\circ}{.}05$

As a check on these results I measured at the same time a small bright flocculus, which could be identified in successive plates during four days; the results are as follows:

DAILY MOTION IN LONGITUDE OF A CALCIUM FLOCCULUS SITUATED IN LATITUDE −8°

Date	G.C.T.	Longitude	Motion in 24 Hours	ξ
1910 March 23........	$2^h 31^m$	41°2 East		
March 24........	2 42	28.2 East	12°9	13°9
March 25........	2 21	14.3 East	14.1	15.1
March 26........	2 24	1.0 East	13.3	14.3

Mean daily sidereal motion = 14°43

The above result, although somewhat rough, is in substantial agreement with Hale's determination of mean motion of the flocculi,[1] and shows that the absorption marking was drifting westward with respect to the flocculus.

It appears, then, that the dark mass of calcium vapor (and hydrogen) near the equator had an angular rotation speed 5 per cent greater than the general surface of the photosphere during the February apparition, and as much as 11 per cent greater during the March apparition. Also that the two apparitions really represent two distinct masses of absorbing gas, emanating from a common origin approximately in solar longitude 75°. Although the February marking could be traced nearly to the western limb on March 3, it must have become dissipated subsequently, as it was not shown near the east limb on some excellent plates taken on March 19 and 20. On March 21 it reappears as a vague and ill-defined dark mass, extending across the equator, and some distance within the east limb. On the following day the bow-shape became evident.

The intermittent character of the marking is also shown in its later phases. On March 25 it had attained its greatest apparent development, extending for a distance of at least 36° of solar latitude, or 250,000 miles (400,000 km). The northern arm can indeed be faintly traced for a much greater distance in a vast circular sweep toward the eastern limb, which it nearly reaches. Notwithstand-

[1] *Astrophysical Journal*, **27**, 227, 1908.

ing the prodigious length of this mass of relatively cool gas, the whole object utterly vanished during the next 24 hours. The plate of March 26 is of the best quality, and shows two small prominences as dark markings on the disk, yet not a trace can be seen of the large marking, nor is it shown on the plates of the 27th or 28th. A portion of the southern branch is seen again on the 29th, but in a more easterly position on the disk; this apparently reappears on the east limb on April 16, and continues visible until near the west limb on the 26th of the month.

From the whole inquiry it seems definitely established that there was a region on the sun, narrowly defined in longitude, and partaking of the normal rotation speed of the photosphere, which was somewhat intermittently giving rise to prominences, and that these prominences drifted westward with about the angular speed of the hydrogen of the chromosphere. The remarkable bow-shape of the absorption marking, with the center over the equator, is very suggestive of a wave surging westward over the photosphere, with a greater speed near the equator than in higher latitudes. Yet the velocity obtained in latitude $+8^{\circ}$ in March is distinctly greater than that observed on the equator a month earlier. The marking is unfortunately too indefinite in higher latitudes for measures of longitude to be made. It is perhaps more probable that the bow-shape is really due to the original disposition of the prominence-forming orifices.

The mean level of the absorbing calcium vapor is difficult to determine. If the whole prominence is effective in producing absorption it may be roughly estimated at some $30''$ above the photosphere, but there is reason for supposing that only portions of the prominence are cooled sufficiently to be effective, and these portions may be in the lower and denser region not extending much above the chromosphere. When these dark markings are seen extending to the sun's limb they are found almost invariably to end in a prominence, but the latter in many cases are somewhat insignificant in height. On the other hand, many large prominences, perhaps the majority of them, seem to produce no absorption on the disk.

The rotational movement of the higher regions of a "quiet"

prominence can be ascertained by spectrographic measures, and this method of investigation has for some time past been on our program of research. Only a few plates have as yet been obtained, but these fortunately include the prominence of March when on the east limb. With a radial slit placed across the limb at the sun's equator the $H\alpha$ line was photographed in this prominence on March 17 and 18. It appears as a sharply defined bright line, about 0.5 mm wide outside the limb, and as an absorption line 1 mm wide within the limb, the scale of the plates being 1 mm = 1.2 Å. I have measured the displacement of the bright line with respect to the dark line, and the result distinctly confirms the westward movement of the prominence. In measuring the plates a single straight thread was used, placed parallel to the spectrum lines, and the error in parallelism which might seriously affect the result was carefully determined and allowed for. The mean results obtained, with the red end of the spectrum to the right and left respectively, are as follows:

Date	Height above Photosphere	$\Delta\lambda$	Km per Sec.
1910 March 17...	17″	−0.015 Å	+0.68
March 18...	30	−0.015 Å	+0.68

The absorption line may be taken to represent the normal chromosphere line, with a velocity of approach of about 2 km per second. The excess velocity of the prominence at a considerable height above the chromosphere amounts therefore to as much as 34 per cent. The consistence of the measures with the red end of the spectrum to right and left, and the undistorted character of the emission line lead me to believe that this relatively high velocity is real and permanent. It would be unsafe, however, to infer from a single instance that the law of increase in angular speed with height discovered by Adams is continued outside the limits of the chromosphere, but this may be taken as a suggestion only.

Solar Observatory
Kodaikánal, South India
November 29, 1910

PHOTOGRAPHIC DETERMINATIONS OF STELLAR PARALLAX MADE WITH THE YERKES REFRACTOR. II

By FRANK SCHLESINGER

Having described the process at the telescope by which the plates were obtained, it is now in order to consider the methods to be employed

AT THE MEASURING ENGINE.

The choice between polar and rectangular co-ordinates for parallax work is easily made: the former are more difficult to measure, they entail more cumbersome computations, and they restrict the choice of comparison stars. The last of these disadvantages is especially marked when distances only are considered, but it is of consequence even when both position-angles and distances are measured. In rectangular co-ordinates a comparison star in one position is practically as good as in any other, if only the mean position of all the comparison stars (sometimes referred to as their "center of gravity") is not too far from the parallax star.

Having then decided to use rectangular co-ordinates, we must next say whether we shall measure in one or in two directions. If a star is near the ecliptic its parallactic ellipse is narrow, and little additional information is gained by measuring the displacement in latitude. Furthermore, by observing early in the evening and late into the morning, plates may be obtained at the times when the star is at either extremity of the major axis of the parallactic ellipse. But it is impossible, except in very rare cases, to obtain observations at both ends of the minor axis, since one of them corresponds to the time when the longitude of the sun is equal to that of the star, and therefore when the latter culminates not far from noon. This fact alone reduces the weight of the parallax as derived from latitude displacements nearly in the ratio of 1 to 4, as compared with that from longitude displacements. For eighteen of the stars here discussed we have therefore oriented the axis parallel to the ecliptic and have measured longitude co-ordinates only.

This is in general the direction that yields the maximum weight for the deduced parallax.

It seemed, however, particularly desirable that, for at least some of the stars, the parallax should be derived independently from two co-ordinates; this has accordingly been done for the seven stars in the present list that are best suited to the purpose. But instead of measuring in longitude and latitude, it is a little preferable to measure in right ascension and declination. The results show that measures in declination have somewhat smaller probable errors than those in right ascension, and therefore the two should be kept separate. This difference in accuracy is doubtless due to the guiding error, the following in declination being independent of the behavior of the clock.

Whatever direction may be chosen for the axes, the combined weight for the two resulting determinations of a parallax is always the same. To prove this, let l and b represent the "parallax factors" in longitude and latitude at the time t; that is, the two displacements due to the parallax (π) will be $l \cdot \pi$ and $b \cdot \pi$. Each plate will, therefore, offer equations of this form:

$$\begin{array}{cc} c+t_1\cdot\mu+l_1\cdot\pi=M_1 & k+t_1\cdot\nu+b_1\cdot\pi=N_1 \\ c+t_2\cdot\mu+l_2\cdot\pi=M_2 & k+t_2\cdot\nu+b_2\cdot\pi=N_2 \\ \text{etc.,} & \text{etc.,} \end{array}$$

where c and k are constants and μ and ν are the proper-motion components. The zero of time may be so chosen as to make the sum of the times for all the plates equal to zero. Accordingly the normal equations for the longtitude measures are

$$\begin{aligned} n\cdot c+O\;\mu+[l]\quad\pi&=[M] \\ +[t^2]\mu+[t\cdot l]\pi&=[t\cdot M] \\ +[l^2]\quad\pi&=[l\cdot M] \end{aligned}$$

where n is the number of plates or equations. The weight of π derived from this solution is

$$[l^2]-\frac{[l]^2}{n}-\frac{[l\cdot t]^2}{[t^2]}\,;$$

and similarly the weight from the measures in latitude is

$$[b^2]-\frac{[b]^2}{n}-\frac{[b\cdot t]^2}{[t^2]}\,.$$

Now let us suppose that the plates are measured anew, with axes rotated through any angle ϕ; then the two displacements due to parallax will be

$$l\cdot\cos\phi-b\cdot\sin\phi \quad \text{and} \quad l\cdot\sin\phi+b\cdot\cos\phi.$$

The observation equations in one direction are

$$\begin{gathered} c^{\mathrm{I}}+t_1\cdot\mu_1{}^{\mathrm{I}}+(l_1\cdot\cos\phi-b_1\cdot\sin\phi)\pi=M_1{}', \\ c^{\mathrm{I}}+t_2\cdot\mu_2{}^{\mathrm{I}}+(l_2\cdot\cos\phi-b_2\cdot\sin\phi)\pi=M_2{}', \\ \text{etc.,} \end{gathered}$$

with similar expressions for the other co-ordinate. Deriving as before the two weights for the parallax, and adding them together, we obtain, after some simple reductions,

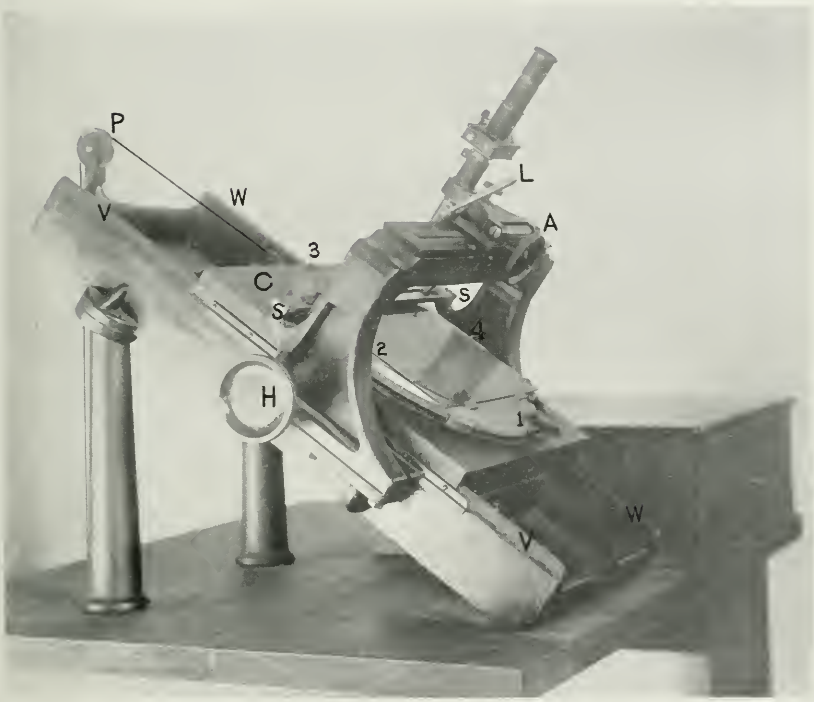
$$[l^2]+[b^2]-\frac{[l]^2+[b]^2}{n}-\frac{[l\cdot t]^2+[b\cdot t]^2}{[t^2]},$$

which is equal to the sum of the weights derived from the measurements in longitude and latitude.[1] There is thus no loss of total weight in measuring the displacements in right ascension and declination. This procedure yields two determinations that are in general more comparable in reliability than the two that would follow from measures in longitude and latitude.

Specifications for a measuring engine to meet the requirements just outlined were drawn up by the writer, and intrusted to Wm. Gaertner & Co., of Chicago, by whom they were carried out in a satisfactory manner. The photograph to be measured is clamped to the carriage *C* (Plate V) in such a way as to bring the film always into the same plane. To put a star under the micrometer microscope the carriage may be moved to and from the observer, and the microscope itself may be racked to the right or left. The former motion is executed by means of the handle *H*, which has its duplicate on the right side of the engine so that either hand may be employed for the purpose. It is necessary that the motion should be accurately in a straight line, and the V-shaped guiding way (*VV*), 67 cm long, was scraped with this in view. On the lower side of the carriage and to the left of the observer are two V-shaped

[1] As this demonstration makes no use of the relation between l or b and the time, it is obviously applicable to other than parallax measurements.

PLATE V



Measuring Machine for Stellar Parallax Plates

lugs, 30 cm apart, that just fit over the guiding way. On the right side of the carriage is a third lug, distant 31 cm from the line joining the two on the left side; the surface of this lug is slightly rounded and it rides upon the plane way *WW*.

The carriage, which is inclined 35° to the horizontal, is accurately counterpoised as shown in the photograph. The point at which the counterweight cord is attached was fixed by experiment so as to make the pull go through the center of gravity of the carriage; and the sheave *P* is so placed that the cord is nearly parallel to *VV*. There is thus little tendency for the carriage to twist as it is moved toward and from the observer.

The microscope objective is half as distant from the plate as from the cross-wires, and the image at its focus is therefore double the size of that upon the plate. The micrometer screw has a pitch of half a millimeter and consequently four turns correspond to one millimeter upon the plate. The micrometer head is graduated into one hundred parts, so that estimated tenths of these spaces are equal to quarter-microns, or 0.″0027 on plates taken with the 40-inch telescope. The scale with which the plates are compared is shown at *SS*. It is divided into millimeters but numbered with every fourth number, thus: 0, 4, 8, 12, etc. This was done in order that we may add the micrometer reading directly to the reading on the scale and obtain the measurements in quarter-millimeters, without the awkward multiplication by 4 that would otherwise be necessary.

In order to compare the plate with the scale, the device employed by Repsold[1] has been adopted. This consists in tilting the microscope through a small angle around a horizontal axis parallel to the scale. The observer sets upon an image and records the micrometer reading; he then depresses the lever *L*, which rotates an eccentric not in view in the photograph, and thus tilts the micrometer around the axis *A*, which is approximately parallel to the scale. The latter is so placed that it now appears, in good focus, in the field of the microscope. The micrometer wire will in general appear between two divisions and the observer makes a bisection and reading upon the one with the lower number, even though it

[1] See Scheiner's *Photographie der Gestirne*, p. 149.

is the more distant of the two; and so for the other images that it is desired to measure.

The micrometer eyepiece has a focal length of about 8 cm; as the objective doubles the size of the stellar image, the net magnifying power is therefore about seven. This is sufficient for plates taken with the 40-inch telescope, even the smallest images (0.07 mm) being large as compared with those obtained with smaller telescopes. In order to eliminate the considerable error depending upon the diameter of the images to be measured, and arising from the tendency to set the wire always too far to the right or always too far to the left, the eyepiece is provided with a reversing prism. Two stops are so placed that the prism may be rotated just 90° and back into the original position. The field is thereby apparently reversed 180°. All bisections, whether upon the stellar images or upon the scale, are made in both positions of the prism, and only the means are used.

The complete operation of measuring an image is as follows: Bring the image into the center of the micrometer field, bisect it, and record the micrometer reading, 0.054. Tilt the microscope, bisect the next lower line, 268, and record the reading, 3.241. Reverse the prism, read again on scale, 3.247. Lower the microscope to view the plate and read again on the image, 0.062. The measured co-ordinate is therefore 271.186 quarter-millimeters. This unit has been retained until the very last step in the computations, when the parallax, its probable error, etc., are transformed into arc by applying the factor 2.66, the number of seconds corresponding to a quarter-millimeter on plates taken with this telescope.

Cases may arise in parallax work where it would be necessary to rotate the plate exactly 90° and to measure accurately the corresponding co-ordinates of the comparison stars, even though the parallax is to be deduced from the displacements in only one of these directions. Suppose, for example, that only two comparison stars are available, or that all the comparison stars are nearly in the same straight line; then it is impossible or impracticable to deduce the plate constants (orientation and scale-value) from measures of only one co-ordinate; but they may be computed, after the application of refraction corrections, if both co-ordinates have

been measured. Means for rotating the plate 90° were provided in the construction of the measuring engine, and, although none of the stars in the present list requires measurements of this kind, it will be well to complete the description of the engine in this respect.

The usual method for effecting this rotation is by the use of an accurately graduated circle read by two micrometer microscopes 180° apart. The same circle permits also of the measurement of position-angles, but these were not contemplated with this engine. As the circle complicates the construction and adds considerably to the expense, the following plan was adopted instead:[1] four posts, shown at 1, 2, 3, and 4 in the photograph, are placed at intervals of nearly 90° and with their upper surfaces in the plane occupied by the sensitive film, so that they are in the focus of the micrometer microscope. Four delicate dots, small and round, are punched into these surfaces, and form the vertices of a quadrilateral that is nearly square. The first of these dots is adjustable, so that the line joining it and (3) can be made, once for all, nearly perpendicular to the line joining (2) and (4). The exact angle between these diagonals is ascertained by a method to be given later. The circular casting upon which these posts are mounted may be rotated by means of a tangent-screw with respect to the carriage *C*. Within it a second circular casting is placed, also provided with a tangent-screw, and it is to this inner circle that the photographic plate is clamped.

This device is used as follows: by means of the outer tangent-screw, the line joining (1) and (3) is first made approximately parallel to the guiding way *VV*. Then by rotating the inner circle with its tangent-screw, the plate is oriented in the direction in which *X* co-ordinates are to be measured. The amount by which the line (1) to (3) fails of parallelism with the guiding way is now measured by setting upon (1), moving the carriage along the way until (3) comes into view, and setting upon it, without having disturbed the microscope between the two settings. The difference between these two readings is recorded, and we proceed to measure

[1] I learn that a coarse graduated circle of diameter 30 cm, reading to 0°.05 by a vernier, has recently been added to the machine, particularly for measuring position-angles of double stars and proper-motion stars.

X co-ordinates. The *outer circle* is then unclamped and rotated until the line joining (2) and (4) is nearly parallel to VV. Readings are taken as before upon these dots: if the difference is equal to that for (1) and (3), and if the lines joining the opposite dots are accurately perpendicular, it is evident that we have turned the photograph exactly 90°; it is equally evident that, in any case, we have all the data necessary to say how much the angle differs from 90°, and to apply to the measures the small corrections that are appropriate.

As compared with the usual method, in which a graduated circle is employed, this device has the disadvantage of not permitting the measurement of position-angles. It has the advantages of simplicity, cheapness, and, above all, greater accuracy. For with a graduated circle we have the additional uncertainty due to division errors, which it would be very laborious to determine with sufficient accuracy; whereas with the four posts, we have in effect to determine only one such error, and this we may do at frequent intervals, as the necessary measurements require little time.

We now consider in order the determination and elimination of the various errors to which measures with this engine are liable.

Deviations of the guiding way from perfect straightness.—A long hair was stretched across a brass frame, and the latter clamped in the measuring engine in the position ordinarily occupied by the plate. It was then rotated to make it nearly parallel to the guiding way, and readings upon its center were made at intervals of about 10 mm throughout its entire length. The frame was then rotated 180° around the axis of the hair, and bisections were again made at exactly the same points as before. The means of each pair of these measurements are free from any sinuosities or irregularities in the hair, and would all be equal to each other if the hair were parallel to the guiding way and if the latter were exactly straight. The application of appropriate linear terms therefore gives the deviations of the way.

This procedure was varied by graving a fine line in the film of a photographic plate and measuring it in a similar way, the plate being turned around the line and the reverse measurements being made through the glass. The two determinations gave very

accordant results. These experiments, made in December 1903, were repeated after an interval of fifteen months in March 1905, to ascertain whether the way had suffered appreciable wear. As may be seen from the table the two sets of corrections are practically

TABLE I

CORRECTIONS IN QUARTER-MILLIMETERS FOR DEVIATIONS FROM STRAIGHTNESS OF THE GUIDING WAY

Scale	Corrections December 1903	Corrections March 1905	Adopted Means
mm			
2	——	+0.032	+0.030
10	+0.023	+ 23	+ 23
20	+ 17	+ 18	+ 18
30	+ 11	+ 11	+ 11
40	+ 7	+ 6	+ 7
50	+ 2	+ 2	0
60	− 1	− 4	0
70	− 3	− 6	0
80	− 3	− 5	0
90	− 3	——	0
100	− 2	− 5	0
110	− 2	− 6	0
120	− 1	− 5	0
130	0	0	0
140	+ 2	0	0
150	+ 4	+ 1	0
160	+ 4	0	0
170	+ 4	+ 2	0
180	+ ·4	+ 5	0
190	+ 4	+ 7	0
200	+ 2	+ 6	0
210	+ 1	+ 2	0
220	− 1	+ 6	0
230	− 2	+ 4	0
236	− 4	+ 1	0

the same. Those determined at the earlier date were adopted as definitive and applied, with the signs shown, to all the measures. The millimeter scale referred to is mounted near the right-hand edge of the carriage *C*, and is parallel to the guiding way. The corrections as actually determined are small for scale-readings 50 to 236 and they were assumed to be zero; the errors that this assumption entails in the position deduced for the parallax star from one plate do not reach 0."01 for any region in the present list. As a matter of

fact no appreciable error would have been incurred had we neglected the deviations of the guiding way altogether, as each plate is put into the machine in a standard position, and nearly the same portion of the way always comes opposite any one star in a region. So long therefore as the deviations from straightness are not erratic, they are automatically eliminated from the relative positions of the parallax star, when we compare one plate with another. The same remark applies with equal force to the other instrumental corrections, except the "runs" of the micrometer screw, which may vary from day to day.

Division errors of the scale.—Star images can best be bisected with a single micrometer wire, while fine graduations upon a scale are better measured with two parallel wires close together. To retain the advantages of both, the graduations were made double, with components separated by about 0.12 mm. The micrometer wire used was a coarse one (as it had to be with an eyepiece of so low a power) and it nearly filled the space between the two components of each line, leaving two narrow strips of the bright German silver on either side; these two strips could be made very exactly equal and enabled the settings to be made with great accuracy.

The calibration of the scale, that is, the determination of the errors of its 502 lines, was very kindly undertaken for us at the Bureau of Standards through the courtesy of Director Stratton, who thus describes the method employed:

First, every tenth of the total length was determined by Hansen's method (*Travaux et Mémoires du Bureau International des Poids et Mesures*, Vol. 5), thus giving every 25 mm space, or according to the numbering, the points 100, 200, 300, etc. As a check, the bar was also calibrated into 200 mm spaces by Hansen's method and the values found for the 200 points were combined with the values found for these points by the previous method. Each 25 mm space, or, according to the numbering, each 100 space, was again calibrated by Hansen's method into 5 mm spaces. The separate millimeters of each 5 mm space were then measured with the micrometer screw. All the work referred to above applies to the main or unprimed lines. The positions of the primed lines were next determined by measuring with the micrometer the distance from the main to the primed lines. The micrometer screw used in this work has a value of 43.95 μ per revolution. The total length of the scale was determined by comparison with a calibrated Geneva Society scale whose total length is known in terms of the international meter. The probable errors are

only given for every 5 mm space, but they may be readily computed for other lines by combining the probable error $\pm 0.3\,\mu$[1] (the average probable error of the intermediate spaces) with the probable error of the 5 mm lines nearest to them. This refers to the main or unprimed lines. The probable error of the spaces between each pair of lines is $\pm 0.2\,\mu$ and must be considered in computing the probable errors of any space defined by a primed line. For example the probable error of the space 0 to 72′ is

$$\sqrt{(0.2)^2+(0.3)^2+(0.2)^2} = \pm 0.4\,\mu.$$

In some cases the probable errors will exceed the limit of $0.5\,\mu$, but the lines of the scale are not sufficiently good to make it possible to keep the probable errors below in every case.

The following is a specimen of the form in which the Bureau communicated the errors thus determined:

Graduation	Measured Distance from Zero mm	mm
0	0	
0 to 0′	0+	0.1102
0 to 4	1−	.0035
0 to 4′	1+	.1097
0 to 8	2−	.0051
0 to 8′	2+	.1088
0 to 12	3−	.0081
0 to 12′	3+	.1078
etc.		etc.

These are the absolute errors of each graduation; but in work like the present only the relative errors are important, and we are therefore at liberty to apply any linear terms that we please. We have accordingly taken the means of the errors for the two components of each line, and then computed

$$4(mean - 0.00007\,n - 0.029),$$

n being the number with which the line is graduated. The factor 4 serves to reduce the corrections to quarter-millimeters. These are the errors or corrections[2] that appear in Table II and they have been applied to all the measures. It will be noticed that they are all positive up to line 616 and all negative thereafter. This circumstance (with which in view these particular linear terms were

[1] This symbol denotes the *micron*, or 0.001 mm.

[2] It is ordinarily the case that errors and corrections have opposite signs; but division errors of a scale are an exception to this rule.

added to the Bureau's corrections) makes their application a little more convenient and lessens the chance of error.

TABLE II

DIVISION ERRORS OF THE SCALE, IN QUARTER-MILLIMETERS

Line	Correction	Line	Correction	Line	Correction	Line	Correction	Line	Correction	Line	Correction
0	+.104	168	+.010	336	+.020	504	+.023	672	−.022	840	−.074
4	+.095	172	+ 8	340	+ 16	508	+ 22	676	− 26	844	− 80
8	+ 89	176	+ 10	344	+ 18	512	+ 26	680	− 26	848	− 83
12	+ 80	180	+ 10	348	+ 25	516	+ 20	684	− 24	852	− 84
16	+ 72	184	+ 12	352	+ 25	520	+ 20	688	− 30	856	− 85
20	+ 69	188	+ 10	356	+ 24	524	+ 23	692	− 30	860	− 82
24	+ 62	192	+ 9	360	+ 22	528	+ 21	696	− 29	864	− 85
28	+ 62	196	+ 10	364	+ 20	532	+ 18	700	− 29	868	− 87
32	+ 58	200	+ 6	368	+ 25	536	+ 18	704	− 32	872	− 85
36	+ 56	204	+ 6	372	+ 27	540	+ 16	708	− 36	876	− 89
40	+ 49	208	+ 4	376	+ 25	544	+ 10	712	− 36	880	− 90
44	+ 46	212	+ 6	380	+ 27	548	+ 5	716	− 38	884	− 96
48	+ 40	216	+ 6	384	+ 20	552	+ 11	720	− 41	888	− 97
52	+ 35	220	+ 8	388	+ 22	556	+ 10	724	− 42	892	− 97
56	+ 36	224	+ 10	392	+ 23	560	+ 6	728	− 44	896	−.104
60	+ 33	228	+ 9	396	+ 28	564	+ 5	732	− 46	900	−.104
64	+ 26	232	+ 10	400	+ 24	568	+ 10	736	− 47	904	−.103
68	+ 19	236	+ 10	404	+ 25	572	+ 15	740	− 44	908	−.104
72	+ 14	240	+ 6	408	+ 23	576	+ 10	744	− 49	912	−.102
76	+ 14	244	+ 3	412	+ 20	580	+ 10	748	− 46	916	−.107
80	+ 15	248	+ 3	416	+ 20	584	+ 11	752	− 50	920	−.108
84	+ 18	252	+ 6	420	+ 20	588	+ 12	756	− 52	924	−.112
88	+ 16	256	+ 5	424	+ 20	592	+ 12	760	− 54	928	−.112
92	+ 16	260	+ 6	428	+ 19	596	+ 8	764	− 50	932	−.114
96	+ 20	264	+ 10	432	+ 23	600	+ 7	768	− 53	936	−.118
100	+ 18	268	+ 5	436	+ 23	604	+ 6	772	− 58	940	−.127
104	+ 20	272	+ 4	440	+ 19	608	+ 4	776	− 56	944	−.131
108	+ 16	276	+ 4	444	+ 19	612	+ 2	780	− 60	948	−.129
112	+ 14	280	0	448	+ 14	616	+ 1	784	− 64	952	−.130
116	+ 14	284	+ 5	452	+ 16	620	− 1	788	− 59	956	−.130
120	+ 16	288	+ 2	456	+ 20	624	− 6	792	− 62	960	−.129
124	+ 14	292	+ 6	460	+ 23	628	− 5	796	− 63	964	−.133
128	+ 16	296	+ 5	464	+ 18	632	− 8	800	− 64	968	−.138
132	+ 17	300	+ 10	468	+ 18	636	− 8	804	− 66	972	−.142
136	+ 16	304	+ 4	472	+ 22	640	− 9	808	− 69	976	−.134
140	+ 14	308	+ 6	476	+ 20	644	− 12	812	− 65	980	−.130
144	+ 9	312	+ 5	480	+ 18	648	− 12	816	− 66	984	−.133
148	+ 11	316	+ 6	484	+ 21	652	− 13	820	− 65	988	−.131
152	+ 14	320	+ 7	488	+ 17	656	− 19	824	− 68	992	−.122
156	+ 10	324	+ 12	492	+ 18	660	− 21	828	− 66	996	−.121
160	+ 14	328	+ 14	496	+ 20	664	− 23	832	− 71	1000	−.124
164	+ 11	332	+ 13	500	+ 23	668	− 23	836	− 73		

The division errors are large, and in this respect the scale is a poor one. But the errors have the merit of running very smoothly; thus in only four cases does the error on any line differ by more than one micron from the mean of the two adjacent lines, the greatest

difference being one and a half microns. Probably even these small differences are for the most part apparent only, since the probable error of one such deviation is nearly four-tenths of a micron.

Correction for runs.—The amount by which four turns of the micrometer screw failed of exact equality with one space on the scale was determined once or twice each day, and the corresponding corrections applied to all the measures of that day. In order that the runs should be the same for all parts of the scale, it is necessary that the latter should be equally distant from the micrometer objective in all its positions. A change in this distance of one-twentieth of a millimeter is appreciable in the runs, but cannot be detected (in an optical system like the present one) by the lack of exact focusing or the presence of "parallax." At the outset of the work,[1] the runs were accordingly determined at several places along the scale. Pieces of very thin paper were inserted between the scale and the casting to which it is screwed, until the runs came out the same for all its parts. Due allowance should have been made in this connection for division errors, but the latter were not forthcoming at the time this adjustment was made. Fortunately the error from this source proves to be negligible, thanks to the smoothness with which the division errors run.

Collimation of the micrometer microscope.—For present purposes the line of collimation of the microscope is defined as joining the optical center of the objective with the zero of the micrometer screw. Bisections upon star images were always made with the micrometer set near zero. Ordinary glass was employed in the photographic plates, and consequently the film does not lie in one plane. For this reason it is necessary that the line of collimation should be closely perpendicular to the general plane of the plate, in at least the right-and-left direction. Otherwise the stellar image will be projected upon the scale in different places, depending upon whether the image is above or below the general plane of the plate. Suppose, for example, that the microscope leans toward the right by the

[1] Of the plates here discussed, eighteen were measured before the necessity for this precaution occurred to me. For these, a small additional correction for runs was applied, depending upon what part of the scale had been used.

angle a; then the measure upon a point that is too high by a distance h will be too great by $h \cdot \tan a$, the zero of the scale being toward the left. As h may easily be a quarter of a millimeter, it follows that a must be made less than $0^\circ.3$, if we wish to avoid errors as great as one micron, which corresponds to $0''.01$. No means were provided in the design of the measuring engine for actually tilting the microscope, nor would it have been advisable to do so. But the position of the micrometer head may be changed with reference to the screw itself, so that the zero point can be brought to that part of the micrometer screw that gives a line of collimation in the desired position.

To effect this adjustment we measured the distance (M_1) between two points known to be at somewhat different heights, h_1 and h_2. The plate was then reversed by turning it 180° in its own plane, and again we measured the distance (M_2). If we call a_1 the angle between the line of collimation and a plane perpendicular to the plate and the scale we have

$$s+(h_2-h_1)\tan a_1=M_1,$$
$$s-(h_2-h_1)\tan a_1=M_2,$$

where s is the true distance between the two points. The zero of the micrometer was now changed by three turns of the screw and the distance measured a third and a fourth time:

$$s+(h_2-h_1)\tan a_2=M_3,$$
$$s-(h_2-h_1)\tan a_2=M_4.$$

We now have all the data necessary to determine a_1 and a_2 and therefore to state where the zero of the micrometer should be taken.

Periodic and progressive errors of the errors of the micrometer screw.—An auxiliary micrometer was mounted upon the carriage with its wire in the plane ordinarily occupied by the photographic film and with the screw parallel to that of our own micrometer. The method of Gill and Lorentzen (described by Jacoby in the *American Journal of Science*, [4] **1**, 333, May 1896) was employed, yielding the errors of the two screws. Both the periodic and the progressive errors of our screw proved to be entirely negligible.

Determination of the angle between the posts.—In the quadrilateral formed by the dots we may readily measure the four sides,

a_1, a_2, a_3 and a_4; a_1 being between (1) and (2), a_2 between (2) and (3), etc. Theoretically we need another dimension (for example, the angle at one of the vertices or the length of a diagonal) in order to compute the angle between the diagonals. But in practice it comes out that we do not have to know any dimension beyond those of the sides. For, if $90°+\theta$ is the angle between the diagonals on the side toward (1) and (4), and $90°+\delta_1$, $90°+\delta_2$, etc., the angles at the vertices, then we have

$$\tfrac{1}{2}\tan\theta=\frac{-a_1+a_2-a_3+a_4}{a_1+a_2+a_3+a_4},$$

in which we have neglected only second order and higher terms in δ_1, δ_2, etc., and similar terms in the differences between the sides. So long therefore as the quadrilateral does not differ too much from a square, we may compute θ from this expression with sufficient accuracy.

Direct measurement of the two diagonals enabled us to compute within a minute of arc the angles δ_1, δ_2, etc., and thus to prove that the sides of the quadrilateral are all parallel or perpendicular to each other within 6′. With practically no error we have then the following co-ordinates of the four dots referred to (1) as origin and (a_4) as the axis of X:

$$(1)\begin{cases}0\\0\end{cases}\quad(2)\begin{cases}-a_1\cdot\sin\delta_1\\a_1\end{cases}\quad(3)\begin{cases}a_2-a_1\cdot\sin\delta_1\\a_3\end{cases}\quad(4)\begin{cases}a_4\\0\end{cases}$$

Consequently

$$\tan(314)=\frac{a_3}{a_2-a_1\cdot\sin\delta_1},\quad\text{and }\tan(241)=\frac{a_1}{a_4+a_1\cdot\sin\delta_1}$$

$$\tan\theta=\cot\{(314)+(241)\}=$$
$$\frac{a_2\cdot a_4-a_1\cdot a_3+a_1\cdot a_2\cdot\sin\delta_1-a_1\cdot a_4\cdot\sin\delta_1-a_1^2\cdot\sin^2\delta_1}{a_1\cdot a_2+a_3\cdot a_4-a_1^2\cdot\sin\delta_1+a_1\cdot a_3\cdot\sin\delta_1}.$$

Let us replace a_1 by $M+\gamma_1$, a_2 by $M+\gamma_2$, etc., M being the mean of the four sides. In the present case none of the quantities γ exceeds 0.0007 M and we may therefore neglect terms of the order of γ^2/M^2. Similarly terms in δ_1^2 may be neglected, δ_1 being less than 3′. Consequently

$$\tan\theta=\frac{-\gamma_1+\gamma_2-\gamma_3+\gamma_4}{2M},\quad\text{or }\tfrac{1}{2}\tan\theta=\frac{-a_1+a_2-a_3+a_4}{a_1+a_2+a_3+a_4}.$$

This expression yields a value of θ that is correct within less than $1''$. A rotation of this amount corresponds to only $0''.01$ in an arc of $35'$ upon the plates.

An investigation of the errors of the microscope way, that is, the one upon which the microscope travels right and left, is hardly necessary in the present connection. Its deviations from straightness would have to be large in order to change the orientation of the axis A enough to make an appreciable difference in the projected position of an image, the distance between the image and the scale being small. Furthermore, any error of this kind would be eliminated by the insertion in the engine in a standard position of all the plates of one region.

The orientation of the first plate of a region was effected through the aid of catalogue-positions of the parallax star and one or two of the comparison stars; or, more commonly, by means of a fourth image, or *trail*, of the parallax star. This orienting image was secured by unclamping the driving clock, and then, after the lapse of perhaps a minute, allowing it to drive long enough to secure a measurable image of the parallax star. Corrections for curvature[1] of path were applied whenever necessary, but no time was wasted in attempting to orient the first plate of a region (closer than a few minutes of arc) to the exact direction of the equator or the ecliptic. It is obvious that unless the parallax were very large, no material error would be incurred if all the plates were measured as much as half a degree from the direction for which the parallax factors are computed. It is, however, of some importance that the second and succeeding plates should be oriented just like the first. To do this we rotated the plate until the difference between the co-ordinates of any two comparison stars came out substantially the same as in the first instance. The plate was then moved to the right or the left so as to bring the middle image of the parallax star opposite a predetermined division on the scale. We might have adjusted the plates so as to bring the *comparison* stars always opposite standard places on the scale, which is not quite the same thing, owing to the large proper motions of some of the parallax stars here included. But the alternative actually adopted is a little preferable, since

[1] *Contributions from the Observatory of Columbia University*, No. 16, p. 62.

outstanding uncertainties in the division-errors, in the screw-errors, etc., are thus more likely to be accidental instead of systematic in their effect upon the relative position of the parallax star.

The images of the parallax star were measured twice and those of the comparison stars, of which there were ordinarily about five, usually only once. But in a few cases where, owing to its position, one of the comparison stars assumed unusual importance,[1] its images were likewise measured twice.

Up to March 1905, nearly every plate was measured both by Miss Ware and myself, but preliminary reductions showed that this was unnecessary and uneconomical. In other words we found (and this seems to be the case with all photographic work) that the mere errors of bisection are small as compared with those that are inherent in the plate itself, or possibly, in the case of photographs taken with smaller telescopes, those that have their origin in the measuring engine. After March 1905, all the measurements were accordingly made by Miss Ware alone. Another reason for coming to this decision was that, owing to my removing from Williams Bay at this date, it would have been very inconvenient, though not prohibitively so, for me to have continued with the work of measuring.

While no systematic difference between the two observers came to light, care was taken to combine the two sets of measures in such a way as to eliminate, in every case, the effect of a possible difference of this sort upon the deduced parallax.

This portion of our subject may properly be terminated with a brief reference to other methods for securing and measuring the plates, with a statement of the reasons that have led me to prefer those just described.

The following procedure was suggested many years ago by Kapteyn and has since been put into practice by him and others: A plate is exposed to the desired region and is then stored without development for half a year. A second exposure is now secured one millimeter to the right of the first and the plate is stored for an additional half-year; then a third and final impression one millimeter to the right of the second is obtained, and the plate is

[1] An exact criterion for the importance of a comparison star will be given later.

developed. From the relative positions of the three images of each star we may deduce parallaxes and proper motions. Assuming that precautions have been taken against hour-angle error and optical distortion, the only known sources of error that remain are those due to uncertainties in the bisections and the guiding error.

After giving the matter some thought, I decided that the advantages of this method, great though they may be in some cases, are outweighed by its disadvantages under the conditions that actually confronted me. The first of these is the additional time consumed at the telescope. A particular plate must be selected without chance of mistake, from perhaps one hundred that are stored in a dark room; the plate must be inserted in its holder, and the holder in the carrier at the telescope, in standard positions. Compare this with simply taking any plate from a box of fresh ones and inserting it in the holder in whatever position it chances to assume. With Kapteyn's method the holder, the carrier, and the plate itself must be such that the second and third exposures shall be oriented very closely the same as the first. Otherwise the relative distances between the three images will change from star to star and uncertainties in the screw-errors, etc., may affect the deduced parallaxes. The relative positions of the three images should be the same within 0.1 mm, or even 0.05 mm in some cases. While there is no real difficulty in doing this, it must always involve a loss of valuable time.

With a telescope of so great a focal length as the 40-inch there is considerable liability of failure for some of the exposures. When the seeing is poor, and to a less extent when the atmosphere is only imperfectly transparent, it is a matter of difficulty to gauge the length of exposures required. It was my practice, whenever circumstances called for it, to develop the first two or three plates, and to expose the following plates accordingly. This cannot be done with Kapteyn's method unless the observer makes exposures for this exclusive purpose.

Even if the exposure is not a failure outright, but is, let us say, entitled to half weight, it can be shown that Kapteyn's method always involves a loss of weight for the deduced parallaxes. For illustration, suppose that we have two Kapteyn plates upon each of which two exposures have been impressed at the middle epoch.

On the first plate suppose that these *middle* exposures have half weight, and that on the second plate the exposures at the *first* and the *third* epochs have half weight. Then if the parallax factors are all ± 1 and if the intervals are just half a year, each plate will yield 2.67 as the weight for the parallax. If now the plates are measured in the ordinary way and combined into one solution, the parallax comes out with a weight of 6.00, which is considerably greater than the sum of the Kapteyn weights. This example is by no means an extreme case; we shall see later that some of the plates here discussed are entitled to several times the weight that can properly be assigned to others.

With Kapteyn's method the observer must incur one of two other disadvantages. If he elects to measure in right-ascension there is a further loss of weight for the deduced parallax as compared with measuring in longitude. If, on the other hand, he decides to measure in the latter direction he must set his plate-carrier at the telescope accordingly; for, since the parallax is deduced from measurements of the *distances* between the images, it is necessary that the plate should be shifted, from one exposure to the next, in the direction for which the parallax factors are to be computed. In either case there is not the same freedom at the telescope as with the ordinary method, and the observer cannot revolve his plate-carrier into any position that may aid him to reach a suitable guiding star or to secure a better distribution of comparison stars. This advantage might be retained, however, by providing a third set of slides in the construction of the plate-carrier.

In point of accuracy the chief difference between the two methods is that one of them eliminates distortions of the film and therefore yields a somewhat smaller probable error for a plate. It is therefore necessary to know the average size of these distortions in order to make a choice between these methods in any particular case. When this work was begun, astronomers were in general of the opinion (that has since proved to be abundantly justified) that the distortions are very small. But shortly afterward some doubts upon this matter were expressed, and I decided to investigate it for the same plates and other conditions that apply to the

work itself. In this investigation, begun at the Yerkes Observatory, and completed at Allegheny,[1] an effort was made to separate this error from all others. The outcome was to show that the mean value of the distortions (in a sense analogous to mean error) is of the order of one micron. This corresponds to a probable error of only 0."007 on these plates. The mean of the three exposures on a plate is probably affected by an even smaller quantity, as the images of a star extend over ten millimeters, and it is not likely that the distortion persists, both as to amount and direction, over such distances. But waiving this point, this source of error can in any case form only a small fraction of the total plate-error. In the most favorable case, a plate containing three good exposures that have been measured by two observers has a probable error of 0."020, which would be reduced by at most 0."0013 if the distortion of the film had been entirely eliminated. The reduction is of course less if the plates are not so good, that is, if their probable error is greater.

By storing the plates in a dark room for long intervals of time, the observer keeps himself in the dark as well. He does not know how his work at the telescope is coming out and he is not free to modify in any way the conditions with which he set out, as for example the length of exposure, the choice of a guiding star, etc. Nor can he begin to measure his plates until a year has elapsed—an important consideration in the present case, where the time available for the whole research was limited.

The use of a *réseau*, like those employed in connection with the *Astrographic Catalogue*, was also carefully considered, but it was rejected for reasons that are very much the same as some of those just explained. The *réseau* would hamper the work at this telescope, though not to the same extent as with Kapteyn's method. It is doubtful even now whether plates measured with or without the *réseau* are the more accurate, and its use would not simplify the work of measuring where only a few images are to be located on each plate.

The reader will understand that I have criticized these methods from the single point of view that was forced upon me by the circumstances with which I had to deal. Some of the objections to

[1] *Publications of the Allegheny Observatory*, 1, 1, 1907.

which I have called attention are altogether inapplicable to other cases. For example, with smaller telescopes the guiding is usually not done with a double-slide plate-carrier but by following the field with an auxiliary telescope and moving the two tubes as a unit. Again, distortions of the film and instrumental errors are small in the present work, the scale of the plates being so large, and the guiding error is the largest item of inaccuracy. But with shorter focal lengths these sources of error are absolutely and relatively more important, and may in some cases equal the guiding error or at least become more nearly comparable with it. The observer must decide from the circumstances in each case how to expend the time and energy at his disposal, so as to secure the most accurate results for the work in hand. Just where the various methods cease to be advantageous I do not presume to say, but I may venture the opinion that, on the one hand, the dividing line will be found close to work of the character to which Kapteyn has applied his method; that is, the derivation of the mean parallaxes of a large number of stars within a restricted area. On the other hand, it seems to me that the usefulness of the *réseau* is nearly limited to work like that of the *Astrographic Catalogue*, where the object is to measure as rapidly as possible both co-ordinates of numerous star images.

Allegheny Observatory
January 1911

THE PRODUCTION OF LIGHT BY CANAL RAYS

By GORDON SCOTT FULCHER

The two phenomena of greatest significance in connection with canal rays are well known to be the simultaneous electrostatic and magnetic deflection of the rays, first obtained by W. Wien, and the Doppler effect, which J. Stark discovered to be shown by light from the path of the canal rays. The Wien experiment enables us to determine the velocity and mean value of $\frac{e}{m}$ of the constituent rays; the Stark effect gives us directly the speed in the line of sight of the sources of the light showing the effect. Two years ago, from a summary and comparison of the experimental results reported by W. Wien, J. J. Thomson, J. Stark, F. Paschen, and others,[1] it seemed evident that the distribution of velocities among the canal rays is quite different from that among the sources of the light showing the Stark effect. It appeared to be necessary to conclude that the canal rays cannot themselves be the chief sources of the light in question, as has been generally assumed.[2]

THE HYPOTHESIS

An alternative hypothesis was suggested. The only other molecules present with velocities great enough for them to be the sources sought are those gas molecules which have been hit by the canal rays and thus have acquired a velocity great or small according to the squareness of the collision. If we suppose this molecular phenomenon to obey the laws of ordinary impact between solid bodies, the momentum given the gas molecule will vary up to a maximum value of the same order as that of the bombarding ray, depending on the relative masses of the two molecules and on the coefficient of restitution. Will the Doppler effect to be expected if these hit molecules emit light agree in all essential

[1] G. S. Fulcher, "Our Present Knowledge of Canal Rays," *Smithsonian Misc. Collections*, **52**, 295–324, 1909.

[2] *Ibid.*, p. 322.

details with the Stark effect as observed? A statistical computation, explained in detail below (p. 41), showed that a satisfactory agreement was obtained if the following assumptions were made:

1. That the intensity of the light emitted as a result of each impact is proportional to the energy transmitted to the hit molecule, but

2. That the hit molecule emits no light unless the energy so transmitted exceeds a certain minimum supposed to be equal to that necessary to produce ionization; and

3. That the hitting molecules emit no light of the kind showing the Stark effect as a result of the collision. Of these, the first has since been experimentally verified (see below), the second is rendered probable by other experimental evidence, and the third, rather more difficult to accept, will be shown later to be a necessary assumption which perhaps can be made more plausible by the consideration that most of the hitting charged rays may be neutralized by the collisions which ionize the hit molecules. Hence if the hitting molecules thus neutralized emit any light, it is not the series-line spectrum which is emitted by the ionized molecules and which alone shows the Stark effect.

It is possible then to reconcile the results of the deflection experiments with the Stark effect if we assume that the sources of the light showing this effect are the gas molecules hit and ionized by the canal rays rather than the canal rays themselves.[1]

EXPERIMENTAL EVIDENCE

If this hypothesis is correct the intensity of the light from a canal-ray beam should vary directly as the number of collisions per unit time, per unit length of path, that is, with the pressure of the gas, providing the number and velocity of the canal rays are kept constant.

The following apparatus, shown diagrammatically in Fig. 1, has been designed to test this deduction. It is so arranged that the pressure of the gas in the canal-ray chamber back of the cathode can be varied in the ratio of one to twenty without changing the pressure in the discharge chamber, which determines the

[1] *Ibid.*

cathode-fall of potential, that is, the number and velocity of the canal rays. In one experiment hydrogen gas from the reservoir where the pressure is maintained at a certain constant value of about 5 cm of mercury, passes slowly and continuously through

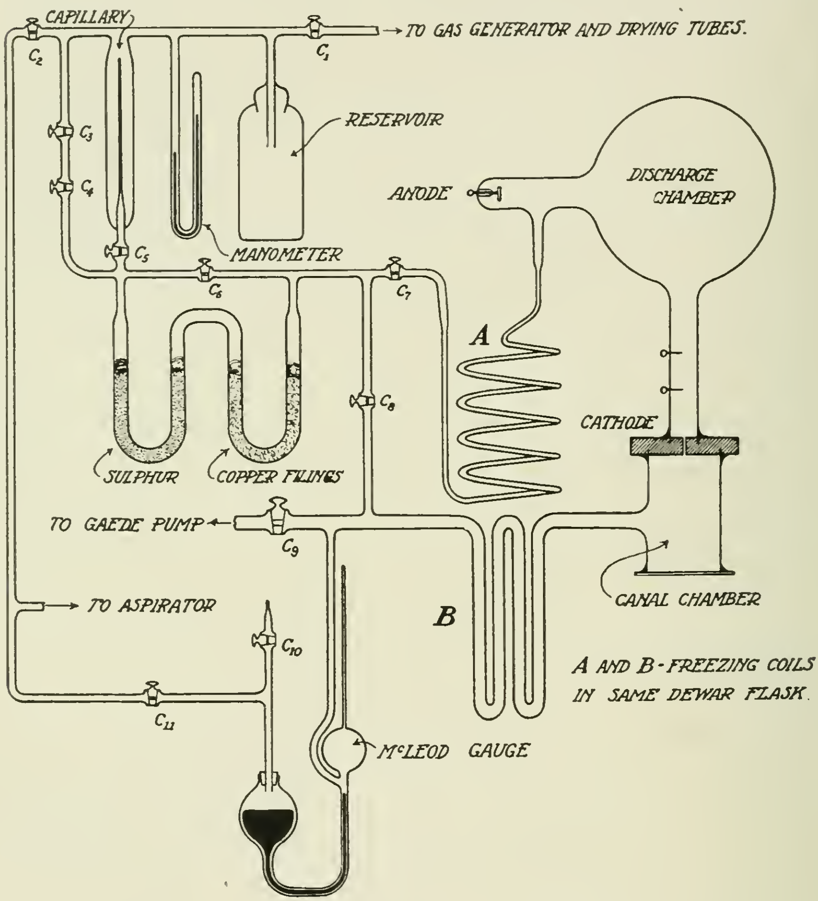


Fig. 1

a capillary tube (10 cm long and with a bore about 0.009 cm in diameter) and some purifying tubes to the discharge chamber. The pressure here is about 0.1 mm of mercury. From this chamber a single hole in the aluminum cathode (0.05 cm in diameter and 1 cm long) leads to the canal chamber, which is

maintained at a pressure of about .005 mm of mercury by a Gaede mercury pump to which it is connected by large tubes. The hole in the cathode thus behaves like a porous plug of a single pore. After equilibrium is established, these pressure relations can be maintained indefinitely, and the intensity of the light emitted when the canal rays pass through gas at the low pressure maintained in the canal chamber can be measured at leisure.[1]

On the other hand the pump may be stopped and the pressure allowed to become the same on both sides of the cathode and equal to the pressure formerly existing only on the discharge-chamber

FIG. 2.—Photographs of the canal-ray beam
a, Beam through gas at low pressure (0.005 mm)
b, Beam through gas at higher pressure (0.1 mm)
c, Extra-focal image of beam, higher pressure

side. Conditions in the discharge chamber will now be the same as in the first experiment; hence the current and the cathode-fall of potential, and therefore the number and velocity of the canal rays, will be the same in both cases, though the pressure in the canal chamber is quite different. Hence the intensity of the light from the canal-ray beam should vary in the same ratio as the pressure of the gas in the canal chamber, if our hypothesis is correct.

To measure the intensity of the light, the beam was photographed with an ordinary camera. The time of exposure was adjusted so that in all cases images of about the same density were obtained. The camera was thrown slightly out of focus so as to secure broader images whose density could be more easily compared

[1] To avoid confusion, the discussion of certain details of the apparatus is postponed to the end of the paper.

by the use of a spectrophotometer (see Fig. 2). The intensity of the light was assumed proportional to the density of the image divided by the time of exposure. The pressure was measured with a McLeod gauge connected to the canal chamber. The results for two cathode-falls of potential are given in the following table.

TABLE I

Cathode-Fall Volts		Pressure mm of Hg	Time of Exposure t	Density of Image D	Intensity of Light $I=\frac{D}{t}$	$\frac{I_1}{I_2}$	$\frac{p_1}{p_2}$	$\frac{\frac{I_1}{p_1}}{\frac{I_2}{p_2}}$
3900	1	0.102	60 sec.	0.41	0.0068	17.5	19.2	0.91
	2	0.0053	1200 "	0.47	0.00039			
4400	1	0.096	50 sec.	0.44	0.0088	22.6	21.9	1.03
	2	0.0044	1200 "	0.47	0.00039			

If the hypothesis is correct $\frac{I_1}{p_1}$ should be approximately equal to $\frac{I_2}{p_2}$. The agreement, though unsatisfactory, is within experimental errors.

It is here assumed that light of a certain intensity acting for twenty minutes will produce the same density of image as light of twenty times the intensity acting for one minute. To test this, half a plate was exposed to light from a point source at a distance of 125 cm for 45 seconds and the other half was exposed for 720 seconds at a distance of 500 cm from the same source. Here the ratio of intensities is 16 and equal to the inverse ratio of the times of exposure. No difference can be observed between the densities of the two halves of the developed plate—proving that for the intensities used the assumption is sufficiently accurate.

I gladly acknowledge my indebtedness to Professor C. E. Mendenhall for help in designing the apparatus. The use of a capillary in combination with a Gaede pump to secure equilibrium conditions and thus make the experiment quantitative was suggested by him.

The conclusion from this experiment is, therefore, that the intensity of the light from the canal-ray beam is proportional to

the pressure of the gas through which the canal rays pass, that is, to the probable number of collisions per unit length of path, per unit time, between the canal rays and gas molecules, provided that the number and velocity of the canal rays is unchanged.[1]

Next the question arises, how does the intensity of the light emitted vary with the velocity of the rays? Does it vary directly as the total energy-flux of the rays times the gas pressure, that is,

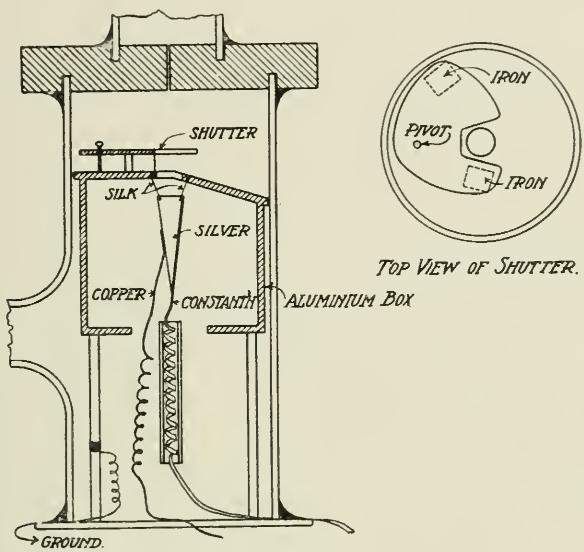


FIG. 3.—Apparatus for measuring the energy-flux of the canal rays

as the mean energy of each ray times the number of collisions per unit length of path per second? To answer this it became necessary to measure the energy-flux of the rays as a function of the cathode-fall of potential. To this end, they were allowed to strike inside a light silver cone so that their kinetic energy might be transformed to heat energy and measured from the temperature changes produced.

In the canal chamber the apparatus shown in Fig. 3 was placed. The silver cone weighing only a fifth of a gram (3 cm long and 0.5 cm in diameter at its base) was suspended in the path of the rays,

[1] This result was reported at a meeting of the Am. Phys. Soc., November 27, 1909.

by means of silk threads, from a grounded aluminum box which served to shield the cone from sudden changes of temperature. To measure the instantaneous temperature of the cone a thermocouple was used. Along one side of the cone was soldered a constantan wire rolled flat, along the other a copper wire. The other junction (see Fig. 4) was maintained at a constant adjustable temperature by immersion in kerosene in a Dewar flask provided

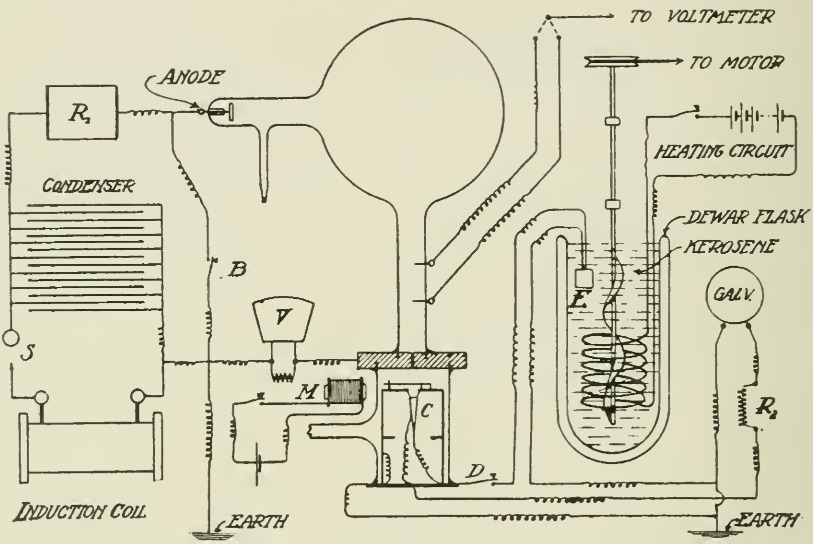


FIG. 4.—Diagram showing the electrical connections of the apparatus

with a heating coil and stirrer.[1] To measure the thermo-electric current a Thomson galvanometer was used whose sensitiveness of 4×10^{-9} amperes per mm deflection could be decreased by introducing resistance R_2. For the most sensitive adjustment used ($R_2=0$), 1 mm deflection corresponded to about 0.001°C increase in temperature of the cone or a net addition to the cone of 1.2×10^{-5} calories. Two telescopes were mounted so that through one the observer could follow the second hand of a watch while noting the galvanometer reading with the other eye. Thus a series of observations at intervals of 5 seconds could be made and

[1] To avoid confusion, the discussion of other details of the apparatus is postponed to the end of the paper.

the instantaneous temperature of the cone as a function of the time determined. Typical curves are shown in Fig. 5. On starting the discharge through the tube the temperature rises rapidly at first, then more and more slowly. The curve never becomes horizontal because of the radiation received from the back surface of the cathode, which increases with the time as the cathode is

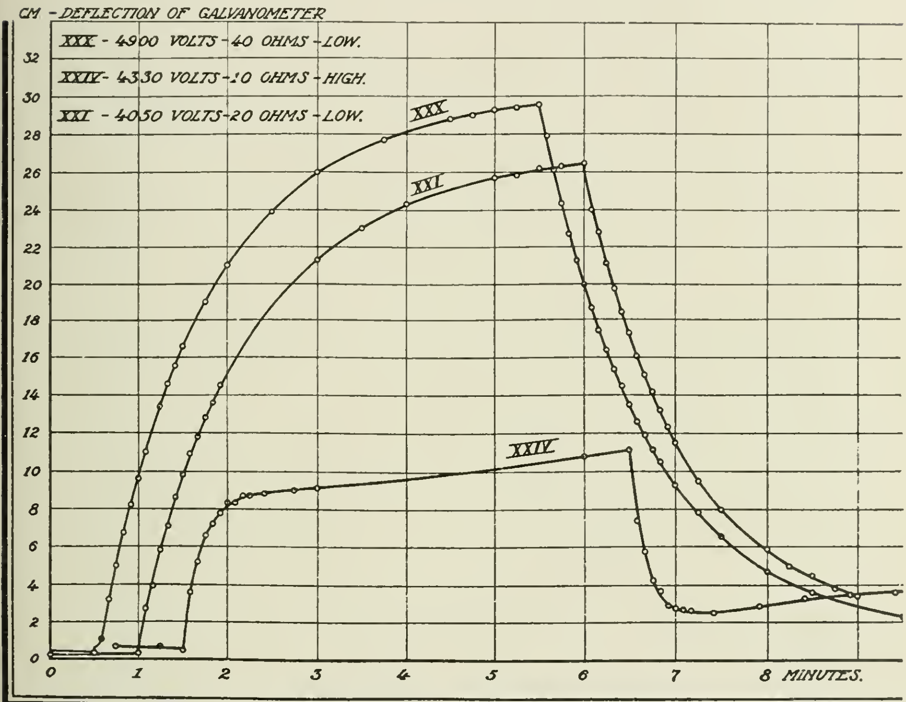


FIG. 5.—Temperature of the cone as a function of the time

heated by the discharge. At the instant when the discharge is stopped, the quickness with which rapid cooling begins is noteworthy. Curves XXI and XXX were taken when the pressure in the canal chamber was low (0.005 mm), while curve XXIV was obtained with a much higher pressure (0.1 mm). The sensitiveness is twice as great for XXIV as for XXI and that in turn twice as great as for XXX.

The problem now is to determine the value of the energy-flux

corresponding to each of these curves. It is easily shown that the temperature is in no case a simple exponential function of the time, as would be the case if the heating of the cone by the rays and the radiation of heat to a constant temperature envelope alone were involved. The radiation from the heated cathode is an important disturbing factor. If we let

x = temperature of the cone,
y = temperature of the surrounding gas and case,
z = temperature of back side of the cathode,
c = heat capacity of the cone,
E = energy received by the cone from the canal rays per second,
A and B = radiation constants, and
C = convection constant depending on the gas pressure,

then

$$c\frac{dx}{dt}=E+A\ (z-x)-(B+C)(x-y) \tag{1}$$

Just preceding the instant ($t=5$) when the discharge is discontinued by short-circuiting the tube we have

$$c\left(\frac{dx}{dt}\right)_{-5}=E+Az_5+(B+C)y_5-(A+B+C)x_5 \tag{2}$$

while immediately following the cessation of the discharge

$$c\left(\frac{dx}{dt}\right)_{+5}=Az_5+(B+C)y_5-(A+B+C)x_5 \tag{3}$$

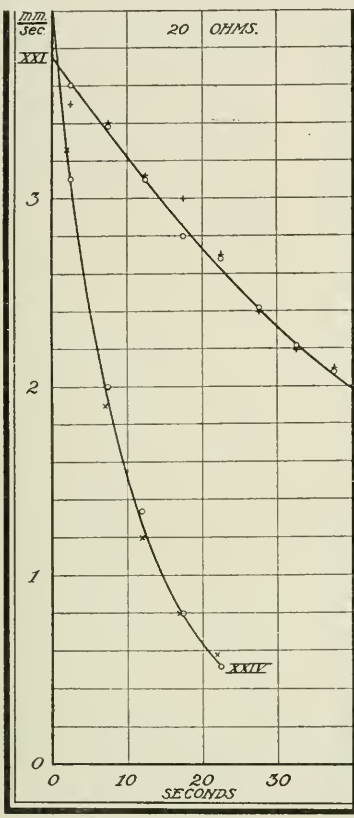
since there is no discontinuity at that instant of either x, y, or z. Hence

$$E=c\left(\frac{dx}{dt}\right)_{-5}-c\left(\frac{dx}{dt}\right)_{+5} \tag{4}$$

an equation which serves most readily for the determination of E from the curves.

The difficulty then was to determine $\left(\frac{dx}{dt}\right)_{+}$ accurately, since the cooling was so rapid at first that the galvanometer readings were uncertain immediately after the discharge was stopped. The method employed was a graphical one. A smooth curve was drawn through the galvanometer readings plotted as a function of the time (Fig. 5), making due allowance for the fact that the

current carried by the rays, which in part passed through the galvanometer, ceased at the same instant as the discharge. The ordinates of the curve were read at regular intervals of time and the values of the time derivatives thus obtained were plotted as a



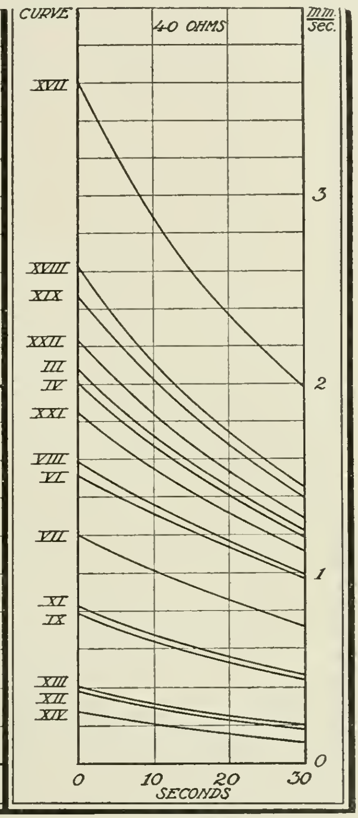


Fig. 6 Fig. 7

Time derivative of the temperature of the cone as a function of the time

function of the time. Fig. 6 shows two such derivative-time curves corresponding to two of the temperature-time curves of Fig. 5. The circles were obtained from the cooling part of the curves, the crosses from the heating part. The former are the more reliable. It is seen that a smooth curve can be drawn through the points with considerable certainty for XXI (low pressure)

and the value of the derivative for $t=0$ can be determined with some accuracy. A number of such derivative-time curves are shown plotted to the same scale in Fig. 7. These curves were each drawn independently. The fact that they fit in with each other so well is evidence for their accuracy.

The results obtained for the energy-flux of the canal rays as a function of the cathode-fall of potential are plotted in Fig. 8. It

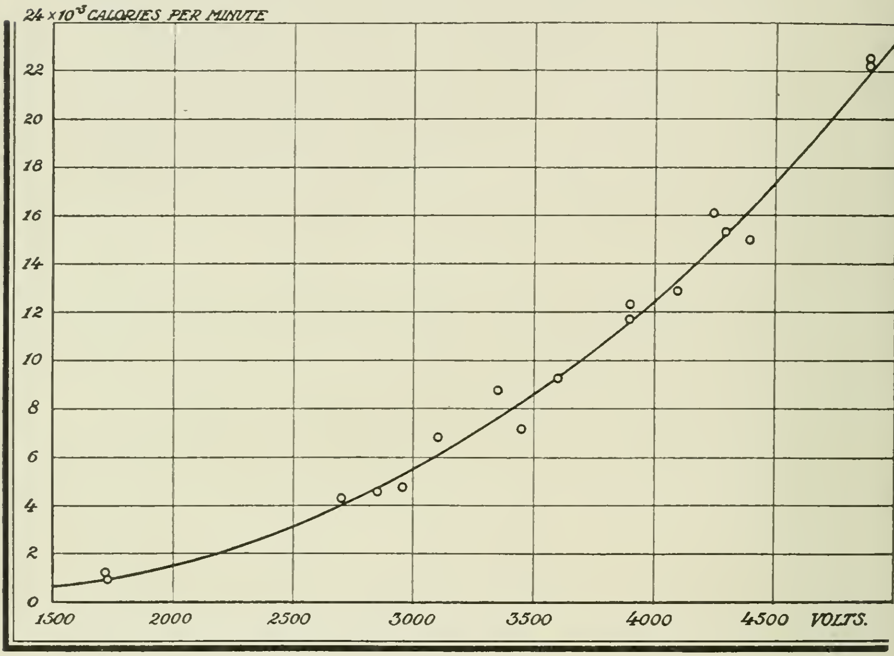


FIG. 8.—Energy-flux of canal-ray bundle as a function of the cathode-fall of potential

is to be regretted that the agreement between the various points obtained is not better. The trouble does not lie in the determination of the energy-flux from the curves, since that is probably accurate to a few per cent, but rather in the measurement of the cathode-fall of potential. The Kelvin electrostatic voltmeter could be read to 1 per cent, but even though special precautions were taken, as will be described later, to secure a continuous discharge through the tube, the cathode-fall of potential was seldom quite constant, so that readings could not be made as accurately as

otherwise. The effect of slight impurities in the gas, such as were doubtless given out by the cathode during the discharge, is very marked, though in no cases were these impurities sufficient to affect the spectrum to any noticeable extent. It is seen that a change of less than 5 per cent in the abscissas of the points plotted in Fig. 8 would bring all of them on the curve drawn through them. We

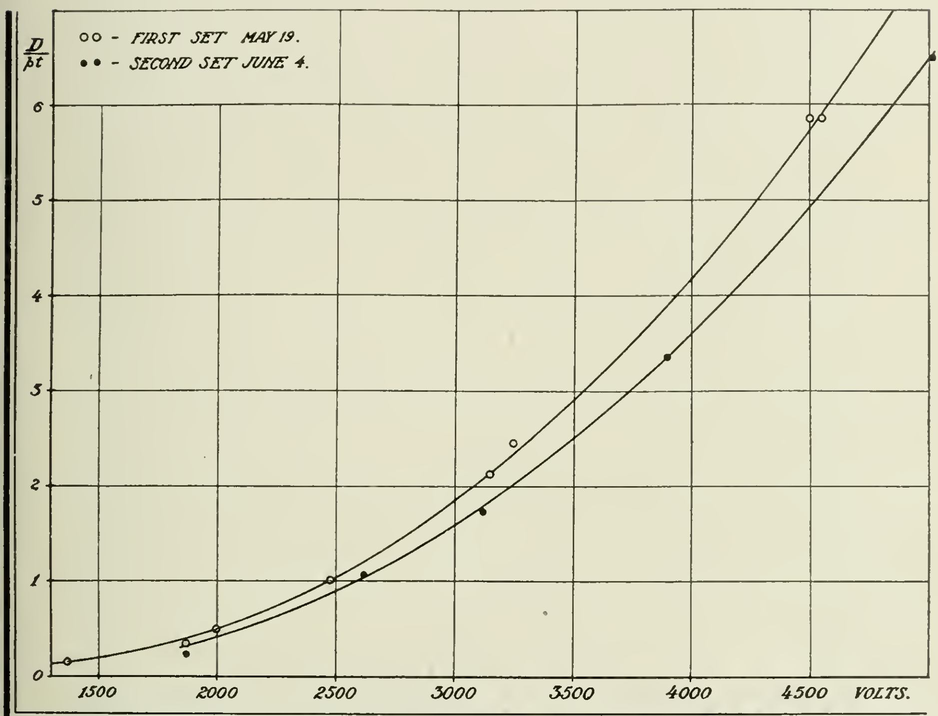


FIG. 9.—Ratio of light-intensity to gas pressure as function of cathode-fall of potential

must conclude, therefore, that the curve represents the experimental results as closely as possible and that the deviations are less than possible experimental errors.

The variation of the intensity of the light from the canal-ray beam as a function of the cathode-fall of potential was determined precisely as in the first experiment by photographing the beam, adjusting the times of exposure so as to obtain images of about the same density. The results are plotted in Fig. 9. The ordinates

are proportional to $\frac{D}{pt}$, where D is the density of the image measured with a spectrophotometer, p is the gas pressure, and t the time of exposure. Two sets of determinations were made. The two curves drawn through the points obtained are each identical with the curve shown in Fig. 8 except for a constant factor of proportionality. The ordinates of Fig. 8 are proportional to the mean energy of the individual rays times the number of rays striking the cone per second when the pressure in the canal chamber is low; that is, they are very closely proportional to the mean energy of the individual rays times the number of rays emerging from the hole in the cathode per second when the pressure is the same on both sides of the cathode. The ordinates of the curves in Fig. 9 are proportional to the mean intensity of the light emitted as a result of each collision times the number of rays emerging from the hole in the cathode per second when the pressure is the same on both sides of the cathode. The fact that through the range of cathode-falls used these curves agree shows that the mean intensity of light emitted per collision is proportional to the mean energy of the individual rays, that is, to the mean energy of each collision. It is not to be expected that this law holds rigorously, and deviations may be expected to increase as the minimum voltage necessary to produce a discharge is approached; but within the limits of voltage and velocity used here (1,500 to 5,000 volts, 6 to $10 \times 10^7 \frac{\text{cm}}{\text{sec}}$) the law seems to be verified within the limits of experimental error, that is, within a few per cent.

DISCUSSION AND STATISTICAL CALCULATIONS

What bearing have these experimental results upon the theory of the production of the light in question? First, it must be pointed out that in the case of canal rays in pure hydrogen the light producing the displaced lines in the Stark effect is several times as intense as that producing the rest lines, hence the experimental results which apply strictly only to the whole of the light from the path of the rays may be taken without serious error to apply to the light from the moving sources alone. The experi-

ments therefore tend to prove that the light showing the Stark effect is emitted only as a result of the collisions of canal rays with gas molecules and that the intensity of the light emitted per collision is proportional to the mean energy imparted to the hit molecules by the collisions.

Can the details of the Stark effect be explained on this basis? If so, what additional assumptions are necessary? In answering these questions a calculation made three years ago, the results of which were reported in part in the article referred to above, is of some importance. By a statistical method a computation was made of the Stark effect to be expected if the following assumptions are true:

1. That canal rays all with the same velocity enter a gas whose molecules are identical with the rays and have velocities negligible in comparison with that of the rays;

2. That the momenta after collision are the same as if the colliding molecules were perfectly elastic spheres;

3. That the intensity of the light emitted by each hit molecule is proportional to the kinetic energy given it as a result of the collision, but

4. That no light is emitted by a hit molecule unless the energy transmitted to it exceeds a certain minimum, which is supposed to be equal to the energy necessary to produce ionization; and finally

5. That no light of the kind in question is emitted by the hitting rays—the supposition being that they are for the most part neutralized by the collisions, hence do not emit the same spectral lines as the ionized hit molecules.

Of these assumptions the first two are made for the sake of simplicity. Their disagreement with the actual facts in the case of hydrogen rays in hydrogen gas is probably not sufficient to affect the qualitative value of the computation. The third assumption has since been proved by the experiment described above. The last two are equivalent to the single assumption that the series-line spectrum is emitted only by the molecules which become positively charged as a result of the shock of the collision. To find out whether this assumption is necessary to explain the details

of the Stark effect, as reported by J. Stark, F. Paschen, and B. Strasser, on the basis of the emission of light only as a result of molecular collision (necessitated by the first experiment above), is the purpose of the following computations.

The method employed was necessarily statistical, as the problem is too complicated to yield to direct mathematical treatment. Using the second assumption, we can readily determine the velocity

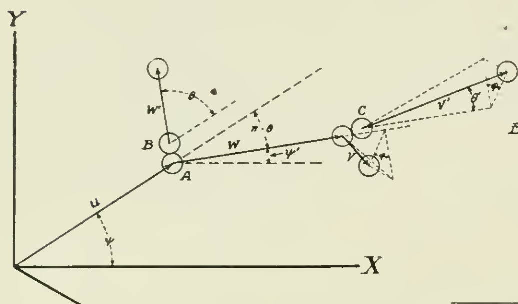


Fig 10

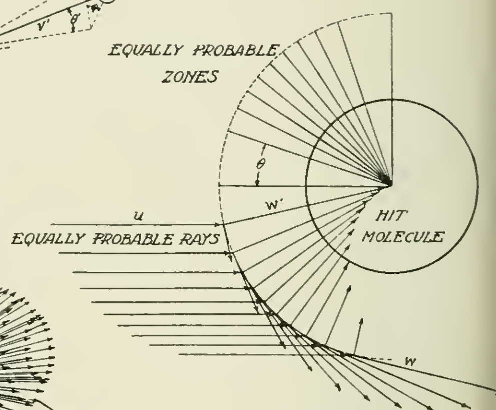


Fig 11

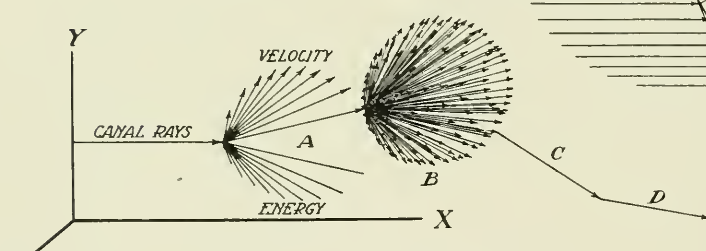


Fig 12

of the hit and of the hitting molecule after each impact. If a molecule with velocity u (Fig. 10) hits a molecule at rest so that the line of centers at the instant of collision lies in the XY plane and makes an angle θ with u, the resulting velocities are:

(for hit molecule) $W' = u \sin\theta$; $W'_x = W' (\cos\theta \cos\psi - \sin\theta \sin\psi)$
(for hitting molecule) $W = u \cos\theta$; $W_x = W (\sin\theta \cos\psi + \cos\theta \sin\psi)$

If the line of centers is not in the XY plane but its projection on a plane perpendicular to the velocity of the hitting molecule before collision (W) makes an angle ϕ with the XY plane, and if

θ' is the angle it makes with W and ψ' is the angle W makes with the X-axis, then after the collision,

$$\text{(for hit molecule)}\quad V' = W \sin\theta' ;$$
$$V'_x = V'(\cos\theta' \cos\psi' - \sin\theta' \sin\psi' \cos\phi) .$$
$$\text{(for hitting molecule)}\quad V = W \cos\theta' ;$$
$$V_x = V (\sin\theta' \cos\psi' + \cos\theta' \sin\psi' \cos\phi) .$$

Now consider Fig. 11. The center of the hitting molecule at the instant of collision must lie on the dotted spherical surface. Suppose its velocity is u. It can readily be shown that if the

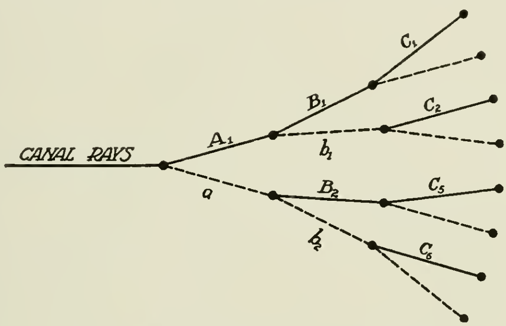


FIG. 13.—Various groups of secondary rays

spherical surface is divided into zones by coaxial cones whose angular apertures are given by the equations $\cos^2\theta = \frac{a}{10}$, where a is an integral parameter varying from 1 to 10, the probability that the center of the hitting molecule will lie within any one zone is the same since the areas of projection of the zones on a plane perpendicular to u are all equal. It is further evident that if each zone is divided into two equally probable zones, the values of θ corresponding to the dividing cones will be equally probable values of θ. If then we assume ten collisions for a given value of u such that the line of centers at collision makes angles with u given by the equation $\cos^2\theta = \frac{1}{20} + \frac{a}{10}$, where a has values 0, 1, 2, ... 8, 9, the resulting velocities after collision may be taken to represent fairly the actual distribution of velocities as far as θ is concerned.

These equally probable rays with the velocities of the hit and hitting molecules after collision in each case are shown in Fig. 11.

The general method of treating the problem in hand is shown in Fig. 12. Canal rays are assumed to enter the gas all moving with the same velocity u in the direction of the X-axis, which is also the line of sight. Let each ray collide with a gas molecule. The distribution of velocities among this first generation of secondary rays (A) can be fairly represented for our purpose by the ten

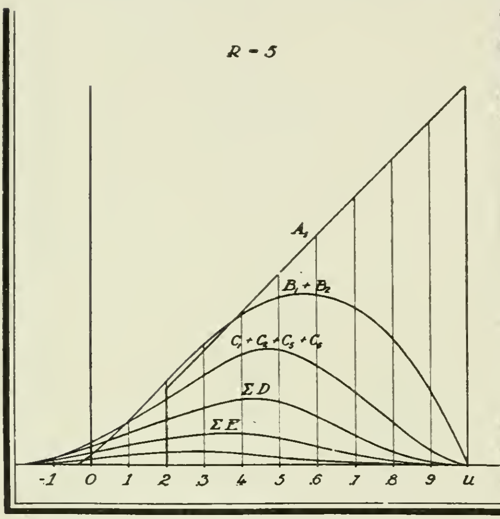


FIG. 14.—Doppler effect due to the various generations of secondary rays

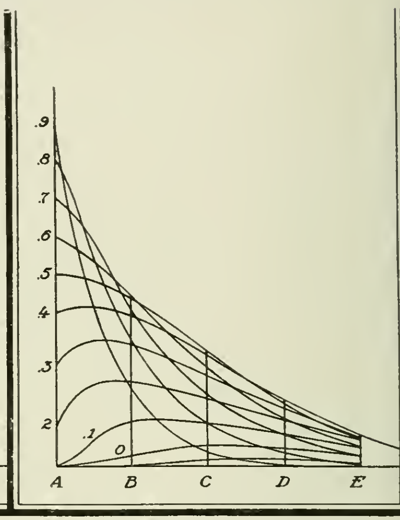


FIG. 15.—Extrapolation curves

bundles of rays shown in the XY plane, since the Doppler effect depends in this case only on the latitude angle θ and not on the azimuth angle ϕ. The energy given the hit molecules is shown in the lower half of the same diagram. It is seen that by far the most light is emitted by the molecules having a considerable velocity in the line of sight. Let us assume that the minimum energy necessary to produce ionization is some fraction, say one-fifth of the original energy of each canal ray. In that case two of our bundles of hit molecules will emit no light.

Now let each of these secondary rays strike gas molecules and produce another generation (B). Here the Doppler effect will

depend on both ϕ and θ; hence to represent this generation 1,000 bundles of hit molecules were chosen, ten values of ϕ (9° to 171° at intervals of 18°) for each value of θ and ten values of θ for each value of ψ, the angle which the hitting molecules made with the X-axis before collision. One hundred only are shown in perspective in the drawing. The energy and the x component of the velocity of each of these bundles was computed and the energy (taken equal to the square of the velocity) of all those whose

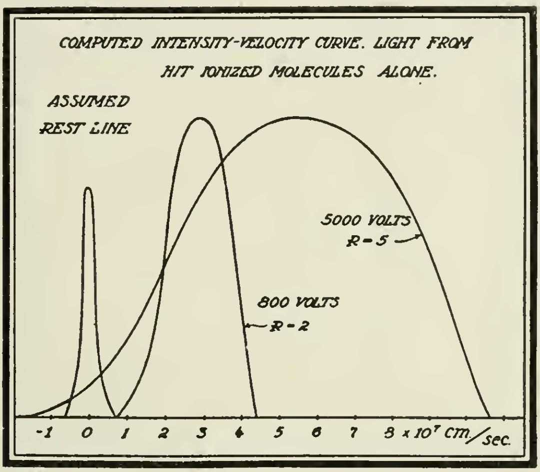


Fig. 16

velocity along the X-axis lay between certain limits, say 0.50 to 0.55 u, was added up and this sum was taken as proportional to the intensity of light corresponding to that Doppler shift. Thus the intensity of light as a function of the Doppler shift to be expected from this generation was obtained (B_1). In getting the effect of the third generation of hit molecules a multiplication by 100 was avoided by further extending the principle of representation. The cosine of ψ', the angle which each of the 1,000 rays of the second generation made with the X-axis, was computed by dividing the velocity of each ray along the X-axis by its actual velocity. Then these values of the cosine were plotted as a function of the velocity, one point for each ray. These were divided into groups of 20 and one ray picked out to represent each 20.

The values of the velocity and of the angles ψ' for each of these 50 representatives were obtained directly from the diagram. As in the case of B, ten values of ϕ for each of ten values of θ for each ray had to be considered, making 5,000 in all, for each of which the energy and velocity along the X-axis were computed. The intensity of light as a function of the Doppler shift was then obtained as before (C_1).

We have considered so far only the secondary rays in one direct

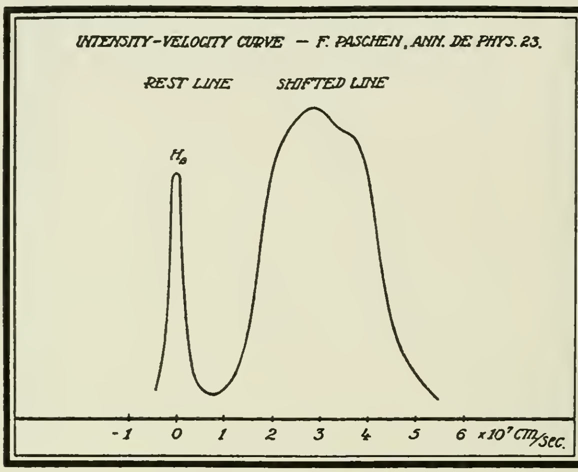


FIG. 17.—Stark effect for hydrogen

line of descent, both in the emission of light and in the production of other secondary rays. But surviving each generation of collisions there are in addition to the ionized hit rays some positively charged hitting rays which failed to ionize the molecules they hit but were merely deflected. In the diagram (Fig. 13), the full lines represent hit ionized molecules which emit light; the dashed lines represent ionized hitting molecules, emitting no light but capable of producing other ionized secondary rays which may emit light. In Fig. 21 the Doppler effect due to each of these groups of rays, on the basis of the assumptions made at the beginning, is shown (A_1, B_1, B_2, C_1, C_2, C_5, C_6) for the special case when the energy of each canal ray is taken to be five times the minimum energy necessary to produce ionization ($R=5$). The total Doppler effect

for each generation for the same case is shown in Fig. 14. The distribution of the light due to the later generations was obtained by extrapolation, using the curves shown in Fig. 15. The sum of them all shows discontinuities at 0.2 u and u due to our artificial assumption that all the canal rays have the same velocity and that the unshifted line has no width. By modifying these assumptions to agree more closely with the actual facts the discontinuities are eliminated. The results for the cases when $R=2$

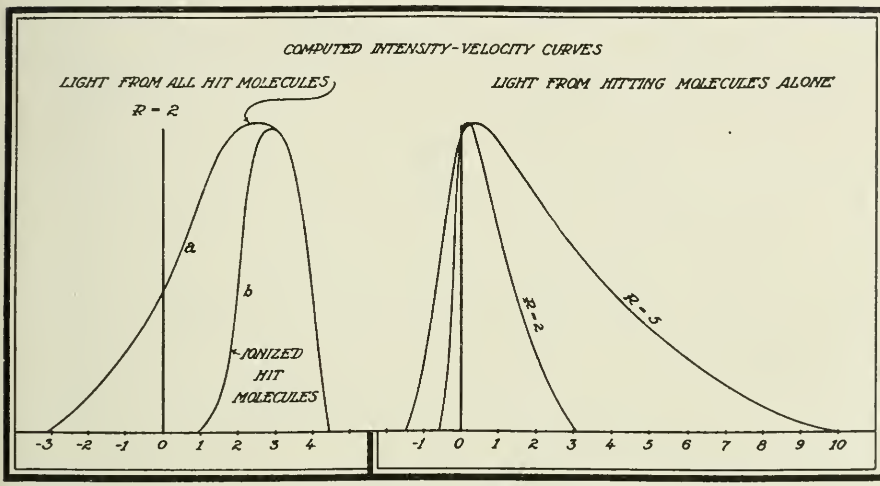


FIG. 18.—Testing the fourth assumption

FIG. 19.—Testing the fifth assumption

and $R=5$ are given in Fig. 16. It was assumed in getting the scale of abscissas that the minimum energy necessary to produce ionization corresponds to a velocity of 2×10^7 cm per sec. in the case of hydrogen. For comparison with the Stark effect as actually observed, an intensity-velocity curve has been reprinted from Professor F. Paschen's paper[1] (Fig. 17). This curve is obviously the sum of two, each having a marked intensity minimum and agreeing in general form with the two computed curves shown in Fig. 16. A better agreement could hardly be expected.

To show the necessity of introducing the fourth assumption regarding the minimum energy necessary to produce ionization,

[1] *Annalen der Physik*, **23**, 250, 1907.

a calculation was made without this assumption. The result for the case when $R=2$ is shown in Fig. 18 (*a*). Curve (*b*) is reproduced from Fig. 16 for comparison. Evidently the presence of the intensity minimum demands this fourth assumption.

The fifth assumption, that no hitting rays emit light as a result of the collisions, seems rather arbitrary and it appeared desirable to determine to what extent this assumption is necessary. If we assume that each of the charged hitting molecules emits light whose intensity is proportional to the energy of the collision, a calculation of the Doppler effect due to this light gives curves shown in Fig. 19. Here again the presence of the intensity-minimum in the Stark effect proves that, for the most part at least, the charged hitting rays do not emit any light corresponding to the series-line spectrum as a result of the collisions.

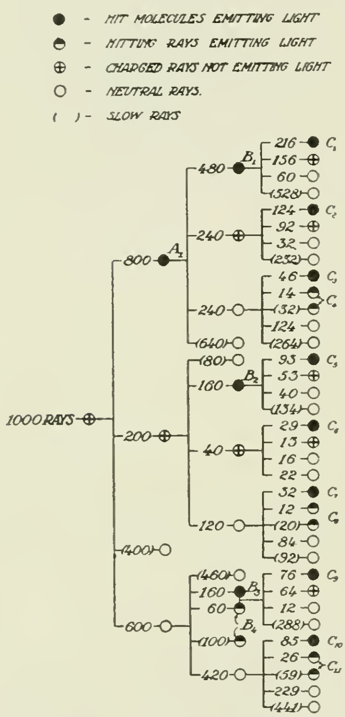


FIG. 20.—Genealogy on the basis of the six assumptions

There is still a further possibility to be investigated. Neutral rays when moving with sufficient velocity doubtless have the power of producing ionization. If so, the hitting molecule is as likely to be ionized as the hit molecule. Nothing is known as to the minimum energy necessary to produce ionization in this case, but a calculation was made adding the following assumption to the five considered above:

6. That when the collision of a neutral ray with a neutral gas molecule involves the transference of more than a certain minimum energy, which is assumed to be equal to the minimum energy necessary for ionization in the case of the collision of a charged

ray with a gas molecule, one will be ionized so that on the average half the hitting and half the hit molecules will emit light.

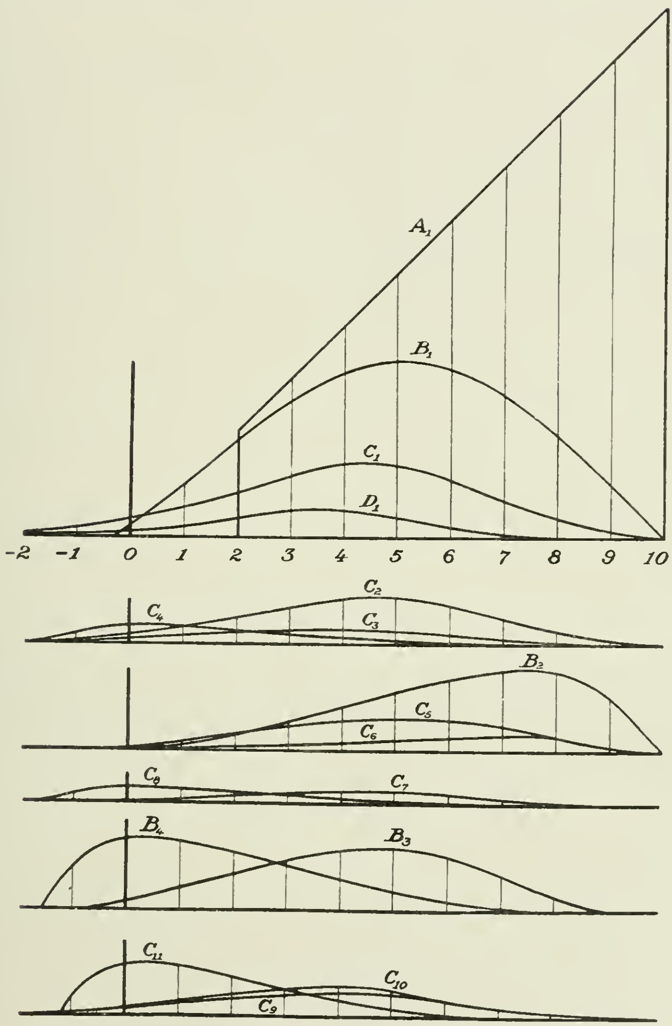


FIG. 21.—Doppler effect due to various groups of secondary rays

The introduction of this assumption adds several groups of active rays to our family. The genealogy for the case of 1,000 positively charged rays ($R = 5$) is given in Fig. 20. The slow rays

are those incapable of producing ionization. The Doppler effect corresponding to each of the active groups of rays was computed for the first three generations. The curves are shown in Fig. 21. The effect of later generations was obtained by extrapolation as above (cf. Fig. 15). The total effect due to 1,000 charged canal rays on the basis of the six assumptions is shown in Fig. 22 (*a*) for the special case when $R=5$. Curve (*b*) is reproduced from Fig. 16 for comparison. Evidently our sixth assumption is incom-

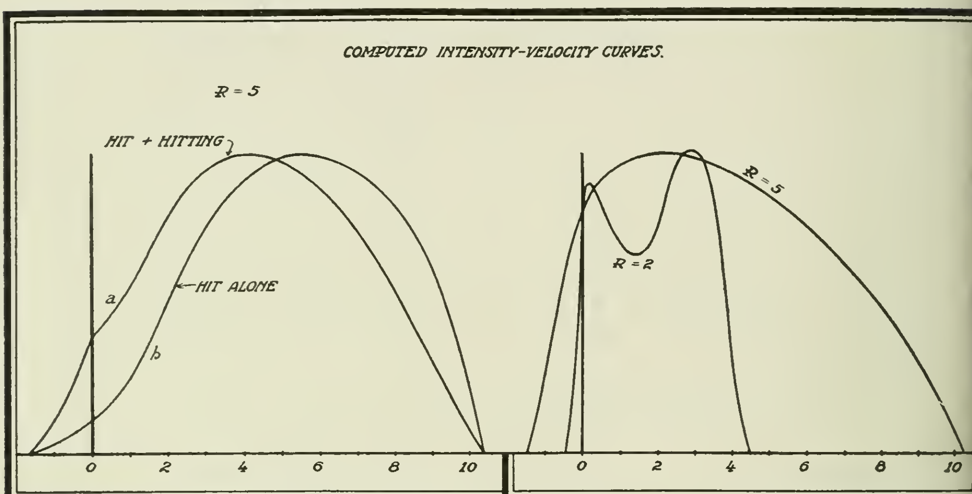


FIG. 22.—Testing the sixth assumption: charged canal rays

FIG. 23.—Doppler effect due to neutral canal rays on the basis of the sixth assumption

patible with an intensity minimum. Probably the minimum energy required for neutral rays to produce ionization is much greater than was assumed and hence the disturbing effect of these neutral rays is much less. Finally a computation was made of the Doppler effect to be expected on the basis of the six assumptions if 1,000 neutral canal rays with the same velocity u enter a gas. The genealogy in this case is similar to that shown in Fig. 20 except that the number in the various groups is quite different. The total Doppler effect due to these 1,000 neutral rays and their offspring is shown in Fig. 23 for the special cases when $R=2$ and

$R=5$. Here again it is evident that the number of neutral rays must be small or assumption 6 must be incorrect. The number of neutral rays, however, is known from deflection experiments not to be small, may be in fact over 30 per cent of the whole number. The other alternative is therefore inevitable.

CONCLUSIONS

As a result of these calculations one seems justified in concluding that the original assumptions regarding the minimum energy necessary to produce the emission of light and the emission of light by the hit ionized molecules alone, represent the true state of the case very approximately. If the neutral rays produce ionization, the minimum energy required must be considerably greater than is necessary in the case of charged rays, so that the light thus produced is insignificant. The hitting charged rays must be neutralized for the most part and particularly when the shock of collision is great, at the same instant as the hit molecule is ionized.

If this analysis of the phenomenon is correct, a far more important conclusion necessarily follows: namely, that the series-line spectrum of hydrogen, which alone shows the Stark effect, is emitted by the positively charged molecule or atom of hydrogen. The reasoning leading to this deduction may seem rather indirect, but the fact that the above assumptions are the only ones, as I believe, which will explain the presence of the intensity minimum in the Stark effect on the basis of the experimentally verified hypothesis that the emission of light in the path of the canal rays is due to the collision of the canal rays and their offspring with gas molecules, is strong evidence for their correctness. Some other experiments will be performed shortly to test some further deductions from this analysis of the phenomenon.

DISCUSSION OF STRASSER'S RESULTS

Last April in the *Annalen der Physik*,[1] B. Strasser published some results of his painstaking experiments on the Stark effect which are of great interest. May I make a few suggestions as to a possible interpretation of these results along the same line as

[1] *Annalen der Physik*, **31**, 890–918, 1910.

in the case of the simple Stark effect in pure hydrogen? Here again one is working rather in the dark, but a suggestion may have value as a working hypothesis even though it later proves to be false.

Strasser has proved the following facts:

1. The intensity of the rest line in the Stark effect for hydrogen canal rays depends on the purity of the hydrogen in the tube. If the gas is sufficiently pure no rest line is obtained.

2. If a definite quantity of another gas is added, the intensity of the rest line is increased and that of the shifted line decreased in proportion to the amount added, so that when a sufficient quantity of the foreign gas is present an intensity minimum is no longer obtained between the two lines.

3. By experimenting with various gases (*N*, *Ar*, *He*) it was found that the effect produced when the partial pressure of the added gas had a certain value was greater the larger the atomic weight of the gas introduced.

From these results it seems evident that the rest line is due to the collision of hydrogen canal rays with molecules of the foreign gas. The larger the number of molecules of the foreign gas present, the greater the number of collisions with them in proportion to the number of collisions with hydrogen molecules which we have assumed produce the displaced line. If we assume that the charged canal rays are not neutralized for the most part by colliding with foreign gas molecules, but, because of the shock of the collision, emit light whose intensity is proportional to the energy of the collision, the Doppler effect due to this light would resemble in general form the curves shown in Fig. 19, though modified by the fact that in this case the hit molecules are larger. Thus we should expect the rest line to be broadened unsymmetrically toward the displaced line as Strasser found. Moreover gases with greater atomic weights may be expected to have larger molecules, hence to exert a greater influence for a given number of molecules by increasing the probability of being struck by the canal rays. The only difficulty is to explain why the hydrogen canal rays are neutralized for the most part when striking hydrogen molecules but not when hitting other molecules. But Strasser has shown that

the Doppler effect produced in the two cases is quite different. It is hard to see how the facts can be explained otherwise.

Strasser also reports that the light from the layer just in front of the cathode shows a weak displaced line with a distinct intensity minimum, whereas immediately behind the cathode a strong, broad displaced line is obtained corresponding to the high velocity of the canal rays. The explanation seems to be that as the canal rays acquire their velocity through the action of the electric field in front of the cathode, and since the potential-gradient is extremely steep right near the cathode, the number of rays having a velocity high enough to produce ionization must increase very rapidly just at the surface of the cathode. This leads to the apparent discontinuity at the surface of the cathode which suggested Strasser and Wien's theory that the canal rays emit light as a result of the electric shock experienced in suddenly passing from a very strong to a weak field—a hypothesis which of course is no longer tenable.

Strasser's observation that the hydrogen lines persist to a greater distance from the cathode than the lines of a foreign gas may be interpreted to show that the minimum energy required to produce the emission of light is greater in the case of the foreign gas than in the case of hydrogen. Finally Strasser reports that the spectrum of the light from the path of the canal rays viewed normal to their velocity shows the same broadening of the lines whether there is a foreign gas present or not. The computation of the effect to be expected on the basis of the five assumptions made above is extremely laborious, but a first approximation shows a fair qualitative agreement with this experimental result. Strasser does not publish quantitative data as to the amount of the broadening. His more recent results for hydrogen canal rays in nitrogen gas[1] can obviously be explained in the same way as these earlier ones and merely add to the evidence in support of this analysis of the phenomenon.

CONDUCTION OF HEAT BY A GAS AT LOW PRESSURE

The curves shown in Figs. 5 and 6 give some idea of the relative importance of convection and of radiation in the cooling of the

[1] *Annalen der Physik*, 32, 1107, 1910.

cone. Curve XIV was made with the pressure in the canal chamber about twenty times as great as when curve XXI was taken. From the derivative curves (Fig. 6) it appears that the energy being received by the cone per second in the first case is even greater than in the second case yet the maximum temperature reached in the first case is only one-sixth of that reached in the second case. By differentiating equation (3) we get

$$c\left(\frac{d^2x}{dt^2}\right)_+ = -(A+B+C)\left(\frac{dx}{dt}\right)_+ \tag{5}$$

since the time derivatives of z and y are negligible in comparison with $\frac{dx}{dt}$ for the cooling part of the curve. This enables a rough determination of $(A+B+C)$ to be made. The results show a surprisingly good agreement. The value increases from 0.00010 calories per second at a pressure of 0.005 mm to .00065 calories per second at a pressure of 0.1 mm of mercury—showing that at the higher pressure convection is at least ten and probably twenty times as important as radiation in cooling the cone. The conduction coefficients computed from these data are about 0.00017 and 0.000013 calories per second per degree C. per cm³ for pressures of 0.1 mm and 0.005 mm respectively. By comparison with the coefficient for hydrogen at ordinary pressures, which is given by Meyer as 0.00040, it is seen that the coefficient must be nearly constant until pressures below 1 mm of mercury are reached.

PROPORTION OF NEUTRAL CANAL RAYS

From the energy-flux of the canal rays it is possible to compute the lower limit to the number striking the cone per second by dividing the energy-flux by the energy a singly charged molecule would have if acted on by the whole cathode-fall of potential. Curve I of Fig. 24 was thus computed from the curve in Fig. 8. A lower limit to the number charged could be obtained by dividing the current carried by the rays to the cone by the known value of e. Thus curve II was computed. The increase in the proportion of neutral rays with the increase in the cathode-fall of potential is to be expected, I think, for the particular form of tube which was used, though it might not be true for another tube.

DESCRIPTION OF APPARATUS (*Continued*)

To admit the gas at a uniform rate a capillary was used. The best dimensions were determined by the use of Knudsen's formula.[1]

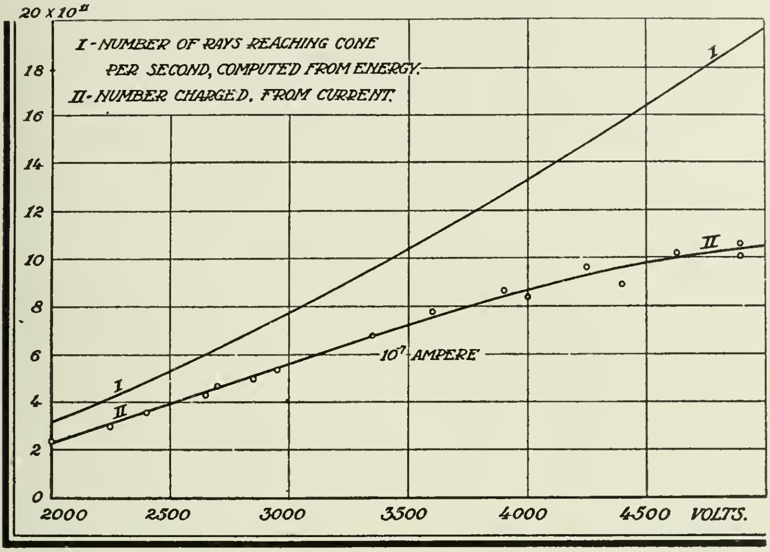


FIG. 24

A number of capillaries were drawn and left attached to tubes of larger bore. The method employed to measure them is illustrated in Fig. 25. Mercury was forced part way into the capillary from the larger tube by means of a rubber bulb, and the capillary was

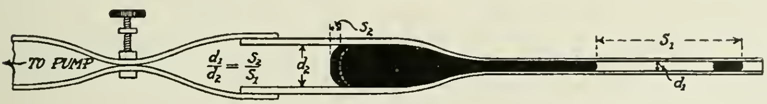


FIG. 25.—Calibration of a capillary tube

calibrated by noting the corresponding distances moved by the two ends of the mercury thread. From the diameter of the larger tube that of the smaller could thus be readily calculated.

To remove the mercury vapor from the gas, it was passed first

[1] *Annalen der Physik*, 28, 75, 1909.

through flowers of sulphur and copper filings to remove the sulphur vapor, then through a coil kept immersed in a freezing mixture of solid CO_2 and ether. All traces of the mercury lines disappeared from the discharge tube with this arrangement.

To secure a continuous discharge the following arrangement was hit upon (see Fig. 4). An induction coil was used to keep a capacity charged by periodically sparking across a point-and-plane gap made sufficiently long to prevent back sparking. This capacity of 0.3 microfarad was allowed to discharge slowly and continuously through a resistance of several hundred thousand ohms (made by rubbing graphite on a plate of ground glass) which was placed in series with the discharge tube. No discontinuities in the discharge could be observed with a rotating mirror. After the induction coil was stopped the luminous discharge would continue sometimes for over a minute. The discharge therefore was probably quite continuous. If the induction coil was connected to the tube directly, producing a discontinuous discharge, a spark-gap of one centimeter would short-circuit the tube if placed in parallel with it, whereas if the condenser set-up was used with the same pressure in the tube, 5,000 volts would maintain a continuous discharge through the tube. Other evidence was secured tending to show that a continuous discharge is an unstable phenomenon unless very special precautions are taken; and that the mean discharge potential is always greater in the case of the discontinuous discharge than in the case of the continuous, though much less than the maximum value reached by the oscillating potential-difference in the former case. A rotating mirror should always be used to test the continuity of the discharge in the case of any quantitative experiments involving the discharge of electricity through gases.

My thanks are due to Professor A. G. Webster for his interest and encouragement during my stay at Clark University, where the above theory was developed.

Madison, Wis.
December 28, 1910

ADDENDUM

By a strange mischance I failed to see, until the above was in type, an article by Professor W. Wien[1] in which he gives results of his measurements of the luminosity of a canal-ray beam at various pressures. He finds that the luminosity increases with the pressure, in qualitative agreement with my observations, but a quantitative comparison would be difficult because of the complexity of the conditions existing with his form of apparatus. His interpretation of the results is quite different from mine.

[1] *Annalen der Physik*, **30**, 349–368, 1909.

THE SPECTRA OF SPIRAL NEBULAE AND GLOBULAR STAR CLUSTERS[1]

SECOND PAPER

By E. A. FATH

The first paper on this subject by the writer describing results of work with the Crossley reflector was published in *Lick Observatory Bulletin*, **5**, 71, 1909. From the evidence presented it was concluded: first, that the spectra of spiral nebulae vary from one in which the bright lines of the gaseous nebulae are the most prominent feature to one closely analogous to that of the sun; second, that the *Hercules* cluster probably contains stars of various types, while the clusters N.G.C. 7078 and 7089 are composed of stars predominantly of the F type.

The present paper will take up the results I have obtained at this observatory with the 60-inch reflector.

The spectrograph used had been designed for other purposes and is not well adapted to this kind of work. Suffice it to say that there is great loss of light due to (1) reflections from two plane mirrors; (2) a large amount of unnecessary glass in the optical system, as the lenses are ordinary commercial portrait lenses; (3) the impossibility of keeping the slit at the focus of the telescope if the focus changed owing to changes of temperature after the exposure had started. Owing to these various factors the exposures were longer than otherwise necessary. It is hoped that all these difficulties will be overcome in a new spectrograph which is soon to be constructed especially for this class of work.

The spectra are but little over 3 mm long from λ 5000 to λ 3700. This small scale and the coarse grain of the Lumière "Sigma" plates, which were used on account of their speed, make the information derived very meager indeed.

The plates were measured by means of a Hartmann spectrocomparator. As standards, stellar spectrograms with well-defined

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 49.

lines taken with the same dispersion were used. The more conspicuous lines found coincident in the standard plate and the one being measured were assumed to be identical, and the remaining lines interpolated between those thus assumed as known. The former are indicated by letters, the latter by their wave-lengths.

The two divisions of the subject will be treated separately.

SPIRAL NEBULAE

The following table gives the data in regard to the exposures on the three nebulae whose spectra will be described.

N.G.C.	Dates of Exposure	Total Exposure
650–51	1909 Sept. 22, 23	10^h47^m
4725	1910 Feb. 15, 16	9 58
4736	Feb. 10	7 00

N.G.C. 650–51.—As stated in my former paper the evidence that this nebula is a spiral is not conclusive, although the presumption in favor of this is fairly strong. At that time the only thing known about its spectrum was an observation by Huggins,[1] who had found the bright nebular line at 501 $\mu\mu$. The present plate shows seven bright lines at

373 $\mu\mu$	486 $\mu\mu$
387	496
397	501
434	

which correspond to lines commonly found in gaseous nebulae. The plate is somewhat underexposed, but so far as can be determined there is no background of continuous spectrum.

N.G.C. 4725.—This spectrum is of the solar type. Lines corresponding to F, G, H, and K are present.

N.G.C. 4736.—The spectrogram of this nebula is not at all like the one obtained at Mount Hamilton. The present plate shows a solar-type spectrum with the following seven lines: F, G, 425 $\mu\mu$, 410 $\mu\mu$, H, K, 387 $\mu\mu$. The Mount Hamilton plate shows a broadening and brightening at 406 $\mu\mu$, which was assumed to be a bright

[1] *Phil. Trans.*, **156**, 381, 1866.

band, and absorption lines at 387 and 400 $\mu\mu$, but the plate is not in good focus. A comparison of the two plates in the spectro-comparator shows certain points of resemblance, yet it is hard to see how a change in focus alone could be responsible for the enormous difference in appearance. The Mount Wilson plate is in good focus and the solar type of spectrum cannot be questioned. As it does not seem probable that the nebula itself changed in the brief interval, about twenty months, between the two exposures, another plate will be taken as soon as possible.

Plates have also been taken of the spectra of other spirals and also of some clusters which are underexposed to such an extent that no reliable conclusion can be drawn from them.

The star-cluster theory of the spirals is thus given additional support by the solar-type spectra of N.G.C. 4725 and 4736.

Although it may appear somewhat premature, it may not be altogether out of place to suggest that we have some evidence, slight though it is, of a progressive change in the spectra of the nebulae with change of form. The beginning of the series would be an irregular nebula such as the great one in *Orion* with a bright-line spectrum. The second type might be a probable spiral like N.G.C. 650–51. The spectrum consists of bright lines and little or no continuous background. The third type is a planetary, such as N.G.C. 6543, with the same type of bright-line spectrum and considerable continuous spectrum,[1] but of quite definite spiral or helical form. This form is partially brought out in Plate 57 of Vol. 8 of the *Lick Observatory Publications*, but is much more clearly shown in a recent negative, taken by Mr. Ritchey with the 60-inch reflector, which is soon to be published and to which I am permitted to refer. The fourth type is a well-developed spiral like N.G.C. 1068,[2] which, while still showing bright lines in its spectrum, has also a strong continuous spectrum which contains absorption lines. The last type, in which the nebula has condensed to stars, at least near its center, is illustrated by nebulae giving solar-type spectra such as the *Andromeda* nebula,[2] N.G.C. 4725

[1] *Astronomy and Astrophysics*, **12**, 51, 1893, and *Lick Observatory Bulletin*, **2**, 51, 1902.

[2] See my first paper.

and 4736. There are great gaps in such a series as outlined. Whether or not these types represent steps in the development of nebulae is certainly a debatable question. If such a series does represent successive stages in the life-history of the nebulae, then star clusters should be added as the final product. This suggestion is advanced with the hope that it may prove of service until something better can be obtained.

GLOBULAR STAR CLUSTERS

Spectrograms of eight globular star clusters have been obtained as follows:

N.G.C.	Dates of Exposure	Total Exposure
5024	1910 May 1	$6^h 45^m$
5272	Apr. 30	5 00
6205	June 15	6 25
6229	June 16, 17	11 45
6341	June 14	3 27
6656	Aug. 11, 12	9 15
6934	Aug. 29, 30	8 00
7078	1909 Aug. 22, 23, 24	16 15

The plates of N.G.C. 6656 and 6934 are considerably underexposed.

The lines found are given in the following table. A line whose presence is very probable but not certain is followed by a question

N.G.C. 5024	5272	6205	6229	6341	6656	6934	7078	7089
$H\beta$	$H\beta$ (?)	$H\beta$ (?)	$H\beta$ (?)	$H\beta$	$H\beta$	....	$H\beta$	$H\beta$
							444	
$H\gamma$	$H\gamma$	$H\gamma$	$H\gamma$	$H\gamma$		$H\gamma$	$H\gamma$	$H\gamma$
418	421	420	420 (?)	419		421	417	
$H\delta$	$H\delta$	$H\delta$	$H\delta$	$H\delta$	$H\delta$	$H\delta$ (?)	$H\delta$	$H\delta$
				402				
$H\epsilon$ or H	H	$H\epsilon$ or H	H	H	$H\epsilon$ or H		H	H
	K		K	K	K (?)		K	K
			$H\zeta$	$H\zeta$			$H\zeta$	$H\zeta$
							$H\eta$	$H\eta$
							$H\theta$	$H\theta$

mark. This uncertainty is due to one of two causes: either the plate is not very strong, and therefore lines at either end of the

the spectrum are weak, or there is a local defect in the plate. N.G.C. 7089, described in the first paper, has been added for purposes of comparison.

A most interesting result is here shown. The globular clusters investigated have almost identical spectra, approximately F type. The hydrogen series predominates, but in every case where the spectrum extends far enough into the violet strong H and K lines of calcium are found. The line near 419 $\mu\mu$, present in nearly all, should more properly be called a band. It is faint and diffuse and undoubtedly corresponds to a large group of lines found in this region in F-type stars. It varies somewhat in position depending on the relative strength of the components. This identification of type must be taken with due precaution. It means simply that the clusters give a spectrum lying between A and G in the Harvard series B, A, F, G, K, M. Furthermore, I am quite certain that only the brighter stars in the clusters are concerned in the spectra obtained. In some instances, when setting on a comparatively coarse cluster, the light from a small group of the brighter stars could be seen coming through the slit. The light from the background of faint stars was almost negligible in comparison.

There is another consideration, however, which must not be overlooked. In the first paper it was stated that from the Mount Hamilton plate the *Hercules* cluster, N.G.C. 6205, gave evidence of containing stars of different spectral types. The Mount Wilson plate does not support this, as it shows only the F type. Both plates must be considered. As the slit in the two cases almost certainly crossed different portions of this cluster the necessity of obtaining more than one plate of each cluster is shown.

I have been able to form a list of 109 so-called globular clusters, some of them very coarse. The nine more condensed ones listed in the table are scattered over a region from 13^h to $21^h.5$ of right ascension and from $+48^\circ$ to -24° in declination. We thus have what might be called a fair sample.

It would be of great interest to know whether there is a connection between the type of spectrum and the number of cluster variables or not. Out of the nine clusters in the above list at least four contain variable stars, namely N.G.C. 5272 with 132 variables,

6656 with 16, 7078 with 51, and 7089 with 10.[1] No direct connection appears; nevertheless it is quite possible that when spectrograms of higher dispersion are available we may be able to establish such a relationship.

In conclusion I wish to express my thanks to Director Campbell of the Lick Observatory for sending me the Mount Hamilton negatives to allow a direct comparison of the two series of plates.

Mount Wilson Solar Observatory
December 23, 1910

[1] *Annals Harvard College Observatory*, **60**, 217, 218.

SOME RESULTS OF A STUDY OF THE SPECTRA OF *SIRIUS*, *PROCYON*, AND *ARCTURUS* WITH HIGH DISPERSION[1]

By WALTER S. ADAMS

Among the accessory instruments to be used with the 60-inch reflecting telescope at Mount Wilson as originally planned by Mr. Hale was included a large spectrograph to be employed in photographing the spectra of some of the brighter stars. The instrument was completed shortly after the erection of the large reflector and has been in use by us for nearly a year. The telescope in this case is employed in the coudé form, and the light is reflected downward through the hollow polar axis to the slit of the spectrograph, the equivalent focal length being 150 feet (45.7 m). The ratio of aperture to focal length accordingly is 1 to 30. It was at first planned to mount the spectrograph on an inclined pier with its axis in the line of the center of the polar axis, but for the sake of convenience in manipulation and constancy of temperature it was decided later to place the instrument in a vertical position in an underground pit. This of course necessitates an additional reflection, which is obtained by means of a small plane mirror supported from the lower bearing of the axis of the telescope.

The spectrograph as used by us at present is of the auto-collimating type. It consists of a lens of 15.2 cm aperture and 5.5 m focal length, used in conjunction with a dense flint-glass prism of 63° angle and a plane mirror to return the light through the prism. Unfortunately, we have as yet been unable to secure a satisfactory prism of sufficient size to utilize the full beam from the collimating lens. The one employed is 20.3 cm long, but has a face only 12.7 cm across. Accordingly, in the position of minimum deviation, it is completely illuminated by a beam only 5.7 cm in cross-section, and there is a loss of light of over 50 per cent. With a prism of full size it will be possible to reduce the exposure times greatly, and thus bring additional stars within the range of the spectro-

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 50.

graph. The approximate linear scale of the photographs obtained is as follows:

$$1 \text{ mm} = 1.4 \text{ Å at } \lambda\, 4300,$$
$$1 \text{ mm} = 2.4 \text{ Å at } \lambda\, 5000,$$
$$1 \text{ mm} = 6.2 \text{ Å at } \lambda\, 6500.$$

The plates used are 43 cm in length and the definition is satisfactory over their entire extent. In the more refrangible part of the spectrum the Lumière "Sigma" plates have been used for the most part, while to the red of λ 4900 Seed "Gilt Edge 27" plates, sensitized by Wallace's formula, have proved very satisfactory. The spectrum of the iron arc has been used for comparison purposes.

We have up to the present time secured with this instrument spectra of the following stars: *Sirius*, *Procyon*, *Arcturus*, *α Orionis*, *β Orionis*, and *α Scorpii*. In this note I wish to refer briefly to some results obtained from a study of the photographs of the first three of these stars. The plates have all been taken by Mr. Babcock and myself.

The material available for investigation includes six plates of *Sirius*, covering the spectrum from λ 4200 to λ 6600; four plates of *Procyon* from λ 4200 to λ 4900; and nine plates of *Arcturus* from λ 4300 to λ 6600. A few of the earlier plates of *Arcturus* were taken with a very wide slit, and are entitled to considerably less weight than those obtained more recently.

The chief object in the study of these spectra has been to test the possibility of detecting difference of displacement for the different lines, and thus obtain some knowledge of the effective pressure in the atmospheres of these stars. Accordingly, the lines used for measurement have been selected with this object in view. In the case of *Sirius* the total number of suitable lines is comparatively small, and so practically all of the lines have been measured for which satisfactory comparison lines for use in the derivation of wave-lengths are available. In the case of *Procyon* and *Arcturus* the selection has been made largely on the basis of the list of lines used by myself in an investigation of the displacements of lines at the sun's limb. Since it seems highly probable that these displacements are due mainly to pressure, this list is very well adapted for the object in view. The fact that the enhanced lines

as a class show larger shifts at the sun's limb than do the ordinary arc lines makes these of particular importance in the study of the stellar spectra, and so practically all of them have been measured upon our plates. As is well known, the enhanced lines are extremely prominent in the spectrum of *Sirius*, a comparatively small number of arc lines being represented, and these as a rule only from among the strongest. In the spectrum of *Procyon* the enhanced lines, though still very strong, are relatively less prominent, and the arc lines are much more numerous. The spectrum of *Arcturus* strongly resembles that of sun-spots, and the enhanced lines are weaker than they are in the sun. The immense number of suitable lines available for measurement in the spectrum of *Arcturus* has made its study of especial interest.

In a summary given below I have collected the results of the measures of the arc and the enhanced lines for the three stars.

TABLE I

Plate	No. Meas.	Region	Enhanced Lines		Arc Lines	
			Δ	Number	Δ	Number
Sirius						
α 20	2	λ 4450	+0.26	37	−0.38	36
25	2	4550	+0.38	36	−0.28	49
27	2	5600	+0.75	6	−0.21	25
34	1	4550	+1.28	37	−0.60	51
41	1	4600	+0.41	33	−0.43	33
Procyon						
α 29	2	λ 4550	+0.37	33	−0.16	108
35	1	4600	+0.11	23	−0.32	60
36	1	4550	+0.49	37	−0.24	132
42	3	4700	+0.42	20	−0.17	49
Arcturus						
α 46	1	λ 4600	+0.13	34	−0.03	156
Mean of 7 plates			+0.03	52	0.00	559

The quantities given under Δ are in kilometers, and represent the differences between the mean of all the lines measured and the enhanced and the arc lines respectively. The positive sign denotes displacement toward the red: thus, in the first case, the value +0.26 indicates that the average velocity given by all the enhanced lines differs from the mean of all the lines measured by 0.26 km. If the measured velocity is negative the enhanced lines show a

smaller numerical value, if positive a larger. In other words, the enhanced lines are displaced to the red relative to the arc lines. The plates have been measured by Miss Lasby, Miss Ware, or myself, and the table indicates whether any given plate has been measured by one or more of us. One plate in the less refrangible part of the spectrum of *Sirius* has been omitted, as the enhanced lines are too few in number to give a determination of any material value.

In the case of *Arcturus* the results from a considerable number of plates are combined in order to obtain a sufficient number of enhanced lines to give a determination of reasonably high weight.

The persistence of the positive sign in the values for the enhanced lines for *Sirius* and *Procyon* is certainly remarkable, and in view of the difficulty of the measurements the absolute values of enhanced—arc lines, with the exception of one plate of *Sirius*, are tolerably accordant. The very large value given by α 34 is probably due mainly to the inferior quality of this plate and the fact that only one measurement is available for it. If we combine the results, assigning weights according to the number of measures and the number of lines, we obtain the following final values:

Sirius: Enhanced—Arc Lines $\Delta = +0.90$ km $= +0.014$ Ångström,
Procyon: Enhanced—Arc Lines $\Delta = +0.58$ km $= +0.009$ Ångström,
Arcturus: Enhanced—Arc Lines $\Delta = +0.08$ km $= +0.001$ Ångström.

The value for *Arcturus* is of comparatively low weight, and mainly interesting as being practically negligible in size.

Before discussing these values further I wish to add the results obtained from an investigation of the lines of different elements in the spectrum of *Arcturus*. As already stated, the spectrum of this star resembles strongly that of a sun-spot, a large portion of the lines of titanium, vanadium, and calcium being greatly strengthened, the enhanced lines weakened, and those of iron and chromium either strengthened or weakened according to their behavior under different conditions of temperature. The only apparent exception seems to be nickel, the lines of which are rather more prominent in the spectrum of the star than in sun-spots.

The following short table shows the values derived for the lines of the different elements from a combination of all the plates.

The weights are obtained by adding together the products of the number of lines measured by the weights of the separate plates. As in the case of the enhanced lines, the quantities represent the differences between the mean values given by all the lines and the mean values for the lines of each element. The positive sign denotes displacement toward the red.

TABLE II

Element	km	Angström	Weight	No. Lines
H.	−1.2	−0.020	22	8
Ca.	−0.70	−0.017	209	78
Mg.	−0.68	−0.011	54	22
V.	−0.24	−0.006	162	58
Ti.	−0.23	−0.006	451	165
Ni.	−0.22	−0.006	107	42
Fe.	+0.25	+0.006	1222	440

According to the results of this table the lines of iron are shifted toward the red with reference to those of all other elements investigated. Among the latter titanium, vanadium, and nickel fall in one group as regards differences from the mean, calcium and magnesium in another, while hydrogen gives the largest value of all.

A definite conclusion as to the reality of these differences, as well as those found for the enhanced lines, must of course be determined by the degree of precision obtained in the measurement of the plates. A calculation of the probable error of a single line from average plates of *Sirius*, *Procyon*, and *Arcturus* gives the following results:

Sirius, 37 enhanced lines $r = \pm 1.00$ km,
Procyon, 26 enhanced lines $r = \pm 0.69$ km,
Arcturus 51 iron lines $r = \pm 0.51$ km.

The probable error accordingly for the mean of a group of 30 lines for *Sirius* will be ± 0.18 km, for *Procyon* ± 0.13 km. If we enter the tables of the probability integral with these values we find approximately: for *Sirius*, the probability that on a single plate two groups of lines, each 30 in number, will differ from each other by 0.64 km owing to errors of measurement is about 1 to 25; for *Procyon*, that two similar groups will differ by 0.43 km the probability is 1 to 17. These values are for single plates, and for the series of plates the probabilities are greatly decreased.

In fact, for the mean values of +0.90 km for *Sirius*, and +0.58 km for *Procyon*, the probability is less than 1 to 1,000 in each case.

A similar calculation in the case of *Arcturus* is of interest as indicating which of the differences from the mean found for the different lines are to be considered as least liable to be due to accidental errors of measurement. If we assume the same degree of accuracy of measurement for the other elements as for iron we find that the probability of the reality of the differences is strongest in the cases of hydrogen, calcium, magnesium, titanium, and iron. While these considerations are largely theoretical, they serve the purpose of indicating the great strength of the presumption that the majority of the differences found for the enhanced lines in *Sirius* and *Procyon*, and among the lines of different elements in *Arcturus*, are to be considered as genuine and not due to accidental causes.

Attention should be called to one point in connection with these results, which I at first thought might have affected the values materially, especially in the cases of *Sirius* and *Procyon*. Since the iron arc was used for comparison purposes, a large number of the stellar arc lines measured in these two stars are direct comparisons with the stronger lines in the comparison spectrum. The enhanced lines, on the other hand, are of course indirect comparisons. If any systematic effects were present in measurements on the stronger comparison lines, they might affect the average of the arc lines more than the enhanced lines. To guard against this possibility the photographs have been reduced by the aid of a residual curve with which the wave-lengths have been corrected. A comparison of the results with those obtained by linear interpolation showed no differences for the mean of arc or enhanced lines on any plate amounting to as much as 0.1 km.

We may now pass to the discussion of these results. Perhaps their most striking feature is the remarkable resemblance they bear to the displacements found in the investigation of the lines at the sun's limb. In the latter case it was found that the enhanced lines were shifted to the red relative to the arc lines. Also that the shifts of the titanium, vanadium, and calcium lines were much less than those of the iron lines, while hydrogen showed no appreciable displacement. A similar shift of the enhanced lines

is found here in the cases of *Sirius* amd *Procyon*, and possibly to a very slight degree in *Arcturus*. In the last-named star, moreover, we find a precisely similar behavior on the part of titanium, calcium, hydrogen, and the other elements referred to. The only exception to the analogy is nickel, and attention has already been called to the somewhat greater prominence of this element in the spectrum of *Arcturus*. It seems altogether probable, therefore, that the cause giving rise to the systematic differences in the stellar spectra is the same as that which produces the displacements observed at the sun's limb. Since the latter have been ascribed mainly to the effects of pressure by those who have investigated the subject, we may consider pressure as the principal agent in causing the displacements observed in the stellar spectra.

According to the laboratory investigations of Humphreys and others, the arc lines of iron are shifted under pressure an average amount of 0.0025 Å per atmosphere. Now at the sun's limb the enhanced lines in the more refrangible part of the spectrum are shifted approximately 50 per cent more than the arc lines. Recent investigations by Gale of the spectrum of the titanium spark under pressure made at this observatory indicate that the enhanced lines of this element are shifted on the average about 50 per cent more than the arc lines at the same pressure. Accordingly, if we may conclude, as seems very probable, that the same relationship holds between the arc and the enhanced lines for other elements as well, we are provided with a means of determining the pressure in the atmospheres of stars whose spectra show such displacements. The enhanced lines in the spectrum of *Sirius* are shifted toward the red relative to the arc lines by 0.014 Ångström. This would correspond to a pressure of 12 atmospheres, or, since solar wave-lengths are used for the comparison lines, the pressure in the atmosphere of *Sirius* would be 12 atmospheres greater than that in the sun's reversing layer. Similarly, in the case of *Procyon* we obtain a pressure 7 atmospheres greater than that in the reversing layer. These results are especially interesting when discussed in connection with the spectra of the two stars. The hypothesis has sometimes been suggested, based on considerations of average density and intrinsic luminosity, that no true photosphere of the solar type

exists in the case of *Sirius*, but rather that the star consists of a vast mass of gas increasing in density toward the center but with no actual surface of condensation. In such a star it is evident that the light could come from regions extending to great depth in which the pressures are high, and this appears to agree with the results found. Similarly, in the case of *Procyon* the character of its spectrum indicates a condition between *Sirius* and the sun, and we find the amount of pressure intermediate between the two.

The results for *Arcturus* are of quite a different sort but of considerable interest. Although its spectrum is almost identical with that of a sun-spot, it is hardly probable that this is due to its being covered with sun-spots, but rather to the fact that the general conditions in its atmosphere are similar to those in sun-spots. Accordingly, we should expect that this star would have a definite photosphere, and that the light emitted by it would come from comparatively shallow depths, and so from regions of relatively low pressure. Furthermore, the results found for the lines of different elements in *Arcturus* indicate a general arrangement of the gases in the star's atmosphere very similar to that in the sun. Thus we know that in the sun hydrogen gas rises to very great heights, and the gases of magnesium and calcium (leaving out of consideration in the latter element the H and K lines) also attain a high level, though somewhat lower than hydrogen. Similarly, titanium is a relatively high-level element, but iron is low. In *Arcturus* we find the lines of all of these elements giving wave-lengths shorter than those of iron, the greatest difference being for hydrogen, the next for magnesium and calcium, and the least for titanium. It seems reasonable, therefore, to conclude that the lines of these elements are subject to less pressure than are those of iron, and consequently that the gases producing them lie at a higher average level.

The latter part of this discussion is naturally somewhat speculative in character, particularly as regards the magnitude of the pressures in the stellar atmospheres. This investigation serves, however, to indicate what may prove to be a satisfactory method of attack on this interesting problem.

MOUNT WILSON SOLAR OBSERVATORY
December 1910

THE CATHODE-RAY FLUORESCENCE OF SODIUM VAPOR

By R. W. WOOD and R. H. GALT

The vapor of metallic sodium, because of its remarkable optical and spectroscopic properties, has been the subject of an extended investigation by one of the present writers, which has furnished much information regarding the mechanics of molecular radiation. When subjecting the vapor to an electric discharge, a new spectrum was accidentally discovered entirely different from anything ever seen in the flame, arc, or spark, and appearing to have no relation to the absorption, fluorescent, or magnetic rotation spectra. Cathode rays were found to excite, in addition to the green fluorescence stimulated by white light, a curious banded spectrum in the red, orange, and yellow region. A report of preliminary work on this spectrum was published in the *Philosophical Magazine* ([6] **15**, 581, 1908) and in the *Proceedings of the American Academy of Arts and Sciences* (November 1908). The present paper is a further study of this spectrum.

APPARATUS

The apparatus was essentially that described in the papers referred to. The vacuum tube was made of steel, 36 cm long and 3.3 cm in diameter, and was connected by a side tube to a mercury pump. As shown in the diagram, the steel tube itself formed the negative electrode; the positive electrode was a nickel wire carried in a glass cap cemented with sealing wax to one end of the steel tube. The electric discharge passed from the end of the nickel rod, inside the steel tube. The opposite end of the steel tube was closed by a glass or qu artz window.

The current was obtained from an induction coil, built by the Roentgen Co. of Philadelphia, capable of sparking ten inches in air. The potential could be adjusted over a very wide range a (from $\frac{1}{4}$-inch spark to a 10-inch one) by almost infinitesimal steps. For

any given spark-gap, the tension on the interrupter could be so adjusted that the sparks were intermittent, perhaps one for every four or five interruptions of the current, dependent on very slight accidental variations in the resistance of the contact points.

Two spectroscopes were used. To obtain the general characteristics of the spectrum, a quartz spectrograph, made by Fuess, was employed; for accurate wave-length determinations, a Rowland concave grating of six feet radius of curvature, 15,000 lines to the inch, was used. The plates used for the red and yellow were Cramer's new "Spectrum" plates; for the green and blue, Wrattan and Wainwright's "Green Sensitive" plates.

The steel tube was heated by means of a single bunsen burner, the use of a hotter flame being prohibited by two facts: the higher the temperature, the longer was the column of vapor, and hence the greater the absorption; and, with too intense heating, it was impossible to keep the ends of the tube cool enough to prevent a fog from forming on the window. Cooling was effected by pouring cold water on cotton batting wrapped around the tube near the ends.

To obtain the spectrum, a piece of clean sodium (about 8 cubic cm) was placed in the middle of the tube, and the pressure reduced to a few millimeters of mercury. The flame was then applied and the pumping continued until most of the occluded hydrogen was removed. When the pressure was 2 mm or higher, if the current was turned on from the coil the ordinary secondary spectrum of hydrogen was obtained whether the tube was heated or cool. If the tube was kept hot at a little less than 2 mm pressure and the pressure gradually reduced, the hydrogen glow began to give way to the sodium discharge. A bright yellow ring of light, touching the inner wall of the tube, appeared over the molten sodium, and as the pressure decreased, this light became more and more intense, contracting toward the center of the tube as the cathode dark space widened. The whole of the discharge took place through the sodium vapor. The sodium light was most intense when it was scarcely possible to draw off any more gas through the pump. Then the discharge, viewed from the window at the end of the tube, appeared as an orange-yellow disk of almost insupportable brilliancy, usually completely detached from the wall of the tube.

This luminous disk evidently was the negative glow seen end-on, it being a solid cylinder along the axis of the tube, over the sodium. An image of the luminous disk was thrown on the slit of the spectroscope by means of a lens.

The nature of the discharge in the tube varies with the temperature, degree of exhaustion, and the potential applied to the electrodes. At a comparatively low temperature, when the yellow light of the sodium first appears, the negative glow had a curious appearance when viewed through the window in an oblique direction. There was an orange-yellow spot near the window, behind which the column of vapor was non-luminous; in the vicinity of the positive electrode on the other side of the dark region there was a green spot

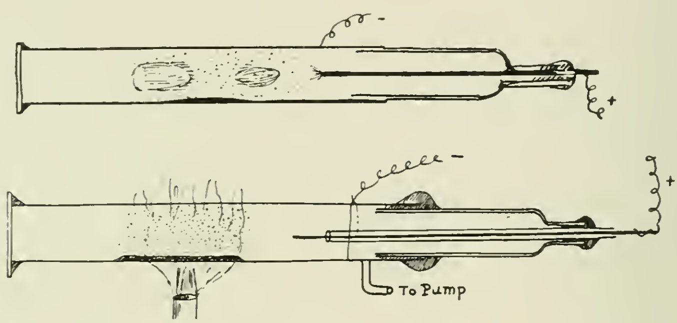
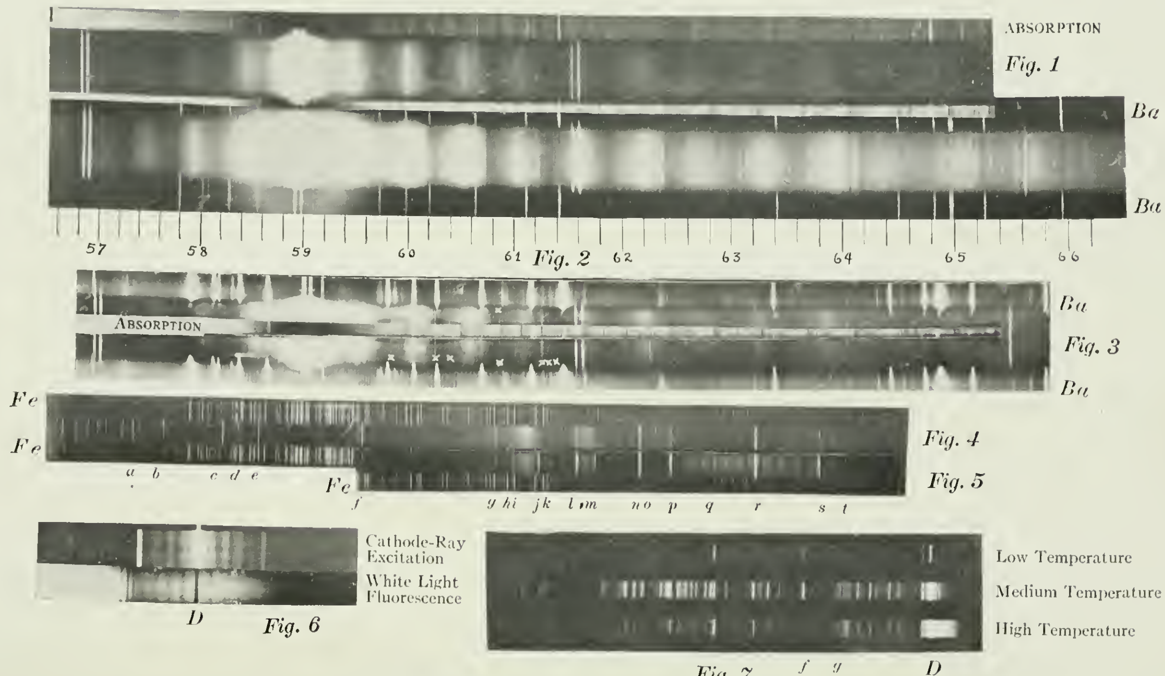


Fig. 1

of light (Fig. 1, upper figure). Viewed from the other end of the tube this spot of light was seen to be orange yellow, its apparent green color when seen through the window being evidently due to absorption. The reason why the central part of the column of vapor was non-luminous is not at once evident; but it may be that the current passed almost entirely through the vapor at the two ends of the column, where the vapor was diluted with hydrogen, the denser vapor carrying practically none of the current. This would presuppose that the sodium vapor conducts better if it is mixed with hydrogen. It is possible too that in the region where condensation is taking place the electrical properties of the vapor differ from those of the superheated vapor. Quite recently (*Physikalische Zeitschrift*, **24**, 1130, 1910) J. Koenigsberger has drawn

PLATE VI



CATHODE-RAY FLUORESCENCE OF SODIUM VAPOR

Wave-lengths corresponding to letters in Figs. 4, 5, and 7:

a, 3285	*e*, 3631	*i*, 4344	*m*, 4544	*q*, 4862
b, 3371	*f*, 3914	*j*, 4392	*n*, 4668	*r*, 4983
c, 3533	*g*, 4278	*k*, 4421	*o*, 4710	*s*, 5154
d, 3576	*h*, 4324	*l*, 4497	*p*, 4752	*t*, 5221

attention to the circumstance that with pure mercury in a very highly exhausted tube no discharge takes place even at a temperature of 150°, with a potential of 100,000 volts, either direct or alternating. It thus appears that metallic vapors do not conduct well unless mixed with small traces of some other gas.

CHARACTERISTICS OF THE SPECTRUM

1. *Red and yellow bands.*—Figs. 1 and 2 of Plate VI are from photographs made with the grating and show that the red and yellow portion of the cathode-ray fluorescent spectrum consists of a number of bands, sixteen of which are distinguishable on the negative. The negative of Fig. 1 was exposed for 30 minutes, that of Fig. 2 for three hours; with a longer exposure more bands could probably be obtained, since faint lines run out as far as λ 6864.

TABLE I

RED AND YELLOW BANDS

Band	Wave-Length	Difference	Band	Wave-Length	Difference
I.	6565		X.	6053	
		50 Å			53 Å
II.	6515		XI.	6000	
		60			54
III.	6455		XII.	5946	
		67			52
IV.	6388		XIII.	5894	
		60			51
V.	6328		XIV.	5843	
		52			51
VI.	6276		XV.	5792	
		58			48
VII.	6218		XVI. . . . *	5744	
		54			44
VIII.	6164		XVII. . . *	5700	
		52			
IX.	6112		Mean.		54.7
		59			

* Bands not well defined.

The distance between the bands is approximately equal in a normal spectrum (see Table I), the mean difference between the centers of intensity of two adjacent bands being 54.7 $\mu\mu$. It is of course impossible to designate the exact center of any band. It is interesting to note that this spacing is of the order of magni-

tude of the spacing of the resonance fluorescent lines[1] excited in this region by the lithium lines λ 6104 and λ 6708.

These red and yellow bands are much less symmetrical than the green fluorescent bands excited by white light. The lines in these cathode-ray bands are irregularly distributed, giving no suggestion of a head or tail.

The best conditions for obtaining these red and yellow bands were: a high vacuum (about 0.1 mm); a fresh supply of sodium in the tube; high potential discharge; high temperature. The high potential was especially important. When the potential was rather low, nothing except the D lines and the lines of the subordinate series appeared; but when the e.m.f. of the coil was gradually increased, a critical value seemed suddenly to be reached at which the banded spectrum flashed out at almost full brilliancy. This e.m.f. was such as to give a spark of about seven inches in air. At high temperatures and low pressures it was sometimes found that the banded spectrum could be obtained with moderate potentials, but the brilliancy was not very great. On gradually increasing the potential, a critical point was reached at which the negative glow suddenly flashed with about tenfold intensity, becoming so bright as to be painful to the eye. If the potential was held at approximately this point, the bright discharge flashed intermittently, say once for every four or five vibrations of the interrupter, showing that the very slight accidental changes in the potential were sufficient to determine the occurrence or absence of the very brilliant discharge. It looks very much as if, for a given condition of the tube, a certain definite potential brings about a very complete ionization of the vapor, which enables it to carry a large current. If the potential drops below this value, by ever so small an amount, the ionization instantly falls to a very low value. No measurements were made of the amount of current flowing through the tube in the two conditions, a matter which should be investigated.

The red and yellow bands are made up of some 600 lines, the wave-lengths of which have been measured as a matter of record. In the present state of our knowledge regarding band spectra it

[1] R. W. Wood and F. E. Hackett, *Astrophysical Journal*, **30**, 339, 1909.

does not appear to be worth while to publish this table. Until such spectra can be decomposed into simple series, such as the resonance spectra discovered by one of the present writers, tables of the wave-lengths of the lines are of little or no value in adding to our knowledge. A comparison of these red and yellow bands with the absorption spectrum shows that there is no relation between the two spectra. Fig. 1 shows the spectra side by side, the barium arc lines being superposed upon the absorption spectrum. Fig. 1, on which the absorption spectrum was recorded with the cathode-ray spectrum, has been mounted in coincidence with Fig. 2, overlapping it a trifle in order to bring the absorption spectrum of Fig. 1 into position for comparison with Fig. 2, which shows the complete band spectrum. Short exposures were necessary to secure detail in the vicinity of the D lines, where the bands have a very great intensity. In Fig. 3 the absorption spectrum runs through the center of the cathode-ray spectrum.

A curious relation exists between the cathode-ray spectrum and the fluorescent spectrum excited by white light when the pressure of the nitrogen in the tube is about 1 cm. Fig. 6 shows that the *dark* bands of the cathode-ray spectrum coincide with the *bright* bands of the fluorescent spectrum. An entirely different fluorescence, excited by white light, in no way related to the cathode-ray spectrum, is obtained from sodium vapor when the nitrogen is at a pressure of only two or three millimeters, a matter which will require further investigation.

2. *Green bands.*—The portion of the cathode-ray spectrum lying between λ 5300 and λ 3053 is shown in Figs. 4 and 5 (Plate VI). A series of fourteen bands lies in the green, between λ 5200 and λ 4750. It is found that these coincide with the green fluorescent bands excited by white light. They appear under the same conditions as the red and yellow bands just described. Fig. 5 was made with a denser vapor than Fig. 4, and has been pasted over the iron comparison spectrum of the latter.

The coincidence of the green cathode-ray bands with the green fluorescent bands, together with the lack of agreement between the two spectra in the red, suggests that the appearance of the green bands in the cathode-ray spectrum may be a secondary effect. It

seems probable that the cathode rays excite the lines of the subordinate series (as shown in Fig. 5), which lines in turn excite by resonance the green fluorescence. The lines of the subordinate series between λ 5200 and λ 4500 are so numerous that the resonance spectrum due to them would appear almost identical with the white-light fluorescence spectrum. The absence of the resonance spectrum in the yellow and red is due to the fact that there are so few strong exciting lines in that region. This hypothesis as to the secondary origin of the green bands is supported by the fact that their intensity is much less than that of the red cathode-ray bands.

3. *Blue and violet bands.*—These bands are shown in Fig. 4; the blue band lying between λ 4541 and λ 4498, the violet band between λ 4390 and λ 4340. A weak continuation of the violet band extends as far as λ 4210. These bands are unresolved, and as we have been unable to identify them with any bands previously known, we feel disposed to refer them to sodium.

Since neither of these bands is resolved, they must be totally different from the bands found by Zickendraht[1] in this vicinity. His bands were in the spectrum of the negative pole of an arc burning in sodium vapor, at a pressure of 26 mm. He was enabled to resolve his bands with a prism giving about one-fourth the dispersion available in the present case.

Fig. 5 shows that these two bands appear under the same conditions as the red and green bands, for though the red bands do not appear in the figure, they are always very strong if there is any trace of the green bands.

4. *Ultra-violet bands.*—There are two other bands in this spectrum which lie at λ 3914 and λ 4273, each one of which consists of 10 or more well-defined lines. These are without doubt the nitrogen bands. In the plates taken with the quartz spectrograph these bands are unresolved. In some of these plates they are present with varying intensity; in other plates they are absent. There is a very strong line a little to one side of each band, which may be the head of the band. Fig. 4 shows that these bands come out strongest when the temperature is not high enough to give the other bands strongly; i.e., they are not present when the sodium vapor

[1] *Annalen der Physik*, **31**, 233, 1910.

is made dense by heating beyond a certain point. Even under these conditions they did not always appear. Their absence cannot be referred to absorption, since sodium vapor is extremely transparent in this region as has been shown by one of the present writers. The variable appearance is doubtless due to the fact that

TABLE II

λ	Intensity	λ	Intensity	λ	Intensity
5221.0	1	4308.7	1	3582	2
5153.7 } 2 S	4	4278.2	6	3576	5†
5149.2 } 2 S	3	4273.0	1	3536.5	2
4983.5 } 1 S	10	71.5	1	3533.3	10
4979.3 } 1 S	10	70	1	3371	8†
4862	1	68	1	3327.6	2
4752.2 } 2 S	5	65 *	1	3318.2	2
4748.3 } 2 S	4	62	1	3304.8 ‡	3
4710	1	59	2	3302.8 ‡	3
4668.4 } 1 S	9	56	1	3285.4	8
4664.7 } 1 S	8	52	1	3274.1	3
4544.8 } 2 S	3	4248	1	3257.9	6
4541.3 } 2 S	2	4236	2	3235.0	2
4497.3 } 1 S	6	3914	10	3225.8	1
4493.8 } 1 S	5	3908.7	2	3213.6	4
4490	1	3907	3	3189.4	4
4484.5	2	3905	3	3169.2	1
4481.5	2	3903 *	3	3163.9	4
4454.5	1	3901.3	3	3158.8	2
4448	1	3898.8	3	3149.0	3
4421.9 } 2 S	2	3896.6	2	3134.9	3
4418.6 } 2 S	1	3894.0	1	3128.9	6
4404.8	1	3884.8	2	3092.3	8
4392.8 } 1 S	4	3882.6	2	3078.9	1
4389.4 } 1 S	3	3805.1	3	3077.4	3
4344.0 } 2 S	3	3754.8	2	3073.9	2
4341.1 } 2 S	2	3710.8	5	3055.4	3
4324.7 } 1 S	3	3631.6	8	3052.9	3
4321.4 } 1 S	2				

* Nitrogen. † Winged. ‡ Principal series.

in some cases air was admitted to the tube during the progress of the experiments.

5. *Lines.*—In addition to the bands we have numerous bright lines in the sodium spectrum. Evidently a large number of these lines belong to the first and second subordinate series.

In the ultra-violet there is a series of lines some of which coincide apparently with the spark lines while others appear to be new.

These ultra-violet lines are shown by Fig. 7 (made with quartz spectrograph) to be strongest when the bands are weak, i.e., at moderate temperatures, a circumstance resulting probably from absorption. The wave-lengths were measured on the grating photograph (Fig. 4) and are as shown in Table II, the lines of the first and second subordinate series being designated by an *S*.

At a certain stage in the warming of the sodium the band spectrum was found to contain a number of strong lines in the orange which turned out to be the secondary lines of hydrogen. When a pressure of about 1 mm was reached, just when the yellow sodium light began to appear, these hydrogen lines came out strongly. Usually they disappeared as soon as a slightly higher vacuum was obtained; but once they were present for a considerable length of time and were recorded on a plate together with the sodium spectrum. This hydrogen spectrum could be obtained with the tube cold as in an ordinary hydrogen vacuum tube. In Fig. 3 these hydrogen lines appear (each marked with an *X*) superposed on the cathode-ray spectrum.

SUMMARY

A new spectrum is obtained from the negative glow of a vacuum tube containing sodium vapor. This spectrum shows the D lines, the first and second subordinate series, the green fluorescent bands excited by white light, a series of symmetrical bands in the red and yellow, two unresolved bands in the blue and violet, and a large number of new lines, some of which may however coincide with the spark lines. The red and yellow bands and the violet bands have been found in no other spectrum of sodium vapor.

Johns Hopkins University
December 1910

ON THE RATIO BETWEEN THE DIAMETER OF THE PHOTOGRAPHIC IMAGE OF A POINT AND THE EXPOSURE WHICH PRODUCED IT

By C. E. KENNETH MEES

It is well known that if a photograph be taken of an extremely small object, such as a star-image, or a spectral line, then the diameter of the image produced upon development depends upon the exposure given. This diameter is, even with short exposures, greater than the diameter of the star-image or spectral line, the increased diameter being produced by the phenomenon known as irradiation, due to the scattering of the light by the particles of silver bromide composing the film of the plate.

This increase of diameter with exposure has been used by astronomers as the basis of a system of photographic photometry, the magnitude of faint stars being deduced from the relation between the diameters of their images and of the images of stars of known magnitude.

The formula which has been generally adopted as expressing the relation between intensity and diameter of image is that known as the Greenwich formula: $m=a-n\sqrt{D}$, which has been modified by various inserted terms to suit special conditions. From this $\sqrt{D}=a+b\log I$, where I is the intensity of the light.

This conclusion—that the square root of the diameter of a star-image is proportional to the logarithm of the exposure given—seemed so opposed to the general laws of the photographic plate, and to other work on irradiation, that the subject was investigated on a laboratory scale.

In the first place it must be premised that the diameter of a star-image is not a thing which can be measured with a high degree of accuracy. The best way of measuring it seems to be by the use of an eyepiece micrometer; deciding by the eye where the image shall be considered to start and where to leave off. The use of traveling microscopes certainly does not give results of equal accuracy.

The object for these experiments consisted of a spectroscopic slit which could be limited as to height by a *V*-plate sliding over it, so that a small, nearly square, opening could be obtained. This slit was photographed from a distance of 3 meters by a lens of 15 cm focus, the reduction on the plate being exactly 20:1. The slit was lighted by a Nernst burner and condenser and exposed for varying lengths of time, the diameter of the images on the developed plate being read as divisions of an eyepiece micrometer in a microscope. One division of this micrometer was equal to 0.0075 mm. The accuracy of reading the diameters is not greater than one division, and for great diameters rather less.

The following results were obtained: slit 7 mm long, 1/10 mm wide. The calculated diameters are inserted, calculated on the assumption that $d = a + b \log E$.

Plate Used	Exposure	Diameter	Differences of Diameters	Diameter (calculated for $\Delta D = 2\frac{1}{2}$)
Fine-grained bromide	5	4		4
Lantern plate	10	7	3	6.5
	20	9.5	$2\frac{1}{2}$	9
	40	12	$2\frac{1}{2}$	11.5
	80	14.5	$2\frac{1}{2}$	14
			(Calculated for $\Delta D = 4$)	
Process plate	5	15		15
	10	18	3	19
	20	21	3	23
	40	24	3	27
	80	28	4	31
	160	33	5	$35\frac{1}{2}$
	320	39	6	39
	640	43	4	43
	1,280	47	4	47
	2,560	52	5	51

This experiment is noteworthy because the range of exposures varies from 1 to 512. The effect of doubling the exposure is not quite constant throughout the range, being apparently 3 at first and then increasing to about $4\frac{1}{2}$ for the remainder of the scale.

Slit 1 mm wide, 7 mm long:

Plate Used	Exposure	Diameter	ΔD	Diameter (calculated for $\Delta D = 2\frac{1}{2}$)
Lantern	1	16		16
	2	18	2	$18\frac{1}{2}$
	4	20	2	21
	8	23	3	$23\frac{1}{2}$
	16	28	5	26
	32	31	3	$28\frac{1}{2}$
	64	32	1	31
	128	34	2	$33\frac{1}{2}$

Slit $\frac{1}{2}$ mm wide, 7 mm long:

Plate Used	Exposure	Diameter	ΔD	Diameter (calculated for $\Delta D = 3$)
Lantern	1	9		9
	2	12	3	12
	4	14	2	15
	8	18	4	18
	16	20	2	21
	32	23	3	24

Square aperture, 1 mm diameter:

Plate Used	Exposure	Diameter	ΔD	Diameter (calculated for $\Delta D = 2\frac{1}{2}$)
Process	$\frac{1}{2}$	18		18
	1	20	2	$20\frac{1}{2}$
	2	22	2	23
	4	24	2	$25\frac{1}{2}$
	8	26	2	28
	16	29	3	$30\frac{1}{2}$
	32	33	4	33
	64	36	3	$35\frac{1}{2}$

Square aperture, 3 mm diameter:

Plate Used	Exposure	Diameter	ΔD	Diameter (calculated for $\Delta D = 3$)
Process	$\frac{1}{2}$	35		35
	1	38	3	38
	2	40	2	41
	4	42	2	44
	8	45	3	47
	16	48	3	50
	32	53	5	53

These results seem to show quite conclusively that the diameter of the small image of a fine slit or point is proportional to the logarithm of the exposure given.

The equation used for astronomical work is therefore not based on a law of the photographic plate, but upon some modification of the true law dependent on the conditions of the formation of images in telescopes.

NOTE.—It has already been shown by Dr. W. Scheffer that, in general, the spreading of light from a point in a turbid medium is an exponential function of the intensity, and in a private communication he states that he has independently found that the diameter of a point-image is proportional to the logarithm of the exposure. His results are, consequently, in accord with those of the author.

RESEARCH LABORATORY OF
WRATTEN & WAINWRIGHT, LTD.
CROYDON, ENGLAND
December 15, 1910

MINOR CONTRIBUTIONS AND NOTES

ADDITIONAL SECONDARY STANDARDS, INTERNATIONAL SYSTEM, IN THE ARC SPECTRUM OF IRON

At the meeting of the International Union for Co-operation in Solar Research held at Mount Wilson, August 31, 1910, the Committee on Wave-Lengths was authorized to publish from time to time wave-lengths which should serve as secondary standards of the International System when these lines had been measured by three independent observers, provided the measurements were satisfactory. The committee reports now that it wishes to add to the list published in the October number of the *Astrophysical Journal* the following lines:

Adopted Wave-Lengths	Fabry and Buisson	Eversheim	Pfund
4352.741	.741	.741	.741
4859.758	.756	.758	.759
4966.104	.104	.105	.103
5302.315	.316	.316	.314
5324.196	.196	.196	.195

These lines all are taken from the arc spectrum of iron.

H. KAYSER
CH. FABRY
J. S. AMES

December 27, 1910

"ON THE TEMPERATURES OF THE STARS"

I am glad to see from Mr. Abbot's note in the November number of this *Journal* (**32**, 319, 1910) that our views have come into agreement up to the point of a title, which is not particularly important. I nevertheless wish to recall to attention that the memoir, *Temperaturbestimmung von 109 helleren Sternen aus*

spectralphotometrischen Beobachtungen, as was expressly stated therein, deals only with the determination of the *effective* temperatures; or attempts to treat of the purely formal question whether the constant T of the Kirchhoff function can be so determined that the observations in the visual part of the stellar spectrum can be properly represented.

As to the solar spectrum, the excellent bolometric measures by Abbot and Fowle on Mount Wilson were represented, in the region between $0.55\,\mu$ and $1.92\,\mu$ (excepting the portion of the energy-curve between $0.3\,\mu$ and $0.5\,\mu$, which is deformed by the numerous absorption lines), with systematic deviations of only about twice the precision attainable in spectral-photometric measurements.

The value for the sun $T=5990°$, or $T=5830°$, according as the constant of radiation c was taken as 14,600 or 14,200, is in good agreement with the results of the pyrometric measurement by Abbot and Fowle, $T=5962°$; with the pyrheliometric measures of Scheiner, $T=6200°$; and with the spectral-photometric measures of Scheiner and Wilsing, $T=5600°$.

J. WILSING

POTSDAM
December 1910

CORRECTION

In our paper on "The Velocity of the Sun's Motion through Space as Derived from the Radial Velocity of *Orion* Stars," in the July number of this *Journal*, this statement is made on p. 89:

> Our result, however, is not explained by the formula of the last-named astronomers, Hough and Halm, for that gives an identical mixture for any two points of the sphere diametrically opposite.

This statement is incorrect, and is quite inexcusable. As a matter of fact the formula of Hough and Halm gives generally a different mixture for two opposite points of the sky. We thank Professor Schwarzschild for calling our attention to the error.

J. C. KAPTEYN AND EDWIN B. FROST

October 1910

Reviews

Handbuch der Spectroscopie, Band V. Von H. Kayser. Leipzig: S. Hirzel, 1910. Large 8vo, pp. 853, with two plates and three figures. M. 48; bound, M. 52.

If the individual volumes of this work had been given separate titles, this one, which marks the fifth step toward completion of the one indispensable work on spectroscopy, would doubtless be called "The Spectra of the Elements."

With characteristic directness the author puts into his very first sentence the plan and purpose of the entire book: "In these volumes has been collected all our spectroscopic information concerning the various elements in order to give the reader the most complete picture possible of what has already been accomplished and of what remains to be done." It is, in fact, the first orderly and comprehensive summary and discussion of results in a science which has already celebrated its jubilee.

The mode of treatment is briefly as follows. The elements are taken up in the alphabetical order of their symbols, the first fifty being covered by the present volume. Of these, six are discussed by Professor Kayser's friend and former colleague, Professor Konen of Münster.

Under each element is given, first, the bibliography of the spectra of that element. Following each citation comes generally a single word—sometimes two—such as "spark," "self-reversal," "banded spectra," "measurements"—giving the keynote of the paper referred to. These references sometimes exceed two hundred for a single element. They are not absolutely, but essentially, complete.

Next comes the discussion of results, occupying for a single element anywhere from one to forty pages. Here, in general, one finds an orderly historical presentation of what is known concerning the line spectra, in arc, spark, and flame, the banded spectra, and the spectra of compounds of this particular element. By means of bracketed numerals references are given, in the text, for practically every statement of fact. The reader is thus at once in possession of the author's authority, a feature which illustrates the impartial, judicial, and scientific spirit which pervades the entire work. On the other hand frank expressions

of opinion and directness of speech are not wanting. Witness the following from a single page: "In my opinion this is entirely incorrect." "The conclusion, that . . . . has two spectra, is thoroughly unjustified." ". . . . . claims. . . . . It is certainly not true."

The whole discussion deals with the successive accomplishments of individual men. Our knowledge of the spectra of each element is presented as a series of human achievements; and in this regard has much of the interest which attaches to political and literary history.

Now and then, though rarely, some hitherto unpublished facts are slipped in, as for instance Eversheim's accurate measures on helium.

To follow Professor Kayser's summary concerning any one of the more important elements, such as potassium, iron, or calcium, is, in a certain sense, to follow the whole history of spectroscopy. For under each such element one is sure to meet remarks on the various sources of radiation, the different instruments of dispersion, the methods of recording and measuring spectra, and the logic of inferences to be drawn from direct observations. All these combine to make the volume much more than a catalogue of the ships. One finds it, on the contrary, a series of sketches representing spectrum analysis from as many points of view as there are important elements. In following these comments, one is forcibly reminded of how much room remains for honest difference of opinion—or, better, for suspension of opinion—upon many of the fundamental questions of spectroscopy: for instance, the origin of the so-called "aluminium-oxide bands," or even the origin of banded spectra, in general.

The third and last step in the treatment of each element is the tabulation of wave-lengths. The superficial reader will here be impressed by the fact that a considerable number of obsolete and imperfect measures are included in these tables. But all these have a qualitative value as showing just what a certain observer saw under his working conditions. Besides it must be admitted that, so far as mere precision is concerned, *all* wave-length determinations previous to the beginning of the present century, with the exception of Michelson's interferometer measures, are obsolete.

Apparently the only element in this entire volume for which the wave-lengths are given consistently and with any completeness in terms of the International Ångström is that of mercury by Stiles.

The appearance of the next volume of this compendium will, in a certain sense, complete the literature of a definite era in the history of

metrical work on spectra. In this era the publication of Kirchhoff and Bunsen's researches in 1860 may be said to form the first chapter.

The appearance of Ångström's map of the solar spectrum, with its introduction of the tenth-meter, in 1868 marks the second step. The third chapter begins with Rowland's Table of Standard Wave-Lengths (1887 and 1889) and the appearance of his solar map.

We stand on the shoulders of our predecessors. And, as we now see it, the fourth chapter began with the introduction of interference methods, and in particular with Michelson's evaluation of the meter in 1892. A delay of ten or fifteen years in the acceptance of Michelson's value by spectroscopists seems to have been due to the fact that the reason for the discrepancy between grating and interferometer methods had not been explained. With the perfection of the interferometer and the adoption of an arbitrary, international standard of a new order of accuracy—say, 0.001 Å or one part in six million—the science of spectral measurement stands at the threshold of the future and at the beginning of chap. v.

Chemists will be interested in three brief preliminary chapters, pp. 11–31, in which the spectroscope is shown to have little value in either qualitative or quantitative analysis in the sense in which these terms are generally used.

A publisher who will undertake the issue of six large volumes devoted to a science so utterly "useless" (from the point of view of "the man on the street") as spectroscopy is at once a patron of, and an honor to, pure science. The joint service of publisher and author is not likely to be over-estimated.

HENRY CREW

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

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The ASTROPHYSICAL JOURNAL is published monthly except in February and August. The annual subscription price is $5.00; postage on foreign subscriptions 62 cents additional. Business communications should be addressed to *The University of Chicago Press, Chicago, Ill.*

All papers for publication and correspondence relating to contributions should be addressed to *Editors of the* ASTROPHYSICAL JOURNAL, *Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.*

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXXIII MARCH 1911 NUMBER 2

ON THE EMISSIVE POWER OF WEDGE-SHAPED CAVITIES AND THEIR USE IN TEMPERATURE MEASUREMENTS

By C. E. MENDENHALL

In practically all investigations of the radiating properties of metallic and other surfaces, the least accurate and most difficult part has been the determination of the true temperature of the radiating surface. This is true even within the range of accurate thermo-electric measurements, but more especially above this range, say from 1700° C. up, where lie the working temperatures of the various incandescent lamps, carbon, tantalum, tungsten, and osmium. The only methods so far found available for use in this extreme range are the radiation methods, total or partial, based respectively upon the law of Stefan-Boltzman or the equation of Wien. As ordinarily applied to a radiating surface, however, these give not the *true* but the so-called black-body temperature, i.e., the temperature of the radiantly equivalent black body.

The device to be described promises to be of some value because it enables one with a calibrated optical pyrometer to determine the *true* temperature of a radiating surface. It is, of course, nothing but a special scheme for obtaining the Kirchhoff black-body conditions—a black body being defined, as usual, by the conditions, a=absorptive power=1; it will have, of course, the maximum possible emissive power at any temperature. The

special scheme referred to is shown in Fig. 1, where F is a flat conducting ribbon, heated by a longitudinal electric current as shown, and folded on a line parallel to the length so that the resulting cross-section perpendicular to the current-flow is a very narrow V—say with about 10° angular opening. If the ribbon is of uniform thickness and width it will be raised to a uniform temperature by a given current, except near the ends. The inside of the V might be then expected to be a close approximation to a black body, or total radiator, since it has but a small opening and uniformly heated walls, and if this were so, observations on it with an optical pyrometer would give the *true* and not the "black-body" temperature of its inside walls. The *outside* of the V will give radiation characteristic of the material of the ribbon, and could be used to study this radiation; but before we can draw conclusions as to the temperature of the *outside* surface we must evidently consider two questions:

1. How closely does the radiation from the inside of the V approximate that of a black body at the temperature of the *inside* walls?

2. How much real temperature difference is there between the inside and outside surface of the wall of the V?

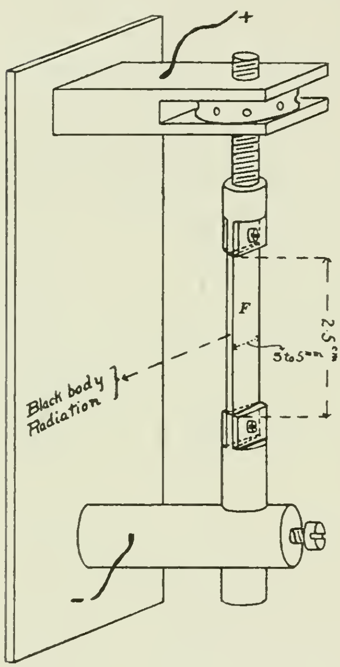


Fig. 1

The most convenient way of answering the first question follows the method used by Ch. Féry[1] in discussing the absorbing power of cones.

If AOB (Fig. 2) represents a section of the V perpendicular to its length, the walls being supposed plane and specularly reflecting,

[1] *Comptes Rendus*, 148, 777, 1909.

then a ray entering in, or nearly in, the plane of the paper will be reversed in direction and emerge after $n=\frac{180}{a}$ reflections. If the reflecting power of the surface is r, the effective reflecting power of the cavity will be r^n, and since there is no transmission the effective absorbing power will be $A=1-r^n$. Thus if $r=0.7$ (about that for a perfect surface of platinum) and $a=10^\circ$, $r^n=0.0016$, and the emissive power $E=Ae=0.998e$, where e=emissive power of a total radiator. This departure from perfect emissivity ($E=e$) would correspond for $\lambda=0.658$ to a temperature difference of less than 0°5 at 1600° C. To this degree of accuracy, then,

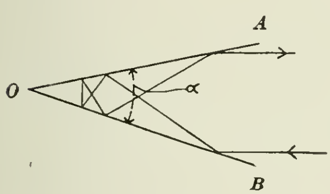


FIG. 2

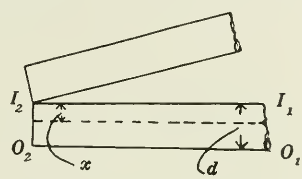


FIG. 3

optical pyrometer observations in the V aperture would give, under these conditions, the true temperature of the inside walls. If the inside surface of the wedge is matt instead of specularly reflecting, the previous conditions will not hold, but an estimate of the mean absorbing power may be obtained by considering the angle subtended by the opening at the middle point of the side, so that if the actual absorbing power of the surface is $A=0.8$ the average effective absorbing power is $A'=0.3+(0.9\times0.2\times0.8)+(0.9\times0.2)^2\times0.8+=0.97$. If $A=0.95$, $A'=0.995$. It is evident that, in order to obtain a given *effective* absorbing power with a matt surface, it must have a very much higher absorbing power (lower reflecting power) than is necessary with a specularly reflecting surface.[1]

The second question may be answered by the following consid-

[1] Féry (*op. cit.*), in computing the effective absorbing power of his cones, treats them as specularly reflecting though they are lamp-blacked. This assumption can hardly be correct, and must lead to too high a computed absorbing power.

erations. Let $I_1\ I_2\ O_1\ O_2$ (Fig. 3) represent a cross-section perpendicular to the edge of one wall of the wedge.

Assume that the heat energy is produced uniformly over the cross-section, and that none of it can escape from the inside $(I_1 I_2)$, but must pass by conduction to the outside $(O_1\ O_2)$; these assumptions will at least give an upper limit to the temperature-difference between I and O. If t represents the temperature over any plane distant x from the inside, k the thermal conductivity at this temperature, w the calories converted into heat per unit length and unit cross-section of filament, then the following equation must hold for the heat passing across unit lateral area of the plane "x":

$$-k\frac{dt}{dx}=w\,x$$

Integrating from $x=o$ to $x=d$

$$-k(t_o-t_d)=\tfrac{1}{2}\,w\,d^2$$

But $w\,d=$ energy dissipated per second per unit length and unit width of filament in calories—hence

$$-k(t_o-t_d)=\frac{1}{42}\ \frac{\text{total watts}}{\text{breadth} \times \text{length of filament}}$$

$$=\frac{W}{b\,l}$$

so that: $t_o-t_d=\frac{1}{2}\frac{d}{k}\ \frac{W}{b\,l}$.

In our case

$W=8.6,\qquad d=0.0035\text{ cm},\qquad b=0.6,\qquad l=2.8\text{ cm};$

data as to k for platinum near the melting point are not available, but using the value $\frac{.18\text{ gr. cal}\times\text{cm}^2}{\text{cm}\times{}^\circ\text{ C.}}$ corresponding to 100° C. the computed value of t_o-t_d is 0°.05 C. It is evident that even with a filament three times as thick, and supposing the conductivity at 1600° to be only $\frac{1}{5}$ as great as at 100°, the error due to this cause would be only 0°.75, which for temperature measurements above 1000° C. is not a serious error. Moreover, if the conductivity is approximately known, this error can approximately

be allowed for. For materials having very much poorer conductivity—for example, carbon—or which must be used of much greater thickness, the conductivity correction as above computed may amount to considerable and, in the absence of data on the value of k at these high temperatures, must be determined experimentally, preferably by observations on wedges with three different thicknesses of walls, graphically extrapolating to the case of zero thickness. Our observations indicate that this extrapolation can be quite accurately carried out, and it also seems probable that by comparison of the observed apparent temperature-differences between inside and outside, for various wall thicknesses with those computed as above, approximate values of the thermal conductivity k may be obtained at high temperatures beyond the range of the usual methods.

In order to test the degree to which the conditions assumed in this discussion could be easily realized in practice, I have carried out, with Mr. Forsythe's assistance, observations upon wedge filaments of pure platinum. We first determined the melting point of platinum by observing with an optical pyrometer on the inside of the wedge as the temperature was slowly raised to the melting point. The mean of several observations gave 1747° C., 8° below the correct value (1755°)—but quite a satisfactory agreement considering the impossibility of getting accurate pyrometer settings at the instant of melting. As a second test we determined the relation between the true and corresponding black-body temperature for platinum, by simultaneous observations on the inside and outside of the wedge. With very thin platinum foil (0.007 mm) it was difficult to get a properly formed wedge, but with a sheet 0.032 mm thick, burnished fairly flat and polished on the outside, the observations shown in Fig. 4 were obtained. They agree almost exactly with those of Waidner and Burgess[1] (shown by the line) which are the mean results of several methods. These results demonstrate, I think, that it is easy to realize the assumed conditions and thus to obtain accurate measurements of the true temperature of a radiating surface.

The device shown in Fig. 1 is a convenient one for use with filaments which can be heated in air, enabling them to be kept

[1] *Bulletin of the Bureau of Standards*, 3, 202, 1907.

straight and taut when heated. Turned on its side it forms an optical meldometer for determining the melting points of substances placed upon the face. Wedges of thin platinum (0.01 mm) used in air showed some difference in temperature between the "open" and "closed" side of the outer surface, due to greater loss

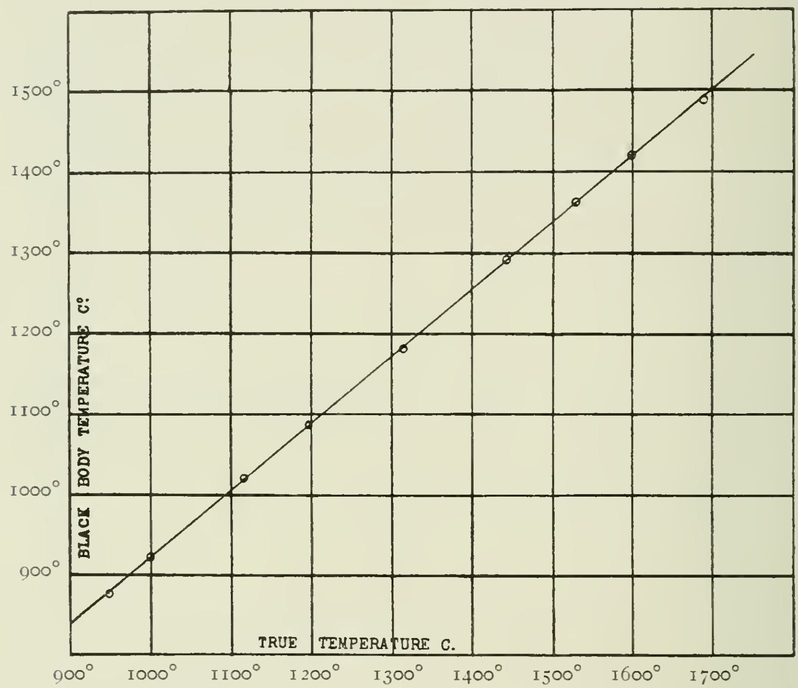


Fig. 4

of heat by convection currents at the opening. With the thicker walls (0.032 mm) this difference had almost disappeared, and was never noticeable in vacuum work. Moreover by sighting, inside and outside, at the middle of the face, this error can be minimized. If made with rounding apex these wedges often show a slight dark streak where the reflecting surface is normal to the opening, as would be expected.

SUMMARY

A discussion of the emissivity of a wedge-shaped aperture, particularly one with reflecting walls, and of the conduction of heat through the walls, leads to the conclusion that optical pyrometer observations on the inside of the wedge would give the true temperature of the outside surface. This is verified by experiment. The device is suggested as a simple means of accurately determining the true temperature of a radiating surface.

University of Wisconsin
Department of Physics
December 1910

THE SPECTRA OF SOME GASES IN THE SCHUMANN REGION

By THEODORE LYMAN

This paper contains the results of an investigation on the nature of the radiation from oxygen, hydrogen, nitrogen, helium, and argon in the region of very short wave-lengths. The writer has already published a preliminary report on the subject.[1] Taken in connection with the measurements of the spectra of hydrogen[2] and carbon[3] monoxide, the investigation is intended to complete the study of gas spectra begun by Schumann.[4]

In work on spectra of gases, even more than in most other physical investigations, the value of the results depends on the excellency of the technique. A detailed account of the methods employed, therefore, will be found at the end of this paper. The results themselves are as follows:

The cases of helium and oxygen are easily dismissed. No spectral lines belonging to these substances have been discovered in the region of wave-lengths more refrangible than λ 1900. That is to say, the radiation from these gases, if any exists, is too feeble to be detected by the methods used in this research.

There are two spectra of nitrogen in the region under consideration: the one consists of faint bands with the heads directed toward the more refrangible end of the spectrum, the other is made up of two pairs of sharp lines. The first spectrum is produced when no condenser is included in the discharge-tube circuit. The second is faintly visible under the same conditions but is very much enhanced by the introduction of capacity. No mention was made in the preliminary report of these spectra, for the band spectrum was so feeble that it escaped detection for a long time, while the line spectrum was wrongly attributed to another element.

[1] *Jahrbuch der Radioaktivität*, 4, 246.

[2] *Astrophysical Journal*, 23, 181, 1906.

[3] *Proc. Am. Acad.*, 45, 315, 1910.

[4] *Smithsonian Contributions*, No. 1413.

Argon yields no lines corresponding to the red spectrum in this region but a considerable number of lines exist in the blue spectrum. In other words, if the discharge tube contains argon at a pressure of 1 or 2 mm, and if there is no capacity in the circuit, then no lines can be observed on the more refrangible side of λ 1900. On the other hand, if a disruptive discharge is used, a considerable number of lines appear which extend throughout this spectral region. If the pressure in the discharge tube is increased to 1 or 2 cm and if the disruptive discharge is employed, the light given by the gas appears white to the eye. Under these circumstances, most of the lines disappear which were present at a lower pressure.

The spectrum of hydrogen gas has already been thoroughly investigated by Schumann and by the writer, but the existence of a primary spectrum, corresponding to that given by Balmer's formula, still remains in question. The writer has been unable to observe such a spectrum, while Schumann[1] states that, with a spark in hydrogen at atmospheric pressure, he has obtained lines which he believes belong to radiation of this type. The writer has repeated these experiments, using a spark 6 or 7 mm long with terminals of aluminum, copper, and iron in an atmosphere of hydrogen. Under these circumstances, with a disruptive discharge, the four-line spectrum of hydrogen is extremely strong and nearly overpowers the metallic lines, at all events in the visible region. Notwithstanding this, no primary spectrum lines have been obtained even after prolonged exposure.

As the writer has already mentioned in another place,[2] theoretical considerations would not lead one to expect that radiation of the Balmer type would exist in the region between λ 2000, and λ 1250, for the chief series given by Rydberg[3] lies on the less refrangible side of λ 2000, and the chief series, as given by Ritz,[4] lies on the more refrangible side of λ 1250. The first line of the latter series has a wave-length, according to Ritz, of λ 1215.3. This is in the region beyond the transparency of fluorite. It has

[1] *Astrophysical Journal*, 11, 312, 1900.

[2] *Report British Association*, 1909, p. 132.

[3] *Astrophysical Journal*, 6, 233, 1897.

[4] *Annalen der Physik*, 25, 667, 1908.

not been possible to push the investigation of gaseous spectra with a disruptive discharge into this region because of experimental difficulties. The existence of this line, therefore, still remains in doubt.

Although the writer has been unable to repeat the observations of Schumann on the lines of the Balmer type, a singular spectrum apparently connected with that of hydrogen has recently come under his notice. If argon, containing a trace of hydrogen at a pressure of 2 or 3 mm, is inclosed in a tube with aluminum electrodes, and if no capacity is introduced in the circuit, a characteristic spectrum is obtained. It consists of five groups, each group containing five lines. These groups begin near λ 1650 and extend to λ 1450. They are all similar in appearance but they are not all identical in constitution. The distance between the lines in a group is of the order of from one to four Ångström units. If the last trace of hydrogen is removed from the argon, this spectrum disappears. Nitrogen, oxygen, and helium containing a trace of hydrogen and examined in a tube with aluminum electrodes do not produce these groups. If the argon and hydrogen are examined in a tube with iron electrodes, the intensity of the groups is very much reduced; if electrodes of copper are employed, the lines are extremely feeble. Under any circumstances, they are destroyed by the introduction of capacity. The most important fact in connection with these groups is that they are always found in the spectrum of pure hydrogen, no matter how this gas is prepared nor what electrodes are employed. In this case they are superposed upon a great number of other lines but they may be readily distinguished from the rest of the spectrum. All the groups can be identified in the reproductions of spectra published by Schumann;[1] the group which lies between λ 1590 and λ 1600 is the most striking, for at this point of the spectrum it is not obscured by the presence of strong lines. That these groups are not due to some impurity common to all the electrodes employed has been proved by using terminals of very pure silver furnished through the kindness of Professor T. W. Richards. The spectrum of hydrogen obtained with these elec-

[1] *Smithsonian Contributions*, No. 1413.

trodes is identical in every respect with the other spectra of this gas. If these groups are due to some impurity in the gas itself, such an impurity must be of a very fundamental character, for it was present in all the hydrogen which the author has employed during the past five years and it was present in the hydrogen used by Schumann.

It is conceivable that groups of this type may be present in the spectrum of argon, containing a trace of hydrogen, in the visible and ultra-violet regions. At first sight, therefore, a suspicion might exist that the "Groups of Four" observed by Rydberg[1] in argon might in reality be connected with hydrogen, but a comparison of the measurements of the argon and hydrogen spectra shows that such a suspicion is without foundation.

Kayser[2] has observed lines belonging to the spectrum of aluminum in the spectrum of argon when aluminum was used for electrodes, but it is obvious from what has just been stated that the Groups of Five observed in the Schumann region cannot be supposed to have a metallic origin.

Such, in brief, are the results of this investigation. Tables of wave-lengths are to be found at the end of the paper. A detailed description of apparatus and methods follows.

In this investigation, the writer's vacuum spectroscope was employed in a manner which has been already described elsewhere. To insure the purity of the gas under investigation, the type of apparatus shown in Fig. 1 has been finally adopted. The glass discharge tube of the usual form has a ground flange at its lower end. To this flange is attached a fluorite window by means of Khotinski cement, care being taken that the cement is applied only to the outside edge of the flange in order that none of it may come in contact with the gas. This window is indicated in the figure at *A*. The tube thus closed is fastened in a brass cone *B* by means of Khotinski cement. This cone fits air-tight into a cup *C* which in turn screws on to the face-plate of the spectroscope. Thus the gas under examination does not come in contact with the brass of the mounting. This is a considerable improvement over other

[1] Kayser, *Handbuch der Spectroscopie*, **2**, 577.

[2] *Astrophysical Journal*, **4**, 8, 1896.

arrangements. If extreme purity in the gas is not a requisite, it is convenient to attach the fluorite window directly to the bottom of the cup *C*, for with this arrangement the window can be readily cleaned by simply withdrawing the discharge tube.

The inlet of the discharge tube was connected with a U-tube 40 cm long which was plunged in liquid air. This device served as an effective trap for mercury vapor and for those carbon compounds whose spectrum has proved such an annoyance in former times. The U-tube in turn was connected with a spiral some 20 cm in length, to regulate the flow of gas, and this tube communicated with a suitable reservoir containing the gas under examination and with the mercury pump.

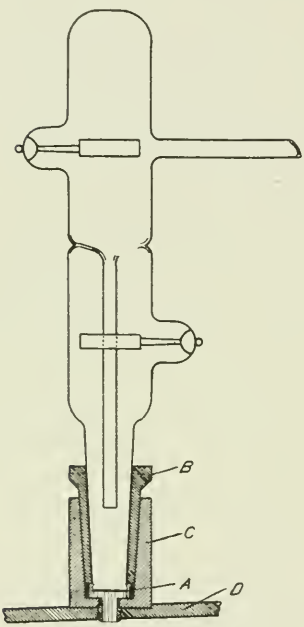


Fig. 1

The tube was excited by a small transformer, made by the Clapp Eastham Company, taking about six amperes in the primary on the 110-volt alternating circuit. The current through the discharge tube was usually between 10 and 15 milliamperes. When a disruptive discharge was desired, a spark-gap was placed in series with the tube and a condenser of 0.026 microfarad capacity was connected in parallel with the coil.

Measurements of wave-lengths were made by comparison with the hydrogen spectrum and the carbon monoxide spectrum, or with known lines of aluminum.

During the course of the experiments the appearance of the light from the discharge tube was frequently examined with a pocket grating spectroscope. The purity of the gas in the tube was estimated by the purity of its spectrum in the visible.

The gases used in this investigation have been prepared for the most part under the direction of Professor Baxter, to whom the writer is greatly indebted.

The oxygen was prepared by heating solid potassium permanganate, the gas was passed through plugs of glass wool, over platinized asbestos, through a wash-bottle and a tower containing potassium hydroxide solution, over solid potassium hydroxide, and finally over phosphorus pentoxide. The apparatus was constructed entirely of glass.

Some trouble was experienced in bringing the discharge tube into a condition which would remain constant from a spectroscopic point of view. A pressure of 5 mm was finally used. In addition to the precautions taken in preparation, the gas was sparked directly before it was admitted to the apparatus. Aluminum electrodes were used. No capacity, beyond that due to the leads, was included in the discharge-tube circuit.

The results were entirely negative, no lines could be attributed to oxygen in the Schumann region. The data collected some years ago in a preliminary investigation confirm this result.

Two specimens of helium were examined; the one was obtained through the kindness of Professor E. P. Adams of Princeton, the other was purchased from Tyrer of London. The latter specimen contained a slight trace of nitrogen which somewhat masked the spectrum of helium in the visible at pressures above 15 mm. At pressures below 5 mm, however, the helium lines appeared without any impurity other than a faint trace of hydrogen. Exposures were made with various pressures from 17 mm to 1 mm; the results of all these experiments were the same, no lines could be attributed to helium in this region; this result is confirmed by all data collected in preliminary investigations on this gas. Aluminum electrodes were used in all cases. Both an uncondensed and a disruptive discharge were employed.

The nitrogen used in the final experiments was made from a mixture of ammonium chloride, sodium nitrite, and potassium bichromate dissolved in water. The gas passed over potassium hydroxide, through concentrated sulphuric acid, and over phosphorus pentoxide, and finally over hot metallic copper.

Owing to the heating of the discharge tube, a gas pressure of 2 mm was selected for the final experiment both in the case of the disruptive discharge and when no capacity was in the circuit.

The feeble character of the band spectrum has been mentioned

already. The persistence of the line spectrum consisting of two pairs must be remembered also, for it was this persistence which prevented these lines from being ascribed to their true cause in the early part of the work. All the gases which have been investigated will show these pairs with a disruptive discharge if they contain a trace of nitrogen or even of air. Beside these pairs, there exist faint lines in this spectrum, but their intensity is so feeble that their wave-lengths have not been accurately measured. They are not included in the table.

Three samples of argon have been examined. The first was from Tyrer, the last was prepared as follows: air was drawn through towers containing solid potassium hydroxide. It was then passed over bright copper rolls properly heated, then through a tube containing two parts of magnesium powder, five parts of freshly ignited calcium oxide, and one-fourth part of metallic sodium free from oxide. After leaving this tube the gas passed over a combustion tube of copper oxide and was then collected over a solution of potassium hydroxide. The combustion tubing was heated electrically, which permitted the temperature to be conveniently controlled. The crude argon thus prepared was repeatedly passed over a mixture of lime and metallic magnesium until no further contraction took place. The gas was then mixed with oxygen and sparked in a tube over potassium hydroxide and mercury for several days. From here it was transferred to a new system and was passed repeatedly over phosphorus pentoxide and over a bright copper roll. The resulting gas showed no trace of nitrogen, hydrogen, or oxygen when examined spectroscopically in the visible part of the spectrum.

The final experiments were made with gas prepared as above described, but the results check very well with those obtained when the specimen from Tyrer was employed. The pressure ranged from 1 to 20 mm, depending on the nature of the discharge which it was desired to produce. Electrodes of aluminum, copper, and iron were used.

The wave-lengths of the lines in the "blue" spectrum of argon are to be found in the table on p. 107. Only three lines have been obtained in the white spectrum: of these, two are also found in

the blue spectrum; the gas at high pressures, therefore, appears to yield but one new line, λ 1650.0.

It is interesting to contrast the behavior of hydrogen with that of argon. With the former gas, a characteristic line spectrum is produced when no capacity is included in the circuit, while with a disruptive discharge these lines almost completely disappear and no new ones take their place. With the latter gas, just the opposite conditions prevail. Argon gives no lines in the Schumann region when the circuit is without capacity; it is only with a disruptive discharge that its characteristic lines appear.

The origin of the Groups of Five remains undetermined. As has been stated, these groups occur in all hydrogen spectra superposed upon a spectrum composed of fine lines. They have been included in all previous measurements. It is only when a trace of hydrogen is mixed with argon that the groups appear without the accompaniment of the lines. The fact that they do not occur as a separate spectrum when a trace of hydrogen is present in nitrogen, helium, or oxygen, would indicate that they are not due to an impurity contained in the walls of the tube. Moreover, experiments have been made in which the discharge tube was heated by the passage of the current until the capillary showed the sodium lines; such treatment in no way influenced the intensity of the Groups of Five when they were present. The nature of the electrodes affects their intensity, but they are not due to an impurity common to the various metals employed. This has been shown by the fact that they persist in the spectrum of hydrogen when the electrodes are of the purest silver.

The construction of these groups will appear from the following table. The lines of which they are composed are very sharp; they do not differ in character from the rest of the hydrogen spectrum.

GROUPS OF FIVE

Group	1	Diff.	2	Diff.	3	Diff.	4	Diff.	5
I..........	1643.0	2.5	1640.5	2.3	1638.2	1.7	1636.5	2.4	1634.1
II..........	1599.4	3.2	1596.2	2.6	1593.6	2.1	1591.5	2.5	1589.0
III..........	1550.6	3.6	1547.0?	3.1	1543.9	2.3	1541.6	2.4	1539.2
IV..........	......	...	1495.5	3.6	1491.9	2.6	1489.3	2.4	1486.9
V..........	1445.2	4.2	1441.0	3.0	1438.0	2.8	1435.2	2.2	1433.0

In order of intensity these groups may be listed as follows, beginning with the greatest intensity: II, V, I, III, IV. Of lines in a single group the more refrangible are the stronger. The strongest line in the strongest group, 1589.0, has the intensity 8. Inspection of the columns which give the differences between successive lines in a group show that though the groups are similar they are not identical.

If these groups are compared with the Groups of Four in the ultra-violet, it is clear that the lines in a single group are much closer together than those in Rydberg's arrangement.[1] Even if the first lines in each group are taken as forming one system, the second lines as forming a second system, and so on, the results do not agree with the formula, for the new systems are not identical, and the frequency differences are not those given by Rydberg.

As to the cause of these Groups of Five there are obviously two opinions: either they are due to a very persistent and subtle impurity in the hydrogen, or they form a new spectrum of that gas.

The hydrogen used in the final experiments was produced electrolytically from a barium hydroxide solution; it was passed through distilled water, over solid potassium hydroxide, and was dried over phosphorus pentoxide. As to the spectrum of hydrogen there is little to add. The presence of moisture in the gas tends to enhance the primary spectrum in the visible; in the Schumann region it tends to weaken the many-line spectrum. The spectrum of hydrogen is sensitive to the smothering effect of a heavier gas. Nitrogen mixed with hydrogen weakens the spectrum of the lighter gas without contributing strong lines of its own.

The use of liquid air does away with mercury vapor in the discharge tube. When mercury is present, however, but one line can be attributed to it in this region, that at λ 1850.0.

In the tables which follow, the wave-lengths are in vacuum. Owing to the feebleness of the nitrogen bands, their position may be in error by four-tenths of an Ångström unit; the errors in the line spectra should not exceed two-tenths of a unit.

[1] Kayser, *Handbuch der Spectroscopie*, 5, 69.

NITROGEN

Band Spectrum				Line Spectrum	
λ	I	λ	I	λ	I
1383.7	3	1687.5	2	1492.8 }	6
1393.5	2	1736.9	2	1494.8 }	5
1416.1	3	1752.9	3		
1431.6	2	1768.5	3	1742.7 }	7
1464.8	4	1804.7 ?	2	1745.3 }	6
1471.1	1	1821.1	4		
1501.1	3	1837.6	5		
1515.4	2	1854.0	3		
1530.6	2	1870.9	2		
1554.4	5				
1611.8 ?	4				
1672.3	3				

ARGON

"Blue" Spectrum

λ	I	λ	I	λ	I	λ	I
1333.7	5	1604.2	4	1827.6	6	1846.9	6
1334.5	7	1607.0	3	1830.6*	10	1850.2	4
1335.8	7	1611.0	4	1831.4*	9	1855.7	9
1460.1	5	1614.8	4	(1834.5)	2	1865.9	8
1463.3	3	1669.7	7	(1835.5)	2	1868.7	8
1465.6	4	1673.5	7	1836.3	9	1873.2	10
1467.9	2	1675.6	7	(1838.1)	2	1877.7	8
1589.5	4	1788.1	5	1839.2	9	1879.7	8
1600.7	5	1807.5	4	(1842.3)	1	1886.1	7
1602.6	2	1820.0	7	1843.1	9		

* Present in the "white" spectrum. ( ) Origin uncertain.

This research has been carried out in great part with the help of a grant from the Bache Fund.

Jefferson Physical Laboratory
Harvard University
December 31, 1910

CIRCULATION IN THE SOLAR ATMOSPHERE AS INDICATED BY PROMINENCES

By FREDERICK SLOCUM

In the course of the study of the solar prominences photographed with the Rumford spectroheliograph of the Yerkes Observatory, so many of the prominences presented the appearance of being driven to one side by a horizontal current that it seemed worth while to make a special study of these cases to see if they might not throw some light upon the circulation of the sun's atmosphere.

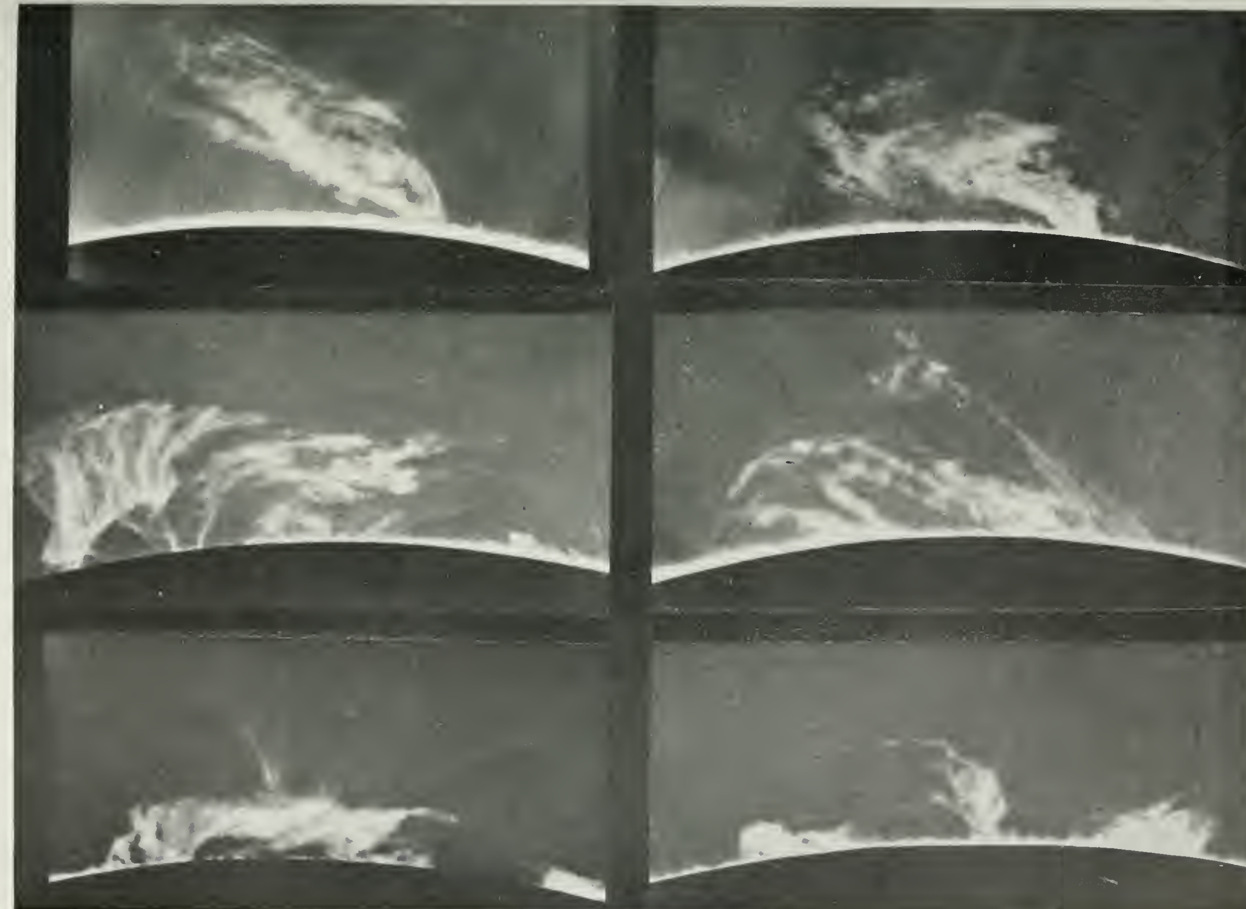
Photographs, in the light of the H line of calcium, of 3323 prominences, taken between January 1904 and December 1910, were examined, and of these, 1094, either by their shapes or movements, indicate a horizontal current. The prominences used as directional in the present investigation may be grouped under the following types:

1. Smoke-like
   a) as if rising from a fire (Figs. 1 and 2, Plate VII);
   b) as if coming from a chimney.
2. Bundles of filaments all bending in one direction (Figs. 3, 4, and 5, Plate VII).
3. Mass with streamers reaching out
   a) horizontally (Fig. 6, Plate VII, and Fig. 9, Plate VIII);
   b) curving down;
   c) curving up.
4. Clouds, detached, floating (Fig. 8, Plate VIII).
5. Treelike forms (Fig. 7, Plate VIII).

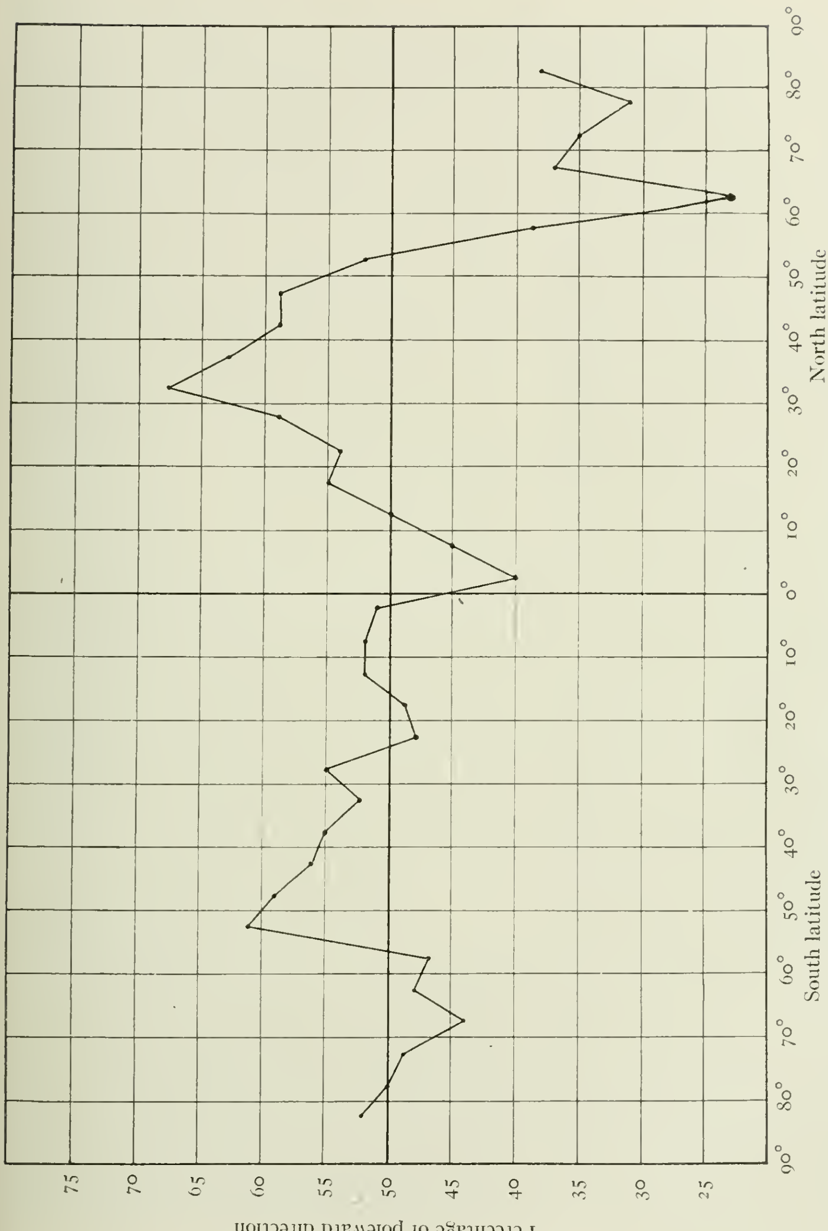
The "sky-rocket" on the extreme left of Fig. 12, Plate VIII, the "meteor" of Fig. 11, and the spike, or jet, of Fig. 10 are due to local explosions or eruptions, and prominences of this type were used only when the normal trajectory is evidently deformed by a lateral action.

The original tabulation contains the limiting latitudes of each prominence, the direction indicated, maximum height, and type of prominence. These values were summarized by counting the num-

PLATE VII



SOLAR PROMINENCES INDICATING HORIZONTAL CURRENTS



Percentage of poleward direction of solar currents as indicated by prominences

ber of directional prominences in each five-degree zone from the north to the south pole. The results are given in the accompanying table. When a prominence extended over two or more zones it was counted in each zone, so that the sum of the numbers in the table is greater than the number of individual prominences. A percentage poleward less than 50 means an average tendency toward the equator, a value greater than 50 means a tendency toward the pole.

The results are shown graphically in the diagram. The abscissas are solar latitudes, the ordinates are percentage of poleward tendency. Points above the X-axis (percentage 50) indicate a tendency toward the pole, below the X-axis a tendency toward the equator.

The 1094 prominences used are distributed in height as follows:

Limiting heights	$0'-1'$	$1'-2'$	$2'-3'$	$3'-4'$	$4'-5'$	$5'-6'$	$7'-8'$
Number	719	330	36	4	3	1	1

In the original tabulation no discrimination was made between high and low prominences. The average height to which the prominences used extend is $0'.7$, or 30,000 km, so that the table and diagram represent the average poleward components of the circulation of the sun's atmosphere from the surface of the chromosphere up to that height.

The following conclusions may be drawn from the diagram:

1. There is a tendency for movement toward the poles in middle latitudes: in the northern hemisphere between latitudes 15° and 55°, with a maximum of 69 per cent in latitude 33°; in the southern hemisphere between latitudes 25° and 55°, with a maximum of 61 per cent in latitude 52°.

2. There is a tendency toward the equator in high latitudes: in the northern hemisphere with a maximum of 78 per cent in latitude 63°; in the southern hemisphere with a maximum of 56 per cent in latitude 68°.

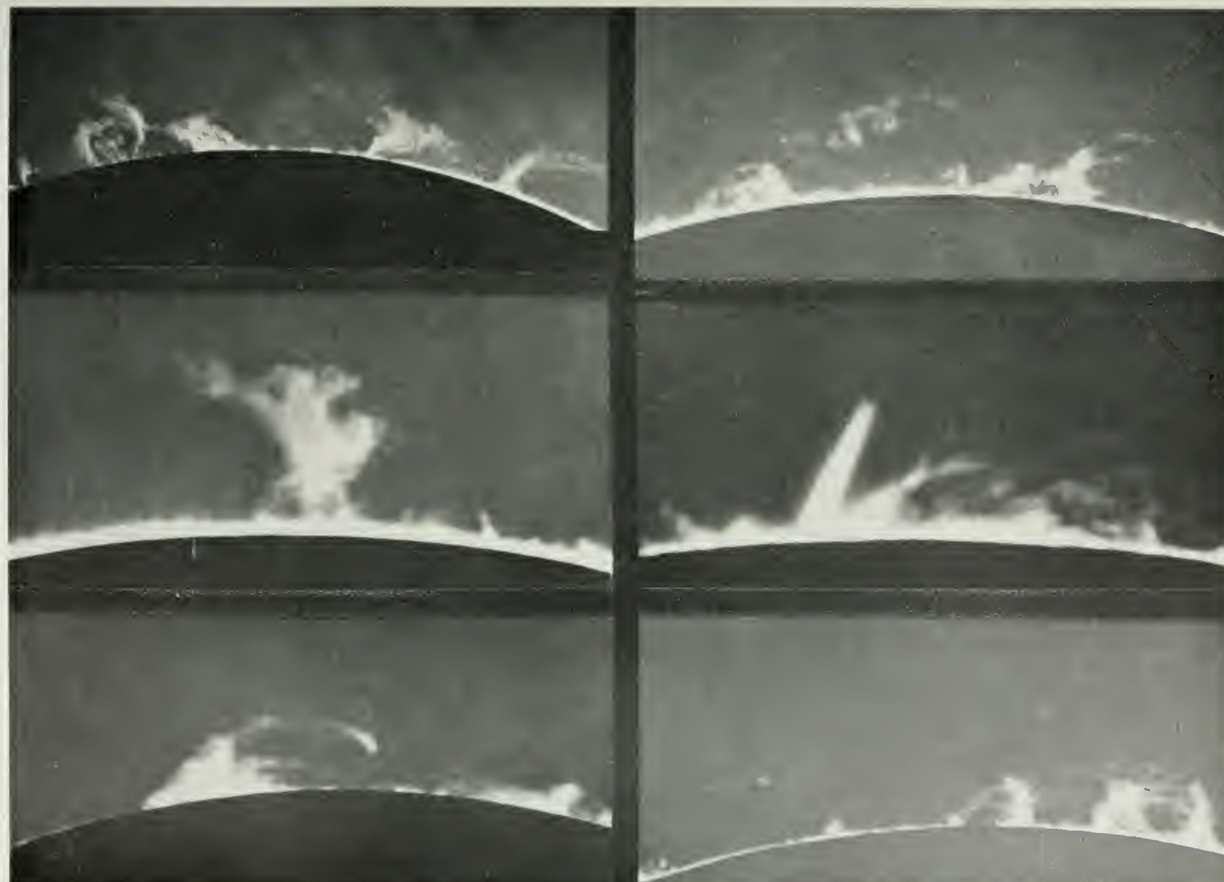
3. The conditions near the equator are practically neutral. There is a very slight apparent motion poleward from 0° to $-15°$ and equatorward from 0° to $+15°$.

PLATE VIII

7 8

9 10

11 12



Solar Prominences Indicating Horizontal Currents

4. The contrast in tendency is greater in the northern than in the southern hemisphere in a ratio of at least 2 to 1.

Very few of the earlier plates afford data for determining velocities, but during the past year at least two prominence plates have been taken each day, if possible. Some of these have already been measured. Although it is not always easy to decide which movements shall be ascribed to local eruptions and which to general

TABLE I

Apparent Currents toward or from the Poles, as Indicated by Prominences

Arranged according to heliographic latitude

Latitude	Northern Hemisphere			Southern Hemisphere		
	Number		Percentage Poleward	Number		Percentage Poleward
	−	+		−	+	
85°–90°	1	0	..	2	1	..
80–85	5	3	38	13	14	52
75–80	9	4	31	20	20	50
70–75	13	7	35	25	24	49
65–70	17	10	37	24	19	44
60–65	42	11	22	28	26	48
55–60	46	30	39	31	28	47
50–55	42	45	52	28	43	61
45–50	27	40	59	36	52	59
40–45	34	49	59	34	43	56
35–40	33	57	63	38	46	55
30–35	33	71	68	49	53	52
25–30	34	49	59	44	53	55
20–25	45	52	54	51	47	48
15–20	39	47	55	48	46	49
10–15	38	38	50	44	48	52
5–10	39	32	45	37	40	52
0–5	39	26	40	33	35	51

+ indicates direction toward the pole. − indicates direction from the pole.

atmospheric drift, there are several of the type shown in Fig. 8, Plate VIII, which retain their form sufficiently to give an approximate value of their velocity, or rather the component of that velocity which is perpendicular to the line of sight. Ten such cases give values ranging from 0.5 km to 10 km per second. One detached cloud floating at a height of 442″, or 320,000 km, shows a velocity of 50 km per second.[1]

In a recent number of the *Astrophysical Journal*, Dr. St. John has

[1] *Astrophysical Journal*, **32**, 128, 1910.

given a résumé of the study of horizontal movements in the solar atmosphere.[1] None of the methods or results there mentioned are strictly comparable with those of the present paper. The older investigations were based upon the drift in latitude and longitude of sun-spots and faculae and therefore concern movements in the photosphere. The more recent observations of Deslandres and St. John involve the determination of radial velocities by the displacement of spectral lines. Both find evidences of a vertical circulation. The former says:[2]

> L'étude simultanée des vitesses radiales montre que la vapeur descend là où sont des plages brilliantes, facules et flocculi; elle s'élève au contraire sur les parties relativement sombres qui les entourent et surtout sur les filaments qui sont les parties les plus noires. Ces mesures décèlent les courants de convection, souvent groupés en tourbillons cellulaires juxtaposés, ainsi que dans les liquides.

St. John finds:

> The calcium vapor producing the absorption line K_3 in the solar spectrum has a descending motion over the general surface of the sun of 1.14 km per second in the mean, as indicated by the progressive increase in the wave-length of the absorption line in passing from the limb to the center of the sun, where the shift toward longer wave-length amounts to 0.015 Å. . . . .
>
> The calcium vapor to which the bright emission line K_2 is due has an ascending motion over the general surface of the sun of 1.97 km per second in the mean, as indicated by the progressive shortening of the wave-lengths of the emission line on passing from the limb to the center of the sun, where it reaches a maximum of 0.026 Å.[3]

These two results are apparently contradictory, but Deslandres has confined his attention to regions of local activity, while St. John, in attempting to represent the general conditions over the surface of the sun, has avoided such disturbances as far as possible.

In order to investigate the horizontal circulation, St. John has measured the wave-lengths of the K_2 and K_3 lines near the limb in the vicinity of the equator and of the poles. From the agreement of the two sets of measures he infers the "absence of currents of

[1] *Astrophysical Journal*, **32**, 59, 1910.

[2] *Annales de l'Observatoire d'Astronomie Physique de Paris* (*Meudon*), **4**, 104, 1910.

[3] *Astrophysical Journal*, **32**, 79, 1910.

appreciable velocity parallel to the solar surface."[1] Near the pole the only current that could be detected by the spectroscope would be a horizontal current along a meridian. In any system of circulation upon a rotating sphere such a component would undoubtedly be a minimum at the poles, so the above test seems hardly sufficient to justify the general conclusion stated.

The equatorial regions should show a strong east and west component. This may account for a part of the consistent difference between the wave-lengths of K_3 derived from measures on the east and on the west limbs. This difference according to St. John is 0.0042 Å for latitude 6°6, and 0.0062 Å for latitude 38°4, the values for the west limb being the greater, indicating a westward motion at the level under consideration.

The extreme height of the absorbing calcium vapor which produces the K_3 line is about 1500 km[2] above the chromosphere, while the average height of the prominences used in the present discussion is 30,000 km, so that the conclusions stated probably apply to an upper current analogous to the terrestrial anti-trades. In a subsequent paper the material on hand will be discussed with respect to different levels, and an attempt will be made to introduce an east and west component by determining the drift of quiescent prominences in the line of sight.

TABLE II

Figure	Date	G. M. T.	Limiting Latitudes	Limb	Direction	Maximum Height		Taken by
							km	
1.....	1905 June 6	$5^h 15^m$	+30°+47°	W	P	187″	134,000	F
2.....	1905 June 5	10 6	+30 +48	W	P	150	108,000	F
3.....	1907 Oct. 2	5 37.6	+16 +44	W	E	126	90,000	F
4.....	1907 May 18	3 15	+15 +37	E	E	168	120,000	F
5.....	1907 Aug. 22	6 2.1	− 8 −25	E	E	48	34,000	S
6.....	1908 July 29	5 47.3	−14 −23	W	E	72	52,000	F & A
7.....	1907 Aug. 10	5 43.5	− 6 −28	E	E	78	56,000	F & S
	1907 Aug. 10	5 43.5	−33 −46	E	P	102	73,000	F & S
8.....	1908 July 14	4 36.4	−38 −64	E	E	96	69,000	F & A
9.....	1910 May 27	5 48.3	+ 4 +14	W	P	105	75,000	S
10.....	1908 Aug. 10	3 47.9	−20 −30	W	P	60	43,000	F & A
11.....	1907 June 29	5 27	−17 −30	E	E	72	52,000	F & S
12.....	1910 Mar. 15	7 36.6	− 4 + 2	W	P	60	43,000	S

Under Direction, P = poleward, E = equatorward.
Observers: F = Fox, S = Slocum, A = Abetti.

[1] *Op. cit.*, p. 80.

[2] *Op. cit.*, p. 81.

The accompanying table II contains data pertaining to the prominences of Plates VII and VIII. The figures are reproduced on quite different scales, but the relative amount of enlargement may be inferred from the curvature of the sun's limb, or may be accurately obtained from the dimensions given in the table. In this connection it should be noted that the data for Fig. 5 pertain to the low prominence. The jet near the center rising to a height of $2'$ is apparently in the background and entirely independent. The data for Fig. 11 apply only to the meteor-like prominence, and in Fig. 12 only the "sky-rocket" shown faintly on the extreme left is considered.

Yerkes Observatory
January 11, 1911

THE EFFECT OF TEMPERATURE ON THE IONIZATION OF A GAS

By J. HARRY CLO

The ultimate purpose of this experiment was to determine whether it is possible to change the kinetic energy of the molecule sufficiently to affect the stability of the atom. The ionization of gases was chosen as a type of the phenomena which involve the separation of the electron from the atom and which therefore depend on its stability.

The experiments made heretofore upon these phenomena may be divided into three classes, namely, those on the ionization of gases, those on the photo-electric effect of ultra-violet light, and those on the emission of electrons from radioactive substances.

H. L. Bronson[1] has shown that upon heating radium salts under conditions which eliminate radioactive transformations, volatilization of products, etc., the ionization of a gas by gamma rays is independent of the temperature of the radium. But the intensity of the gamma rays, and therefore their ionizing power, is generally believed to depend on the number of electrons given off per second from the radium. Hence it would seem to follow that the rate of emission of the electrons is not affected by the temperature. His observations extended from $-180°$ C. to $1600°$ C. and showed practically no variation greater than 1 per cent.

The expulsion of negative electrons from metals under the influence of ultra-violet light was shown to be independent of the temperature of the metal by Millikan and Winchester,[2] Ladenburg[3] and others. Millikan and Winchester made observations on different metals up to temperatures about $350°$ C. Ladenburg experimented on platinum, gold, and iridium, varying the temperatures from $20°$ C. to as high as $860°$ C. With platinum his results

[1] *Proc. Roy. Soc.*, A, **78**, 494, 1907.

[2] *Phil. Mag.* (6), **14**, 188, 1907.

[3] *Verh. der Deutschen Phys. Gesell.*, **9**, 165, 1907.

show no variation greater than about 5.5 per cent from the mean. For gold and iridium his results are even better.

The effect of temperature on the ionization of a gas has been investigated by J. Perrin,[1] McClung,[2] A. Gallarotti,[3] Herweg,[4] and Crowther.[5] With the exception of Perrin none of these experimenters found any systematic variation of the ionization. Perrin measured the ionization produced in air by Roentgen rays. Correcting for the variation of the ionization with density, he found it to be proportional to the absolute temperature. McClung investigated the phenomenon very thoroughly, using Roentgen rays as an ionizing agent. He measured the ionization in air, hydrogen, and carbon dioxide, working both at constant pressure and at constant density. His method allowed him to correct for the variation in the ionizing power of the rays. For air at constant pressure and at temperatures up to 272° C., he found the ionization to be constant to within about 6.5 per cent of the mean value. At constant density he found no greater variation in readings up to 201° C. For hydrogen at constant density his results show no variation greater than about 15 per cent up to 226° and for carbon dioxide no variation greater than 4.9 per cent up to about the same temperature. Gallarotti investigated the effect of temperature on the ionization of air at low temperatures. With X-rays his results show the ionization to be constant to within 2.5 per cent of the mean for temperatures down to −187° C. With radium he obtained measurements of the ionization at −10°, −60°, and −187° C., which did not vary more than 1.2 per cent from the mean.

These results show that there is no variation of the phenomena with temperature such as might have been expected from the results of Perrin on the ionization of gases. They show that for the temperatures considered, the rate at which the electrons are separated from the atom does not vary more than about 10 per cent in the case of solids and about 5 per cent in the case of gases.

[1] *Annales de Chimie et de Physique* (7), **11**, 496, 1897.

[2] *Phil. Mag.* (6), **7**, 81, 1904.

[3] *Atti della R. Accad. dei Lincei*, **16**, 297, 1907.

[4] *Annalen der Physik*, **19**, 333, 1906.

[5] *Proc. Roy. Soc.*, A, **82**, 351.

The kinetic theory, however, leads to the conclusion that the variation of the stability of the atom with temperature must be very slight and may even be beyond the limits of experimental determination. While the above results are consistent with this conclusion, they throw no light on the question as to whether a smaller variation takes place.

In the present experiment an attempt has been made, first, to reduce the observational errors with the purpose of measuring smaller variations than would have been detected in previous experiments, and second, to work at higher temperatures than had been employed in previous experiments on the ionization of gases.

The experiment was made upon gases for the following reasons: (1) According to the kinetic theory the molecular structure of a gas is simpler than that of solids and liquids. (2) The application of the fundamental concepts of the kinetic theory to gases has been more thoroughly demonstrated than in the case of solids or liquids. (3) The previous results on gases are not so accurate as some of the results on solids, and the experiments have not been made at as high temperatures as should be attainable.

OUTLINE OF EXPERIMENT

The observations consisted in the measurement of the ionization current in a gas within a closed vessel, by means of the rate of leak of a charge to an electrometer. An attempt was made to measure the ionization at constant pressure, but owing to variations and disturbances due to the change in density, this method was abandoned and all observations were made with the gas at constant density.

Air and hydrogen were the only gases studied. Radium was used as the ionizing agent. In all the observations recorded here the gamma rays were the only radiation entering the ionization chamber.

The current was measured with a quadrant electrometer of the Dolezelek type, arranged to have a sensitiveness of from 150 to 200 scale divisions per volt at a distance of 150 cm. With this sensitiveness the spot of light from the mirror moved one millimeter in from 0.1 to 0.2 seconds, a rate which varied in different series of obser-

vations. No observations were made in which the light did not move over at least 400 scale divisions.

The temperatures were measured by means of a gas manometer, whose minimum sensitiveness was about one-half millimeter per degree.

DESCRIPTION OF APPARATUS

The apparatus as shown in the accompanying figure is as follows: *C* is the vessel in which the ionization took place and whose temperature was varied. It was made from an iron cylinder of about

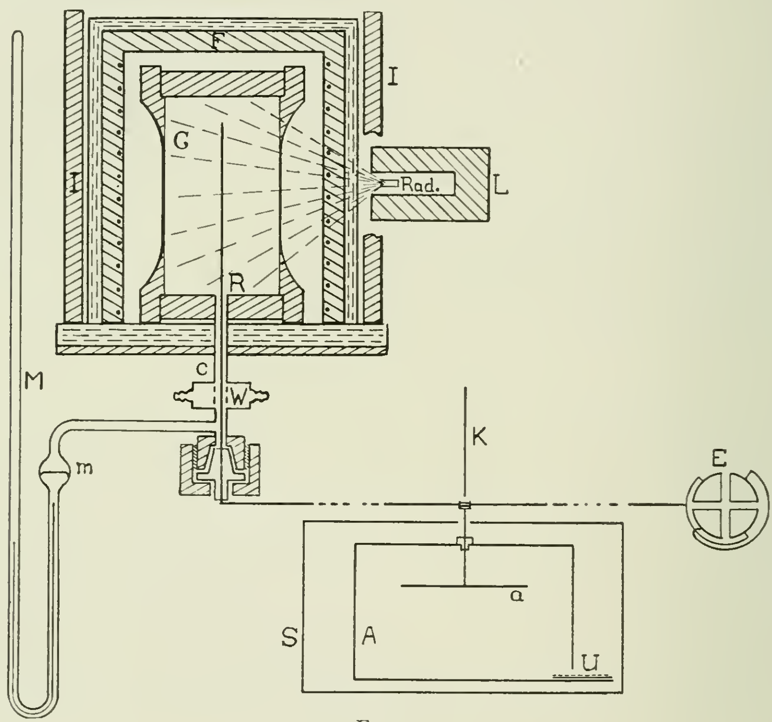


FIG. 1

11 cm internal diameter, by reducing the walls to a thickness of a millimeter or two everywhere except at the ends. The side through which the rays passed was further reduced. Ends of heavy iron plates were brazed to this wall, forming a cylindrical chamber of about 18 cm height.

The tube c leads downward to the manometer. It is surrounded by a water-jacket W. By means of the amber plug p, and the cap which presses the plug into position, it supports the rod R of about 2 mm diameter. This rod forms one electrode. The amber plug gave satisfactory insulation throughout the experiment.

The vessel C rested upon asbestos and an iron plate. Surrounding it was the electric furnace F. The furnace was surrounded by asbestos and the whole inclosed in an iron box I, with walls about 2 cm thick.

The manometer M was made of a graduated capillary tube. It was thoroughly cleaned and filled repeatedly with dry air before using. For convenience in placing it and in reducing the readings to degrees of absolute temperature, a bulb m was blown in the tube to form a reservoir for mercury, thereby keeping that arm of the mercury at very nearly a constant height. The whole air column was inclosed in a water jacket (not shown in the figure) to regulate and determine its temperature.

A is an auxiliary chamber. It consists of a metallic box in which a plate a is held insulated from the box by means of an amber plug. At the top of the rod which holds this plate is a mercury cup which, with the movable rod K, constitutes the earthing key of the system. This insulated system includes the rod R, the plate a, one pair of the electrometer quadrants, and the connecting wires. U is a layer of uranium oxide used to ionize the air in A. S is a metallic shield for A. E is the electrometer.

The radium was held in a lead block L, in such a position as would expose all parts of the gas chamber to the gamma rays. It was of sufficient strength to give, in this position, a measurable rate of deflection of the electrometer through 5 cm of lead.

The electrometer and all connecting wires were surrounded by earthed conductors to prevent leaks and electrostatic disturbances.

For the source of potential small storage cells were used. It was necessary that this potential be fairly constant. By letting the freshly charged cells discharge to that potential which remained constant for the longest time and using them in this condition, they were found to be all that was necessary.

METHOD OF OBSERVING, SOURCES OF ERROR

In taking the observations the method of procedure was as follows: The vessel *C* was repeatedly filled with the gas by exhausting and allowing the gas to flow in through drying agents. The temperature of the gas and the barometric reading were then taken. The reading of the manometer being taken at a known barometric pressure and for a known temperature of the gas in the manometer, the latter was sealed on to the chamber *C*. These readings were again taken. The values under these known conditions gave the constants of a reduction formula which in turn gave the absolute temperature in terms of the readings of the manometer and the temperature of the air in the manometer.

The needle of the electrometer was charged to a potential that would give the desired sensitiveness and at the same time minimize any errors due to the variation of this potential. The quartz fiber suspending the needle was made conductive by coating with a solution of zinc chloride. Although it was necessary to moisten the fiber every week or two, this method proved to be more satisfactory than any other.

The potential of the system was held at zero, while *C* was given a potential above or below this value. This potential was generally about 300 volts, a value well above that which would give a saturation current. *A* was held at a potential of opposite sign to that of *C*, and of sufficient value to give the saturation current caused by the presence of the uranium oxide in *A*.

To take a reading the radium was removed from its position near *C*, the uranium oxide from its position in *A*, and the system, insulated by opening the key *K*. Under ordinary conditions there would be a perfect balance of any small leaks and no deflection of the electrometer would result. If a charge leaked into the system from *C*, the uranium oxide was inserted into *A* sufficiently to cause a charge of opposite sign to pass into the system from *A*. By adjusting the position of the oxide the system could be kept at zero potential indefinitely. The radium was now placed in position and the rate of the deflection of the electrometer needle was measured.

In suitable weather no difficulty was encountered with electro-

static disturbances. It was seldom necessary to use the balancing device on account of failure of insulation. The insulation was always tested by keeping the system at zero potential for a period much longer than the time necessary for taking a reading. The absence of variations due to disturbances was considered sufficiently demonstrated when successive readings at constant temperature were found to agree as closely as it was possible to measure the rate of deflection.

The first difficulty that was encountered was one similar to the disturbance mentioned above as probably due to change in density. It was found that while the temperature of the gas was changing, except when that change took place very slowly, the rate of deflection of the electrometer was not constant but varied irregularly. Since this variation was absent when the temperature remained almost constant, it was considered due to some convective disturbance in the gas and was eliminated by slow and careful heating or by taking the observations at constant temperatures.

A second difficulty was found in the expulsion of ions from the hot metallic electrodes. This could not be overcome completely in the apparatus used for these observations. It was, however, partly overcome. This heat leak[1] was found to begin ordinarily at about 350° C., but, by prolonged or repeated heating at temperatures above this value, the temperature at which the leak first appeared was changed to about 450° C. Above this temperature it was always present. It was to meet this difficulty that the auxiliary chamber *A* was introduced into the system.

To measure the ionization under these conditions the temperature of the chamber *C* was first brought to as nearly a constant value as possible. With the radium removed, the position of the uranium oxide was varied until the system when insulated would remain at zero potential for a period at least several times as long as that required for a reading. The radium was then placed in position, the reading taken, and the balance again tested. If the balance was now disturbed enough to affect the reading by as much as 0.1 per cent the reading was discarded.

As is well known, a temperature is soon reached at which this

[1] Richardson, *Proc. Camb. Phil. Soc.*, 11, 287, 1902.

leak increases very rapidly with the rise of temperature. At a temperature of about 650° C., it became impossible to keep the temperature sufficiently constant to be able to balance the leak for a period great enough to measure the ionization.

At temperatures above 600° C. the escape of the gas from the ionization chamber began to introduce another source of error. This difficulty alone was sufficient to limit the range of observations to temperatures around this value.

DISCUSSION OF READINGS, DATA

The results of some of the observations are shown in the accompanying tables. The rate of ionization of the gas, as represented by the rate of movement of the electrometer needle, is here expressed in millimeters per second. The rate in any one table is not to be compared with that in another table, as the sensitiveness of the instrument was not the same for the different series even when the same gas was used.

In Table I, columns I, II, and III show readings for air. Columns IV and V are for hydrogen. Columns I, II, and IV show individual readings only. It was impossible to work with hydrogen at as high temperatures as those reached in air because the convective disturbances were much greatei than in air, and because the vessel *C* would not hold the hydrogen under as great a pressure as the air.

As one may see from the table, the individual readings for air are constant to within about 0.25 per cent from the mean for temperatures up to about 500° C. For hydrogen column IV shows about the same uniformity, but the readings were taken to about 425° C. only. While the individual readings of column III are not so nearly constant, the mean of the readings at each temperature shows a variation of only about 0.5 per cent from the mean up to about 615° C. In this series the readings at the highest temperature are subject to a slight correction on account of the leak of air from the vessel. Either this correction or the difficulty of keeping the temperature sufficiently constant would account for the irregularity in the readings at this temperature.

In general the readings agree as closely as those taken under the same conditions and at the same time upon the gas at room tem-

perature. Hence the readings agree as closely as the method of observation would warrant, upon the assumption that there should be no variation at all.

TABLE I

Air							Hydrogen				
I		II		III			IV		V		
Temp.	Rate	Temp.	Rate	Temp.	Rate	Mean	Temp.	Rate	Temp.	Rate	Mean
22°....	7.93	20°	7.77	21°	5.66	....	10°	6.35	13°	6.39	....
	7.93	....	7.77	....	5.68	5.67	....	6.35	....	6.39	6.38
	7.90	....	7.78	....	5.65	....	....	....	....	6.37	....
	7.92	28	7.77	....	5.68	....	....	....	....	6.39	....
25.....	7.93	42	7.78	....	....	....	107	6.38	....	....	....
67.....	7.93	....	7.75	....	....	....	....	....	38	6.39	6.37
77.....	7.93	59	7.78	....	....	....	....	....	....	6.35	....
197.....	7.91	81	7.77	....	....	....	....	....	....	....	....
	7.90	94	7.77	....	5.65	....	116	6.38	144	6.35	....
200.....	7.92	106	7.75	396	5.66	....	....	....	....	6.38	....
	7.93	197	7.78	....	5.66	5.66	....	....	....	6.35	6.38
270.....	7.90	228	7.75	....	5.65	....	....	....	217	6.37	....
283.....	7.92	251	7.77	442	5.64	5.64	....	....	....	6.33	....
298.....	7.90	273	7.78	....	5.64	....	198	6.38	....	6.37	6.35
309.....	7.92	302	7.75	....	....	....	....	....	....	6.39	....
318.....	7.90	311	7.78	....	....	....	....	....	....	....	....
326.....	7.92	323	7.75	486	5.72	....	....	....	314	6.35	....
334.....	7.92	332	7.79	....	5.67	5.69	366	6.37	....	6.37	6.37
367.....	7.90	359	7.77	....	5.67	....	....	....	....	6.37	....
438.....	7.92	398	7.75	513	5.62	....	....	....	....	6.37	....
467.....	7.91	424	7.78	....	5.69	5.66	....	....	....	....	....
473.....	7.91	441	7.78	....	5.67	....	402	6.37	343	6.37	6.37
491.....	7.94	466	7.75	....	....	....	....	6.37	....	6.39	....
	....	497	7.75	613	5.47	....	....	6.37	....	....	....
	....	515	7.62	....	5.80	5.63	....	....	....	....	....
	....	....	....	....	....	....	429	6.35	413	6.47	....
	....	....	....	....	....	....	....	....	....	6.27	6.33
	....	....	....	....	....	....	....	....	....	6.27	....
	....	....	....	....	....	....	....	....	....	....	....

TABLE II

Mean Rate from	I′	II′	III′	IV′	V′
0°–100°......	7.924	7.771	5.667	6.375	6.380
100 –200	7.915	7.765	.....	6.380	6.360
200 –300	7.913	7.766	.....	.....	6.365
300 –400	7.914	7.765	5.656	6.37	6.370
400 –500	7.920	7.765	5.663	6.365	6.334
500 –600	.....	.....	5.660	.....	.....

In columns I′, II′, III′, IV′, and V′ (Table II), the mean rate for one range of temperature of 100° is compared with the mean for

other equal ranges. It will be noticed that for the higher temperatures the mean rate is in general slightly lower. This is doubtless due to the difficulty in keeping the temperature sufficiently constant or in varying it slowly enough to avoid the disturbances due to convection currents in the gas.

SUMMARY

The temperatures were varied from room temperature to about 615° C. The absolute temperature and therefore the mean kinetic energy of the molecules was increased to three times its value at room temperature. From the kinetic theory it may be shown that about 1 per cent of the molecules have a probable mean energy four times this mean. Hence the energy of agitation of 1 per cent of the molecules was probably about twelve times the mean energy at room temperature.

Readings were taken nearly 300° C. above the temperature at which electrons are first driven from the metals by heating. Both hydrogen and air were experimented with, the latter furnishing a desirable mixture of gases of different molecular weights.

The individual readings were in general constant to within 0.2 per cent of the mean. In columns I′, II′, and III′ (Table II), which are mean readings for air, the greatest variation is a little over 0.1 per cent.

The ionization of air by means of the gamma rays from radium is therefore independent of the temperature of the gas to within 0.2 per cent up to about 600° C. For hydrogen the same independence is shown for temperatures up to about 430° C.

A variation of over 200 per cent in the absolute temperature of a gas does not affect the stability of the atom sufficiently to change the ionization by more than about 0.1 per cent.

In conclusion the writer wishes to express his thanks for their assistance and encouragement to Professor Michelson and the staff of Ryerson Physical Laboratory, and especially to Professor Millikan, under whose direct supervision this experiment was undertaken.

THE UNIVERSITY OF CHICAGO
January 26, 1911

THE PYRHELIOMETRIC SCALE[1]

By C. G. ABBOT and L. B. ALDRICH

When in 1902 solar constant measurements were begun at the Astrophysical Observatory of the Smithsonian Institution, two copies of Crova's alcohol actinometer were obtained, one of which was made for the Observatory under Professor Crova's own supervision. In a letter to Professor Langley he said of his instrument: "Une longue expérience m'a appris que, lorsqu'il est bien étalonné sur un actinomètre absolu quelconque, il donne des résultats tout aussi exacts que l'actinomètre absolu qui a servi à l'étalonner." He speaks as if absolute actinometers were common, but it is only after eight years of seeking that we are satisfied that we have one.[2]

At first, having no Ångström compensation pyrheliometer, an instrument somewhat like Tyndall's was constructed and used as a standard. This comprised a blackened copper box filled with mercury and having a mercury thermometer inserted to measure the rise of temperature of the box of mercury under solar heating. Numerous determinations of the capacity for heat (water equivalent) of the instrument were made by Dr. Benton (then of the Observatory), and it is on the pyrheliometric scale thus fixed that all of the "solar constant" observations given in Vol. II of the Observatory *Annals* are stated. We never were sure that this scale was within several per cent of the true one, for there were numerous sources of error almost impossible to estimate in employing the mercury pyrheliometer as a standard.

We soon found that it was more convenient for us to use the mercury instrument in daily observations than the alcohol actinometer of Crova; for we never mastered the art of easily keeping a thread of mercury in a good part of the scale of the alcohol thermometer. Furthermore we found, by consulting the tables, that the specific heat of alcohol varies rapidly with the temperature,

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] The earlier experiments made here to devise a standard pyrheliometer are described in Vol. II of the *Annals of the Astrophysical Observatory*, p. 39.

so that an unknown error is introduced by the unknown temperature of the instrument.

About 1904 Mr. Abbot conceived the idea of constructing a standard pyrheliometer comprising the following essentials. A cylindrical chamber of about the form of a large test-tube, and blackened within, has hollow walls adapted for the circulation of a liquid. The sun's rays shine into this chamber through a measured orifice, and heat the walls at the rear end. A stream of water flowing at a measured rate removes the heat as fast as formed, independently of its distribution within the chamber. Such heat as escapes capture at the rear end of the chamber will be gathered somewhere along the sides, excepting the negligible amount escaping at the front orifice. Four platinum wires bathed by the water, two at the entrance, two at the outflow of the stream, are joined with accessories to form a resistance thermometer. This serves to measure the rise of temperature of the water due to solar heating. By increasing the rate of flow the rise of temperature may be diminished until the escape of heat on the *outside walls* of the chamber becomes negligible. To prove that the heating is correctly measured, two electrical heating coils are introduced near the back of the chamber, one of which is far more favorably situated than the other to communicate to the walls any heat which may be formed in it. In test experiments heat may be introduced electrically in either coil in known quantity, and its amount measured, as if it were solar heat, by the resistance thermometer. The full recovery of such test-quantities of heat, whether from the favorably or the unfavorably situated coil, serves to prove the accuracy of the instrument.

Preliminary experiments at length justified the construction of one of these instruments, which is described in Vol. II of the *Annals of the Astrophysical Observatory*, pp. 39–47. Although this instrument worked well, its disagreement with the Ångström instruments of the Weather Bureau was so marked as to lead to a suspicion of its inaccuracy. Accordingly two other instruments of similar essential features, but differing in minor parts and in dimensions, were afterward constructed by Mr. Kramer after Mr. Abbot's designs.

In 1910 a long and thorough determination of all the electrical and other constants of these two new standard pyrheliometers, numbered II and III, was made by Mr. Aldrich. We were satisfied at its conclusion that no error in the readings of either instrument exceeding 1/500 could possibly occur from lack of accuracy in the determination of these constants. Secondary silver-disk pyrheliometer No. 8_{bis} was then compared with each of the two standards. In this comparison Mr. Fowle aided toward the last.

We do not give here the numerous details of the many measurements of electrical constants, and of the many pyrheliometer comparisons. These details will be given in Vol. III of the *Annals* of the Observatory. We remark that in Standard No. II the spiral channel in the hollow walls of the chamber is too restricted, so that about 60 feet head of water is required to force the stream of water at a proper rate (about 1 gram per second). In this pyrheliometer, also, the platinum thermometer resistance wires are too small in diameter and are probably swayed by eddies in the water-flow. In consequence of these defects there was considerable drift of the galvanometer to be contended with in the use of Standard No. II, so that more comparisons were necessary with it than with No. III.

The final results are as follows:

With Standard Pyrheliometer No. II: Sixteen comparisons on four different days with No. 8_{bis} give

$$\frac{\text{No. II}}{\text{No. } 8_{bis}} = 0.6289 \pm 0.0029 .$$

Electrical heating was introduced in No. II twenty-one times on six days. Percentage "found" ranged from 96.3 to 105.7.

$$\text{Mean} = 99.1 \pm 0.3 .$$

With Standard Pyrheliometer No. III: Six comparisons on one day with No. 8_{bis} give

$$\frac{\text{No. III}}{\text{No. } 8_{bis}} = 0.6287 \pm 0.0014 .$$

Electrical heating was introduced in No. III eight times on two days. Percentage "found" ranged from 98.6 to 99.9.

$$\text{Mean} = 99.25 \pm 0.13 .$$

From these comparisons with No. 8_{bis}:

$$\frac{\text{No. II}}{\text{No. III}} = 1.0003 \pm 0.0051\,.$$

In the solar-constant work on Mount Wilson, secondary pyrheliometer No. IV has been read on every observing day since early in 1906. It was compared with secondary No. V at Washington in 1906, and with No. 8_{bis} on Mount Wilson in 1909. No. 8_{bis} was compared with No. V at Washington in 1910. Hence there were two ways of getting the ratio $\frac{\text{No. IV}}{\text{No. } 8_{bis}}$, one direct, the other by $\frac{\text{No. IV}}{\text{No. V}} \times \frac{\text{No. V}}{\text{No. } 8_{bis}}$. In the direct way we found $\frac{\text{No. IV}}{\text{No. } 8_{bis}} = 0.7353$. The other gives $\frac{\text{No. IV}}{\text{No. } 8_{bis}} = 0.7358$. From the mean of these:

$$\frac{\text{No. IV}}{\text{Standard}} = 0.8551\,.$$

In October 1910 Mr. Abbot made a direct comparison on Mount Wilson between No. IV and Standard No. III. He obtained from eight observations on two days:

$$\frac{\text{No. IV}}{\text{Standard}} = 0.8581 \pm 0.00185\,.$$

CONCLUSIONS

We believe of the two standard pyrheliometers, No. II and No. III, that each recovers and correctly indicates the heat received in test experiments with known quantities of heat electrically introduced, within less than 1 per cent.

As the heat electrically introduced must be communicated to the air of the chamber and thence to its walls before being taken up by the water stream, we believe that solar heating, which occurs on the walls themselves, may be correctly measured by both standards, and in our experiments was measured within 0.2 per cent.

This conclusion is supported by the agreement to within $\frac{2}{6290}$ of the comparisons of the two standards with Secondary No. 8_{bis}.

We regret that we cannot feel quite sure that No. 8_{bis} preserved

its constants unchanged in the journey from Mount Wilson to Washington, and we therefore assign the given ratio $\frac{\text{No. IV}}{\text{No. } 8_{bis}}$ little weight. We assign full weight to the given ratio $\frac{\text{No. IV}}{\text{Standard III}}$ obtained by direct comparisons of October 1910 on Mount Wilson.

We believe the scale of the solar-constant observations of the Astrophysical Observatory is thereby reduced to the absolute scale of calories (15° C.) per square centimeter per minute within a probable error of 0.2 per cent.

We incline to think that the unit of pyrheliometry furnished by new Ångström pyrheliometers is about 5.5 per cent greater than the true calory.[1] We do not state this unreservedly until confirmed by further comparisons.

Astrophysical Observatory
Smithsonian Institution
Washington, D.C.
January 30, 1911

[1] See H. H. Kimball, *Bulletin of the Mt. Weather Observatory*, **3**, Pt. 2, p. 84, 1910.

ON A QUANTITATIVE METHOD FOR DETERMINING THE SPECTRAL TYPES OF THE BRIGHTER STARS

By SEBASTIAN ALBRECHT

In 1906[1] the writer published a paper "On the Relation between Stellar Spectral Types and the Intensities of Certain Lines in Their Spectra." The basis for that paper was the discovery that the wave-lengths of certain spectrum lines varied in the different stellar spectral types, the variation being *progressive* as one proceeded from the F to the Mb types. At that time I believed that eventually it would be possible to invert the problem and, by means of the wave-lengths of the variable lines, to determine the spectral types of the stars. The study of the lines of variable wave-lengths was continued with various interruptions, and, when I had practically completed it, at Córdoba, for the region ordinarily used on the three-prism spectrograms on which λ 4340 is central, I decided to attack this inverse problem.[2]

The classification employed is the Draper classification,[3] which uses the letters O, B, A, F, G, K, M, and N to designate the sequence of the spectra. Numerals from 1 to 9 after the letter denote tenths of the interval between two successive letters; thus F8 designates a spectrum about eight-tenths of the interval from F to G.

To review briefly some of the results of the former paper: an examination of Rowland's tables shows that in most cases studied, if not in all, lines apparently single on the stellar spectrograms are in reality blends of two or more close components. The nature of the variations of the wave-lengths found is such as to indicate varying intensities of the same components, rather than the

[1] *Lick Observatory Bulletin*, No. 106; *Astrophysical Journal*, **24**, 333, 1906.

[2] I was engaged in this work when Professor Schlesinger's circular letter of November 7, 1910, reached me. In this letter I received my first information in regard to the appointment by the Fourth Conference of the International Union for Co-operation in Solar Research of a "Committee on the Classification of Stellar Spectra."

[3] *Annals of Harvard College Observatory*, **56**, 66.

presence or absence of different components in the different stellar spectral types. A comparison was also made with Adams'[1] list of sun-spot lines, and the indications were very strong that the physical conditions in the stars as we pass from the F (this may be extended to include the A type) to the Mb type vary roughly in the same direction as from the sun to the sun-spots. The results of the

TABLE I

Observed Wave-Lengths of Lines Varying Progressively with Spectral Type

λ	F	F5	F8	G	G5	K	K2	K5	Ma	Mb
4246.9....	.996	.996		[G to K .972]			.959		.954	
54.5....	.517	.520		.500	.499	.499	.479			
60.6....	.629	.642		.671			.720			
67.1....	.148			.051		.033	.021			
67.9....	.857			.883	.922	.929	.990		.022	
74.9....	.976	.955		.950	.954	.944	.946			
78.4....	.338			.346	.373	.401	.374			
86.1....						.163	.120		.077	
87.1....				.145	.095	.126	.106	.095		
88.1....	.076	.130		.136	.157	.164	.155			
93.2....			.217	.234	.229	.245	.271			
4314.3....	.350	.324		.344	.380	.367	.359			
15.1....	.182	.182		.182	.144	.152	.141			.117
21.0....	.018	.996		.984	.942	.920	.900			
25.1....	.190	.186		.169	.205	.220	.234		.217	
31.8....			.838	.798	.801	.782	.775		.775	
34.0....	.990			.052	.083	.101	.106		.111	
40.6....	.661	.655		.661	.657	.656	.644	.590	.593	
44.6....	.493			.586	.597	.623	.623		.654	
52.0....	.026	.024		.010	.015	.010	.005	.994	.981	.970
52.9....	.935	.938		.949	.960	.972	.990		.009	
62.2....	.299						.173			.135
90.1....	.881			.095		.121	.143	.137	.154	.159
94.2....	.242						.127		.107	
95.2....	.235	.248		.258	.269	.272	.272		.277	.310
98.3....	.333						.262		.206	
99.9....	.951			.941	.900	.912	.902	.851		
4400.6....	[F to G .587]				.672		.670		.680	
12.2....	.096					.284	.319		.302	
16.9....	.994		.994			.762			.740	
25.6....	.582	.599		.640		.887		.908		
28.7....					.736	.721	.652			.656
30.7....	[F to G .791]					.741	.730	.689		
33.4....			.391			.434	.440			
35.2....	.139	.183		.187	.204	.217	.253	.242	.250	
57.6....		.610				.680	.665	.653	.641	
64.7....	.631	.773	.798				.900			
68.7....	.648	.651		.666		.693	.711		.729	
69.5....	.492			.564		.595	.620	.643		
73.0....	.129			.975	.975	.000	.972	.938		.912

[1] *Astrophysical Journal*, **24**, 69, 1906.

former paper, as well as of this, depend not only upon the appearance of the lines but primarily upon quantitative measurements of their positions.

A large number of lines whose wave-lengths vary progressively with the stellar spectral type has been found in addition to those published in the paper referred to above. The attempt to deter-

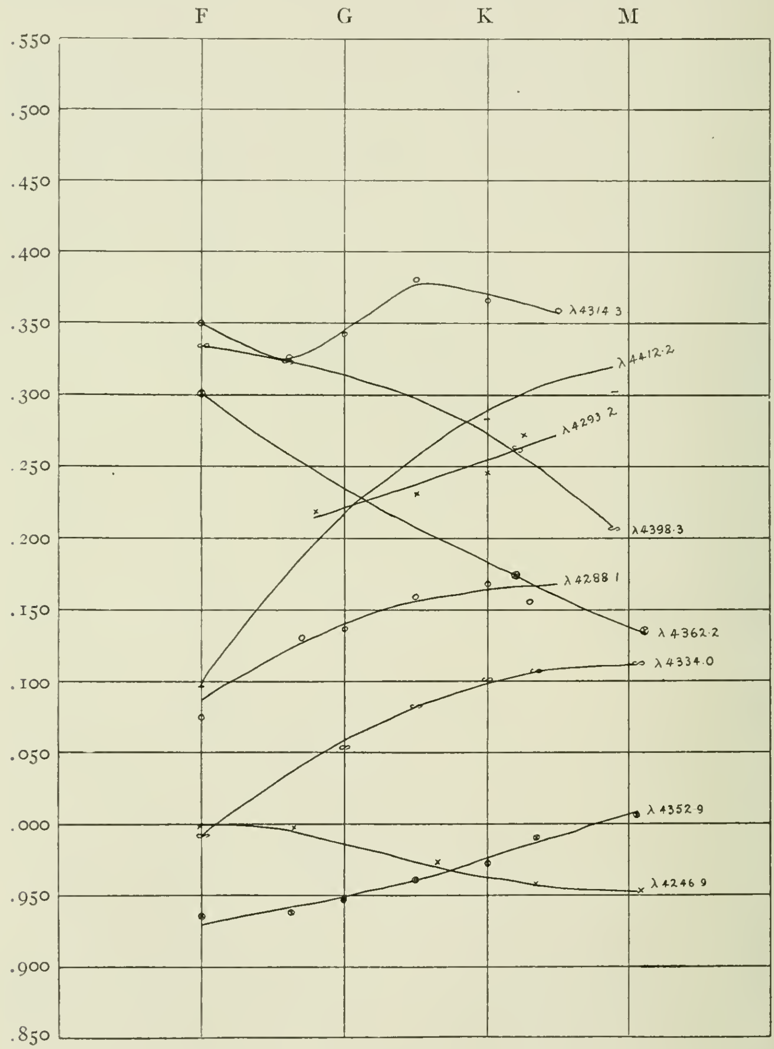


FIG. 1.—Curves for lines whose wave-lengths vary progressively with spectral type

mine the spectral types of stars by means of these wave-lengths was made for some Southern Mills spectrograms on which λ 4340 is central. For this purpose the more important of these lines between λ 4236 and λ 4480 were selected. In Table I are given the observed wave-lengths, in types F to Mb. These values were plotted with wave-lengths as ordinates and spectral types as

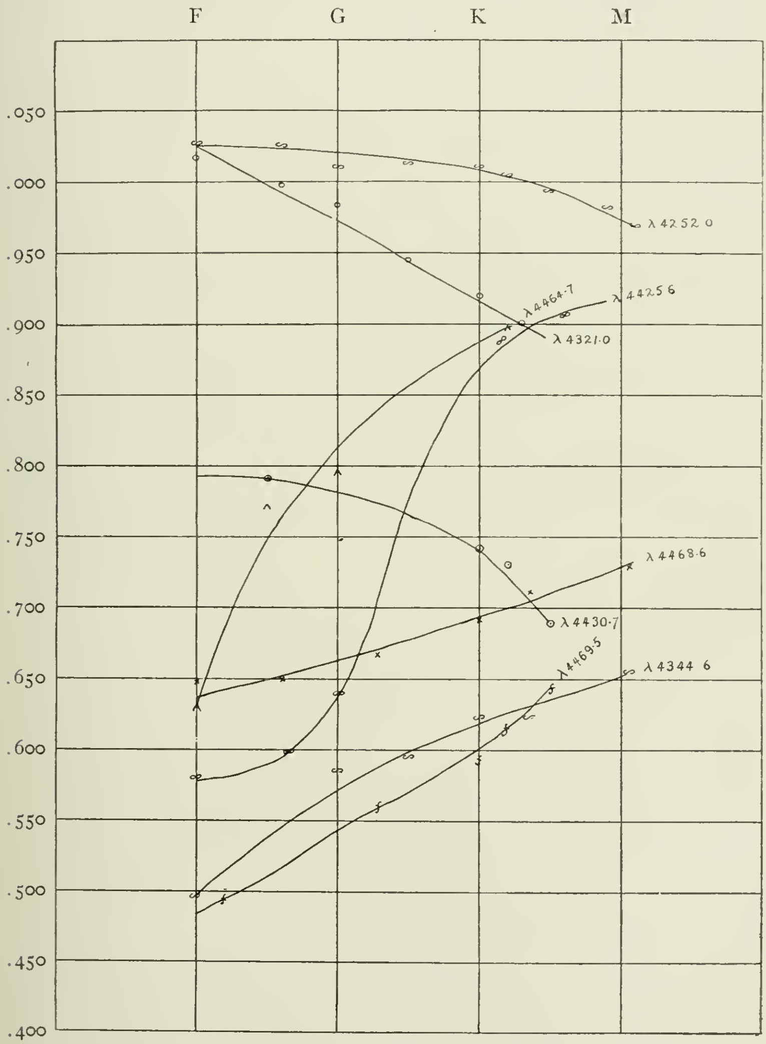


FIG. 2.—Curves for lines whose wave-lengths vary progressively with spectral tyep

abscissas. Some of these curves are reproduced in the figures. Table II gives the wave-lengths taken from the curves for each

TABLE II

WAVE-LENGTHS FROM CURVES FOR EACH TYPE

λ	F	F5	F8	G	G5	K	K2	K5	Ma	Mb	Average Deviation	Largest Deviation
4246.9	.999	.997	.992	.987	.974	.964	.960	.957	.955	.953	.001	.003
54.5	.518	.514	.511	.508	.500	.492	.488				.004	.008
60.6	.625	.643	.654	.663	.685	.710	.720				.005	.008
67.1	.122	.100	.086	.077	.055	.032	.022				.009	.026
67.9	.852	.870	.883	.892	.920	.951	.965	.986	.015	.030	.011	.022
74.9	.973	.960	.955	.952	.948	.947	.947				.002	.004
78.4	.319	.335	.346	.353	.370	.387	.392				.016	.020
86.1						.143	.126	.106	.084	.074	.006	.020
87.1				.145	.128	.112	.105				.006	.033
88.1	.086	.116	.132	.140	.154	.163	.165				.006	.010
93.2			.215	.221	.238	.255	.262				.008	.013
4314.3	.350	.326	.333	.344	.377	.370	.365				.002	.003
15.1	.191	.182	.176	.173	.162	.148	.143	.134	.122	.117	.006	.018
21.0	.025	.999	.982	.972	.945	.917	.905	.889			.004	.012
25.1	.190	.183	.181	.180	.204	.222	.225	.226	.221	.218	.003	.011
31.8			.819	.811	.793	.781	.778	.776	.774	.774	.004	.019
34.0	.990	.026	.046	.058	.083	.098	.103	.108	.111	.112	.001	.005
40.6	.660	.659	.659	.658	.656	.646	.639	.622	.59	.57	.008	.032
44.6	.495	.537	.558	.571	.599	.620	.627	.637	.650	.655	.006	.015
52.0	.026	.025	.022	.021	.017	.009	.004	.995	.980	.971	.001	.011
52.9	.928	.937	.944	.948	.961	.975	.981	.991	.004	.011	.002	.008
62.2	.300	.266	.247	.235	.207	.184	.175	.161	.144	.135	.000	.000
90.1	.88	.996	.035	.056	.096	.124	.132	.143	.154	.159	.004	.011
94.2	.240	.209	.190	.179	.154	.134	.127	.117	.107		.000	.001
95.2	.236	.245	.252	.255	.264	.273	.278	.283	.291	.295	.004	.015
98.3	.333	.325	.318	.313	.296	.272	.260	.239	.206		.000	.000
99.9	.953	.947	.940	.936	.922	.905	.896	.882			.009	.032
4400.6	.575	.595	.612	.622	.652	.672	.677	.682	.680	.676	.012	.026
12.2	.096	.162	.195	.214	.257	.290	.300	.310	.318		.009	.014
16.9	.999	.996	.988	.977	.92	.782	.758	.746	.739	.738	.002	.010
25.6	.577	.592	.613	.638	.770	.870	.888	.906	.916		.002	.004
28.7					.74	.706	.678	.655	.650	.649	.009	.015
30.7	.794	.791	.787	.784	.770	.741	.723	.690			.003	.007
33.4			.385	.395	.416	.434	.440				.000	.001
35.2	.142	.166	.181	.190	.211	.230	.235	.244	.254	.260	.006	.020
57.6			.610	.625	.659	.674	.669	.656	.644	.636	.003	.006
64.7	.631	.746	.790	.812	.855	.886	.899				.006	.028
68.8	.636	.650	.658	.664	.680	.695	.704	.714	.727	.735	.004	.009
69.5	.484	.510	.529	.541	.571	.601	.616	.643			.003	.006
73.0	.129	.050	.005	.978	.975	.994	.980	.949	.91	.90	.003	.017

type. In plotting the curves it was necessary to make an assumption in regard to the relative values of the intervals between types F and G, G and K, and K and M. These intervals were provisionally taken to be equal, and Ma and Mb were arbitrarily

placed 0.1 of an interval preceding and following the position of M in the scale. From the plots of a large number of the curves for the variable spectral lines, it may be possible later to determine the relative values of these assumed intervals. The curves at present available indicate that the intervals are approximately equal. I am now engaged in extending this work to the so-called "earlier" types and to about λ 4650, and shall accordingly delay the readjustment of the horizontal scale until the completion of that work.

Eight stars, one of which is our sun (spectrograms of *Venus* were used), were selected to illustrate the application of the method. The measured wave-lengths of the spectrum lines which vary progressively with the spectral type were compared with the curves for those lines to determine the corresponding values of the spectral type. Thus for each star the measured wave-length of each of these lines will furnish a value for the spectral type of that star. In Table III are given the results of this comparison. Column λ gives the wave-lengths of the lines employed. The quantities m are the values of the spectral type furnished by the individual lines in each star. The + and − signs indicate direction along the curves toward the right and left, respectively, counted from the G type for stars *a Fornacis* and the sun, from the K type for *a Trianguli Australis*, and from the F type for the remaining five stars. Column p gives the weights assigned to each line, and column pm gives the products of the two quantities. The results for each star are summarized at the bottom of the table, the last three lines giving respectively the probable errors in units of a tenth of the interval between the types represented by two successive letters in the Draper classification, the resulting spectrum type, and the type given in the Draper classification. These examples are merely illustrative of the method. The probable errors are somewhat large for several reasons. It is believed that it will eventually be possible to attain a considerable degree of accuracy. It is interesting to note that this method gives the position of the sun in the stellar classification as being very closely that of the G type.

The weights assigned are, to a certain extent, arbitrary, but in

TABLE III

Spectral Type Determined from Curves for Lines Varying Progressively in Wave-Length

λ	α *Fornacis*			*b Velorum*			*Venus*			α *Trianguli Australis*			ι *Carinae*			α *Circini*			π *Sagittarii*			κ *Reticuli*		
	m	p	pm	m	p	pm	m	p	pm	m	p	pm	m	p	pm	m	p	pm	m	p	pm	m	p	pm
4246.9	− 6	1.3	− 7.8				+11	1.3	+14.3	− 3	0.8	− 2.4	+16	0.8	+12.8							+10	0.8	+ 8.0
54.5	+ 6	0.8	+ 4.8	+21	0.8	+16.8	+ 4	1.3	+ 5.2	+ 6	0.8	+ 4.8	+ 5	0.8	+ 4.0	+15	0.8	+12.0	− 4	0.8	+ 3.2	−22	0.8	−17.6
60.6	− 4	2.0	− 8.0	− 3	2.0	− 6.0				+ 4	1.7	+ 6.8	− 4	1.9	− 7.6				−16	1.5	−24.0	+ 5	1.7	+ 8.5
67.1										+ 5	1.5	+ 7.5												
67.9										+ 6	2.0	+12.0												
74.9	+ 1	0.9	+0.9				+ 1	0.8	+ 0.8	− 9	0.8	− 7.2	+11	1.4	+15.4	− 1	1.3	− 1.3	− 1	1.2	− 1.2	+ 6	1.4	+ 8.4
78.3				± 0	1.9	± 0.0				+ 1	0.8	+ 0.8	+ 8	1.4	+11.2									
86.1										± 0	1.7	± 0.0												
87.1							± 0	1.3	± 0.0	+11	1.5	+16.5												
88.1	− 1	2.2	− 2.2	− 1	2.7	− 2.7	− 5	1.7	− 8.5	− 6	0.8	− 4.8	− 3	2.2	− 6.6				+ 3	1.7	+ 5.1	+ 6	1.9	+11.4
93.2	− 1	1.5	− 1.5				+ 1	1.7	+ 1.7	+ 3	1.7	+ 5.1												
4314.3	− 4	0	....	+ 2 or + 9	0	....	± 0	0.8	± 0.0	+ 2	1.2	+ 2.4	− 1	0.8	− 0.8				− 2 or +13			− 6 or +15	0	....
15.1	−13	0.8	−10.2				−12	1.3	−15.6	+ 2	1.2	+ 2.4	+12	0.8	+ 9.6	− 5	0.8	− 4.0	+16	0.8	+12.8	+14	0.9	+12.6
21.0	− 1	2.2	− 2.2	+ 1	2.7	+ 2.7	+ 1	2.0	+ 2.0	− 2	2.2	− 4.4	+ 5	2.0	+10.0				− 2	1.7	− 3.4	+ 4	2.7	+10.8
25.1	± 0	0	....	± 0	0	....	± 0	0	....	− 1	0	....	+ 3	0	....							− 2 or +13	0	.....
31.7	− 5	1.2	− 6.0							+ 5	0.8	+ 4.0												
34.0										+ 1	1.4	+ 1.4												
40.6							−12	0	....				− 6	0	....	−20	0	....	−20	0	....	+21	0	....
44.6				+ 1	2.7	+ 2.7	+ 4	2.4	+ 9.6	− 4	1.9	− 7.6												
52.0	+ 3	0.8	+ 2.4	−30	0.8	−24.0	± 0	0.8	± 0.0	+ 3	1.2	+ 3.6	+24	0	....	−40	0	....	+25	0	....	+ 3	0.9	+ 2.7
52.9	− 8	1.3	−10.4	+32	1.2	+38.3	+ 5	1.2	+ 6.0	+ 4	1.9	+ 7.6	+16	0.8	+12.8							+ 6	1.4	+ 8.4
62.2										+ 5	2.0	+10.0												
90.1										+ 5	2.0	+10.0												
4394.2				+ 5	3.0	+15.0				+ 4	1.5	+ 6.0	+ 1	2.0	+ 2.0									

4395.2	−13	1.4	−18.2				+ 2	1.3	+ 2.6	− 6	1.3	− 7.8	− 3	1.4	− 4.2	+ 4	1.2	+ 4.8	+24	1.2	+28.8	+13	1.5	+19.5
98.3										+ 2	2.0	+ 4.0												
99.9				+16	1.5	+24.0	+ 2	1.3	+ 2.6	−13	1.7	+22.1	+ 1	0.8	+ 0.8							+ 1	1.2	+ 1.2
4400.6				+ 3	2.0	+ 6.0	− 5	1.7	− 8.5	− 1	1.2	− 1.2	± 0	1.7	± 0.0									
12.2										+ 3	1.7	+ 5.1												
16.9	± 0	1.3	± 0.0	−20 or 30	0	....				+ 6	1.2	+ 7.2	+10	0	....	−12	0	....				−20	0	....
25.6	− 2	2.1	− 4.2				± 0	2.9	± 0.0	+ 3	2.0	+ 6.0				+ 5	0.8	+ 4.0				+ 5	1.5	+ 7.5
28.7										+11	0.8	+ 8.8												
30.7										+ 3	2.2	+ 6.6												
33.4	− 2	1.5	− 3.0				− 1	1.5	− 1.5	− 1	1.5	− 1.5												
35.2	− 3	2.5	− 7.5				+ 1	2.0	+ 2.0	+12	1.2	+14.4				− 2	1.9	− 3.8				± 0	2.2	± 0.0
57.6										+ 1 or 2	0.8	+ 0.8												
64.7				+ 1	3.0	+ 3.0	− 1	2.3	− 2.3	− 2	1.7	− 3.4	− 1	2.3	− 2.3				+ 7	2.3	+16.1	+ 7	2.3	+16.1
68.6	− 7	1.3	− 9.1				− 6	1.7	−10.2	+ 6	2.2	+13.2	− 1	1.4	− 1.4	−11	1.3	−14.3	+19	1.2	+22.8	+ 6	1.6	+ 9.6
69.5							+ 4	2.4	+ 9.6	+ 2	2.1	+ 4.2				+ 5	2.2	+11.0				− 5	2.2	+11.0
4473.0				+ 5	2.7	+13.5	+ 2 or +13	0	....	+ 7	0.8	+ 5.6												
[pm]	−82.2			+99.0			+ 9.8			+114.4			+55.7			+ 8.4			+53.8			+96.1		
[p]	25.1			27.0			33.7			55.8			21.7			10.3			12.4			25.0		
$z=\frac{[pm]}{[p]}$	− 3.3			+ 3.7			+ 0.3			+ 2.0			+ 2.6			+ 0.8			+ 4.3			+ 3.9		
Prob. error	+ 0.7			± 1.9			± 0.6			± 0.5			± 1.0			± 1.7			± 2.7			± 1.1		
Type	F 6.7			F 3.7 Pec.			G 0.3			K 2.0			F 2.6			F 0.8			F 4.3			F 3.9		
[Draper catalogue]	(F8)			(F5 Pec.)			(G)			(K2)			(F o)			(F o)			(F2)			(F5)		

a general way account was taken of the slope of the curve at each spectral type, and, in the case of short curves, of the distance of the point from the nearest end of the curve. Other things being equal, points well within the ends of a curve will naturally give more reliable results than will those at the ends. The weights obtained in this manner were combined with weights proportional to the square roots of the number of observations.

TABLE IV

(Summary of Table III)

Star	Type		
	Draper Classification	Value Obtained Above	Probable Error
ι *Carinae*...........	F	F 2.6	±1.0
α *Circini*............	F	F 0.8	±1.7
π *Sagittarii*..........	F 2	F 4.3	±2.7
κ *Reticuli*...........	F 5	F 3.9	±1.1
b Velorum...........	F 5 Pec.	F 3.7 Pec.	±1.9
α *Fornacis*...........	F 8	F 6.7	±0.7
Venus.............	G(0)	G 0.3	±0.6
α *Triang. Austr*......	K 2	K 2.0	±0.5

For the F and M types, which are present at the extremities of the observed portions of the curves, the correction to the type obtained from the individual lines will frequently have to be taken by extrapolation at some distance from the observed parts of the curves. Consequently, the limits of the curves will be given especial attention in a later discussion of this problem.

Generally the steepest parts of a curve should yield more reliable corrections to the spectral type than the portions of the curve having a moderate slope. For example, from outstanding residuals from the steep portion of the curve for λ 4464.7, π *Sagittarii* had for some time been suspected of belonging more nearly to type F5 than to type F. I do not wish to attach much confidence to my value of F4, as the lines measured on the one spectrogram which I used were not especially suitable for this purpose. Frequently, however, at the steepest portions of some of the very steep curves the line was not included in the measures. At these points the relative intensities of the components change most rapidly with comparatively small differences in spectral type, and,

as such blends are comparatively wide and often fuzzy in appearance, the grain of the plate, underexposure, and overexposure will make it difficult to judge the position of the effective center of the blend. This "fuzzy" appearance of lines is, however, not confined entirely to the lines with steep curves.

The special aim has been to avoid the introduction of systematic errors. In this connection it has been especially gratifying that, for the variable lines, the wave-lengths for the G type as taken from the curves are in practically every case in fair agreement with the Rowland wave-length in the sun, giving the components of blends weights equal to their intensities in the sun. In the K types the wave-lengths of such blends are also in fair agreement with the wave-length resulting from blending the components according to their intensities in sun-spots.

The writer wishes to urge that in future measurements for radial velocities a number of favorable lines be included so that the same measures can later be used for the determination of stellar spectra without requiring the remeasurement of the spectrograms. For the region λ 4236 to λ 4474 I append such a list, Table V, both for lines with constant and with variable wave-lengths. This list will be extended later to about λ 4620. Even from the lines of variable wave-lengths the best ones can safely be used for radial velocity work if the proper wave-lengths for the type be selected. The writer's experience has been that such lines will frequently be among the best available. In fact, a knowledge of the behavior of a line in the various spectral types inspires a considerable degree of confidence in its use for radial velocity work.

It is probable that, for most of the brighter stars, the spectral type is known within a few tenths of a unit of the scale, so that the error introduced in taking the wave-lengths from the curves for a few of the lines measured will usually be balanced and will produce an inappreciable effect upon the radial velocity obtained from the plate.

The spectral types thus determined will be relative to the types of the stars used in obtaining the variations of the spectral lines. As it is desirable to have this basis rest upon a fairly large number

TABLE V

SUGGESTED LIST OF LINES TO BE INCLUDED IN THE MEASURES FOR RADIAL VELOCITIES

(The wave-lengths given in part *b* were determined from the stellar spectra)

λ	*a*, Lines Whose Wave-Lengths Vary	λ	*b*, Lines Whose Wave-Lengths Are Constant
4246.9	F to Mb	4245.435	F to Mb
54.5	F to K2	50.293	F to Mb
67.9	F to Mb	50.951	F to Mb
74.9	F to K2	71.319	F to Mb
88.1	F to K5	94.273	F to Mb
93.2	F8 to K5	4313.041	F to Mb*
4315.1	F to Mb	18.867	F to Mb
21.0	F to K5	28.101	G to Mb
34.0	F to Mb	37.223	F to Mb
44.6	F to Mb	39.721	F to Mb
52.0	F to Mb	76.103	F to Mb
52.9	F to Mb	79.364	G to Mb
90.1	F to Mb	83.725	F to Mb
95.2	F to Mb	4401.611	F to Ma
4425.6	F to Ma	06.803	F8 to Mb
30.7	F to K5	07.851	F to Mb
35.2	F to Mb	08.572	F to Mb
64.7	F to K2	27.444	F to Mb
68.7	F to Mb	42.526	F to Mb
69.5	F to K5	47.911	F to Mb

* May vary slightly.

of stars, and in order to eliminate systematic errors which may enter into the results on account of the dispersion, the slit-width, the kind of photographic plates, the grain and photographic treatment of the plates, a request has been sent to a number of observatories for the loan of a suitable series of spectograms. It is hoped that the results of that investigation will be ready for publication at an early date.

CÓRDOBA, ARGENTINA
December 26, 1910

PHOTOGRAPHIC OBSERVATIONS OF PROMINENCES[1]

By GIORGIO ABETTI and RUTH EMILY SMITH

A photograph of the prominences is taken daily by Mr. Ellerman as a part of the regular work of the five-foot spectroheliograph connected with the Snow telescope[2] of the Mount Wilson Observatory. From 1906 to the beginning of 1908 the spectrum line used was the H line of calcium. On the latter date a series of photographs of the solar disk and of the prominences with the $H\alpha$ line was begun; and since April 1908 only the $H\alpha$ series, which gives better results, has been continued. The observations under discussion do not cover this interval completely, but combined with photographic and visual observations made elsewhere they will contribute to a more complete knowledge of the solar activity.

The reduction of the observations was made from the original negatives with the aid of a simple circle divided into degrees for determining the position-angle of the prominences, and for estimating their heights a millimetric scale was used. To determine the areas the method used for the flocculi was followed; that is, the prominences were cut from photographic prints and the total weight of the paper thus cut out was compared with the weight of a unit of paper cut from the solar disk.

A comparison was made between the Mount Wilson photographs and those taken at the Yerkes Observatory by Fox (aided in 1908 by Abetti) with the Rumford spectroheliograph attached to the 40-inch telescope. As the latter photographs were taken through the H line, the comparison with the $H\alpha$ photographs of Mount Wilson is of especial interest. In the two instruments the scale of the solar image is almost the same. On the Mount Wilson plates 1 mm = $11''.2$, and on the Yerkes plates 1 mm = $10''.6$.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 51.

[2] *Ibid.*, No. 7; *Astrophysical Journal*, **23**, 54, 1906.

If P_0 be the position-angle of a prominence measured on the plate and read in the direction N.E.S.W., we must, to obtain the true position-angle of the prominence, apply to P_0 the orientation angle p of the telescope, which is experimentally determined by observing the transits of the solar image when the mirror of the coelostat is rotated. This angle varies by about 1° for different positions of the coelostat and for different hour-angles of the sun. By applying to the sum of P_0 and p the position-angle of the north point of the sun reckoned from the north extremity of the axis, with its appropriate sign, we obtain the heliographic latitude of the prominence. The latitude obtained in this way is but approximate, and agrees with the true figure only when this does not exceed 50° or 60°, according to the heliographic latitude of the center of the solar disk. For values exceeding this limit it is necessary to use the more accurate formula

$$\sin \lambda = \cos D \cos \chi,$$

λ being the required latitude, D the latitude of the center of the sun, and χ the position-angle of the prominence. This computation may be avoided by the use of Riccò's tables,[1] by the aid of which the latitudes given in the following tables have been corrected. The effect of refraction has not been taken into consideration because it is always smaller than the errors of measurement up to a zenith distance of at least 80°.[2] The quadrant to which the prominence belongs is indicated by the sign +, North, and −, South, combined with the column headings, East or West.

The width of the base of the prominence is reckoned in heliographic degrees, and the height in geocentric seconds. For the lower limit of height 30″ was taken in most cases, but at times prominences between 25″ and 30″ were noted. The direction of the prominences was also noted; that is, the inclination of the extremities of those prominences which do not emerge radially from the solar limb. In addition to the cardinal point, it is noted whether the prominence is directed toward the surface of the sun (down), parallel to it (||), or away from it (up). Thus some idea

[1] *Memorie della Società degli Spettroscopisti Italiani*, **10**, 21, 1881.

[2] See, for instance, *Greenwich Photo-heliographic Results*, 1905, p. ix.

is obtained as to the local disturbances to which the prominence is subjected owing to other peculiarities of the solar disk.

Besides the character of the prominences explained by the abbreviations on p. 148, the relation which probably exists between the prominences and the flocculi is given, deduced from an examination of photographs of the disk taken in calcium or hydrogen light. In various cases it is noted that bright flocculi are found on the sun's disk about the region where the chromosphere is disturbed by a prominence; but it is not to be concluded from this that a complete coincidence exists between the flocculus and the prominence because it is difficult to follow the former very close to the limb. This difficulty is still greater in the case of the dark hydrogen flocculi, which almost always, perhaps owing to lack of contrast, become invisible in the neighborhood of the limb. On the other hand, an essential difference must exist between flocculi and prominences if it is true that the maximum and the minimum of the prominence distribution in latitude do not agree with those of the flocculi. For instance, in the interval from July 1893 to September 1894,[1] while the maximum of the flocculi was in the zone 10° to 20° of latitude in both the northern and southern hemispheres, the maximum of prominences as taken from the observations of Tacchini[2] was in the zone 20° to 30° for both hemispheres.

The area of the prominences for each plate was expressed in hundred-thousandths of the visible hemisphere of the sun. This value was obtained by taking the ratio between the weight in paper, as cut from the prints, of all the prominences which were higher than 30″, and the weight of a square centimeter of the same print. This ratio, divided by the number of square centimeters contained in the visible hemisphere of the sun, and multiplied by 100,000, gives the desired value. With these quantities as ordinates the curve of the areas was obtained. The abscissas are dates, the intervals being one synodic rotation period of 27.5 days. The tables contain only those prominences with height and base equal to or

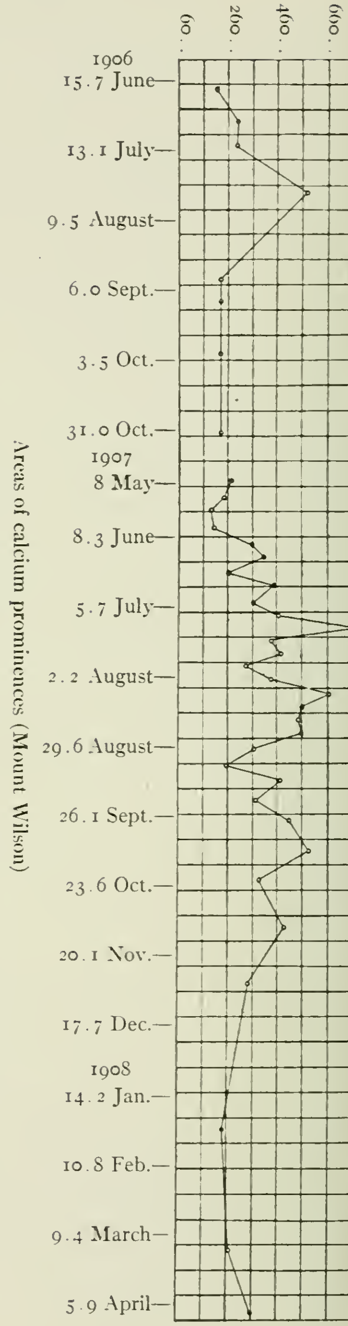
[1] Hale and Fox, "The Rotation Period of the Sun," *Publications of the Carnegie Institution of Washington*, No. 93, p. 46, 1908.

[2] *Memorie della Società degli Spettroscopisti Italiani*, **23**, 59, 67, 94, 121, 1894; **24**, 47, 1895.

exceeding 100″ and 10°, respectively. The record for the calcium prominences extends from June 1906 to April 1908; that for the *Ha* prominences from April 1908 to 1909. The results for the others have been collected for statistical and other studies and will be published when more material is available.

In Table III will be found the measurements made on prints of the Mount Wilson and Yerkes prominence photographs. Photographs of the same date are chosen for comparison and the time given is Greenwich Mean Time. The prominences common to the two places of observation, or assumed to be so, stand opposite each other in the table. The calcium prominences for both places are first compared, then the hydrogen prominences of Mount Wilson with the calcium prominences of the Yerkes photographs. Since the photographs of the two observatories were not taken at the same hour, considerable differences are found.

The mean height of the calcium prominences at Mount Wilson and at Yerkes is the same, 48″, while the *Ha* prominences at Mount Wilson have a mean height of 39″ and those of calcium at Yerkes 45″. Thus it seems that the



calcium prominences reach a greater elevation than those of hydrogen. This is also confirmed by the curve of the areas, which, throughout the period of observation, remains lower for the hydrogen than for the calcium. This result might also be produced by a decrease of the solar activity, but from an examination of the Catania observations this does not seem to be the case.[1] From the first quarter of the year 1908 to the second, in which the hydrogen prominence observations at Catania give very nearly a constant activity, the curve of the areas shows a mean daily difference of about 1/1000 of the visible solar hemisphere between the hydrogen and the calcium prominences, the area of the former being less than that of the latter. Making further a comparison between the Mount Wilson photographic and the Catania visual observations we find that the calcium prominences at Mount Wilson are about 40 per cent more than the hydrogen prominences at Catania, and that those of hydrogen at Mount Wilson are about 5 per cent less than those at Catania. The mean height of the calcium prominences at Mount Wilson (56″) is 7″ greater than that of the hydrogen prominences at Catania (49″), while the heights of the hydrogen prominences are in both places equal. This agrees well with the results of Fenyi[2] and Riccò.[3] As to the structure of the prominences, the filamentous appearance is seldom to be seen in the photographs either in the hydrogen or in the calcium; but it is probable that this peculiar structure is too difficult to be recorded ordinarily, owing to the exceptionally favorable atmospheric conditions together with the extreme regularity in the movement of the spectroheliograph which are required.

The results in the table giving the number of prominences in zones of 10° of heliographic latitude have been used to form curves, with the latitudes as abscissas and the number of observed prominences as ordinates.

Comparing the results of the Catania observations made during the same, or almost the same periods with these curves, it seems probable that the calcium prominences have a slightly different

[1] *Memorie della Società degli Spettroscopisti Italiani*, 37, 87, 1908; 38, 92, 1909.

[2] *Ibid.*, 37, 114, 1908.

[3] *Ibid.*, 38, 75, 1909.

distribution from those of hydrogen, i.e., the regions of maximum and minimum activity do not correspond in the two cases. Taking into consideration the fact that the Catania observations extend only through the second quarter of 1908, the later observations being not yet available, we see a better agreement between the curves for the hydrogen prominences observed at Mount Wilson and Catania than between those for hydrogen and calcium.[1]

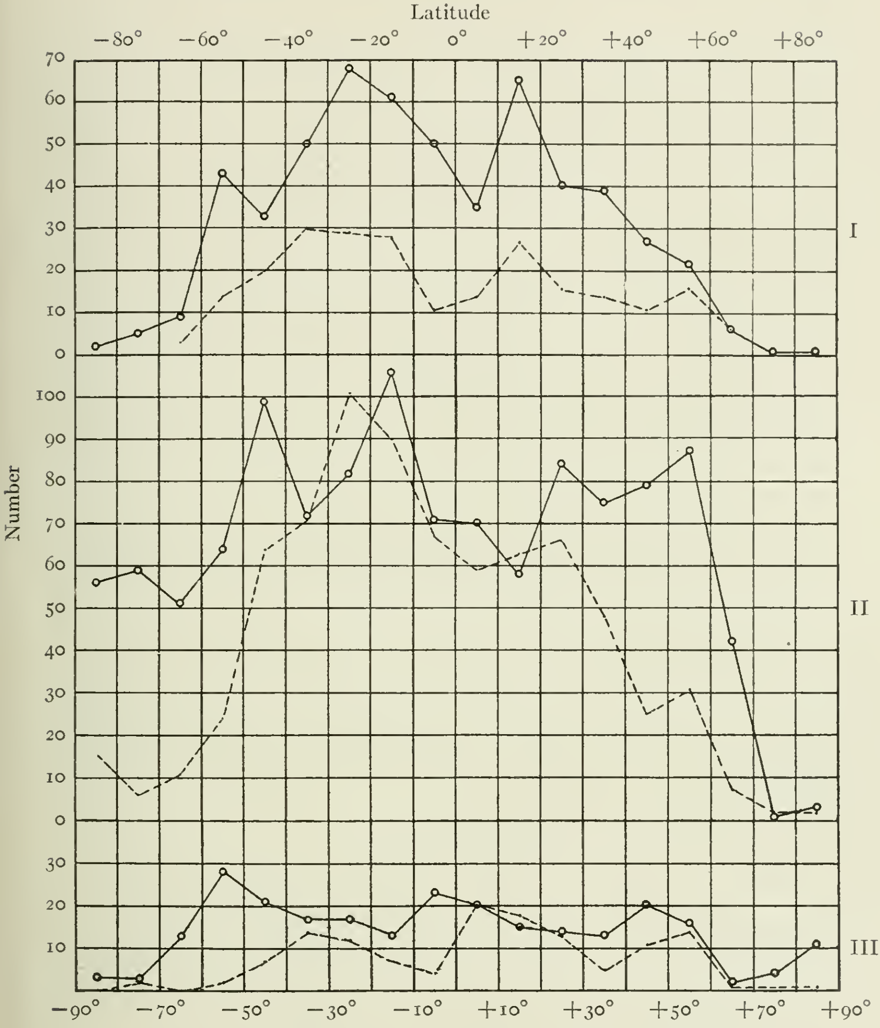
NUMBER OF PROMINENCES

MOUNT WILSON

Heliographic Latitude	June 15, 1906, to November 11, 1906 Calcium	May 15, 1907, to April 25, 1908 Calcium	March 30, 1908, to April 1, 1909 Hydrogen
Northern			
90° to 80°	11	3	1
80 to 70	4	1	1
70 to 60	2	42	6
60 to 50	16	87	22
50 to 40	20	79	27
40 to 30	13	75	39
30 to 20	14	84	40
20 to 10	15	58	65
10 to 0	20	70	35
Southern			
0° to 10°	23	71	50
10 to 20	13	106	61
20 to 30	17	82	68
30 to 40	17	72	50
40 to 50	21	99	33
50 to 60	28	64	43
60 to 70	13	51	9
70 to 80	4	59	5
80 to 90	4	56	2
Days of observation...........	39	157	126

Among the peculiar cases, those of July 17, 1907, Plate 2264, and August 11, 1907, Plate 2501, are worthy of note. In the first, between the two prominences at $-41°$ and at $-73°$ of latitude, East, a large arch is seen suspended above the chromosphere. Since there is a prominence on the plate of the preceding day (Plate

[1] A later comparison with the following Catania observations to the end of 1908 (*Memorie*, **38**, 159, 1909) confirms the above result. The maximum number of hydrogen prominences is at $+15°$ and $-25°$ for Mount Wilson and Catania; for the calcium prominences (Mount Wilson) there are two maxima in each hemisphere, at $+25°$, $+55°$ and $-15°$, $-45°$.



Distribution of the prominences in latitude

—o— Mount Wilson. Hydrogen in curve at top, calcium in lower curves

..... Catania. Hydrogen in all curves

I. Mount Wilson: March 30, 1908, to April 1, 1909. *H*
Catania: April 1, 1908, to June 30, 1908. *H*

II. Mount Wilson: May 15, 1907, to April 25, 1908. *Ca*
Catania: May 15, 1907, to March 30, 1908. *H*

III. Mount Wilson: June 15, 1906, to November 11, 1906. *Ca*
Catania: June 1, 1906, to December 31, 1906. *H*

2255) at $-74°$, it is more probable that the two prominences represent two independent objects, than that the prominence at $-73°$ is the extremity of that at $-41°$ bending down to the limb of the sun. From an examination of the photographs of the calcium flocculi taken on July 16 we see half-way between the two prominences very near to the limb of the sun a V-shaped dark flocculus about $35''$ long; and both prominences seem to be bent toward the place where the flocculus was on the preceding day. Almost a full rotation after this we find a similar case in the same region. At the same latitude, $-41°$, on August 11, 1907 (Plate 2501), there appears a prominence $100''$ high from which a long arm extends toward the east and seems to be falling on to the chromosphere at $-26°5$. In the photograph of the disk taken 3 days later (August 14, Plate 2520) a spot is seen at the same latitude (between $-24°$ and $-31°$) surrounded by very bright flocculi. Of course we cannot be sure that the prominence arch whose extremity is at $-26°5$ has its base at $-41°$, because it may well be possible that the matter itself is coming from $-26°$, but from an examination of the plates the former supposition appears the more likely.

It is through the careful discussion of observations of this kind, made both visually and spectrographically at as many stations as possible, well distributed in longitude, that we may hope materially to extend our knowledge of the varying phenomena of solar activity.

ABBREVIATIONS[1]

u = up
|| = parallel
dw = down
a v = fairly bright
v = bright
v v = very bright
n = nebulous
n n = very nebulous
n f = nebulous and filamentous
f = filamentous
f f = very filamentous
d = faint
d d = very faint

[1] See *Memorie della Società degli Spettroscopisti Italiani*, **1**, 55, 1872.

TABLE I

CALCIUM (H) PROMINENCES

Height $\geqq 100''$, or Base $\geqq 10°$

Plate No.	Date	P.S.T.	No. Prom.	Heliographic Lat. Quadrant East	Heliographic Lat. Quadrant West	Base	Height	Direction	Remarks
	1906								
492	June 29	$6^h 50^m$	1		−36°	2°.7	110″		
			10	+ 8°		10.0	55		bright floc.
500	June 30	6 50	3		+ 7	10.0	30		
552	July 8	17 24	7		+45	10.0	77		
564	July 10	6 49	5		−13	10.0	44		
			6		+ 4	15.0	52		d n bright floc.
			7		+24	10.0	56		bright floc.
575	July 11	6 52	2	−53		2.4	264	Eu	d a long slim prom.
			8	+ 6		0.4	110	Nu	bright floc.
585	July 12	17 35	5	+ 8		12.0	44		bright floc.
633	July 21	6 52	12	+27		10.0	48	Eu	
640	July 22	7 21	12	+23		18.0	81		n bright floc.
713	Aug. 2	6 42	7	+18		10.0	92		
958	Sept. 3	6 58	5		+51	10.0	44		d
1016	Sept. 10	6 55	1	− 1		2.0	110	Su	
			7	+23		10.0	30		small bright floc.
1033	Sept. 12	9 24	2	−55		5.0	121		d f
	1907								
1827	May 21	6 58	1	−48		0.5	198		f very long slim prom.
1837	May 23	6 43	1	−53		0.3	121		d f
1960	June 10	7 9	3	+57		3.0	116		d
1975	June 13	6 55	4	+ 4		—	132		detached cloud above chromosphere
2006	June 16	7 7	3		−58	11.0	48		d d
			10	−18		13.0	108		bright floc.
2012	June 17	7 16	2		−58	22.0	77		
2027	June 19	6 56	7		+38	10.0	55		
2053	June 23	7 0	2	−60		—	121		d detached clouds
2080	June 26	6 49	2		−33	13.0	48		
2099	June 28	7 13	7		+83	—	143		cloud
2117	June 30	6 55	4		+60	10.0	44		
2145	July 4	6 55	4	+65		12.0	103		d d
			5	+42		11.0	121		small dark floc.
2169	July 7	7 29	1		−16	10.0	29		bright floc.
2184	July 9	7 13	2	−21		2.0	132	Su	d
2193	July 10	6 54	1	−29		3.0	132	Su	cloud projected from top of prom. bright floc. spot.
2208	July 11	6 55	1	−32		3.0	121	Su	f spot, bright floc.
2217	July 12	6 53	1	−34		4.0	143	Eu	f tree-shaped prom. large spot 5° E
			3		−23	10.0	44	Su	
2227	July 13	6 45	1	−14		11.0	35		
			2	−34		10.0	121	Eu	

TABLE I—*Continued*

Plate No.	Date	P.S.T.	No. Prom.	Heliographic Lat. Quadrant East	Heliographic Lat. Quadrant West	Base	Height	Direction	Remarks
	1907								
2236	July 14	6^h56^m	1	−16°		10°.0	66″	Su	bright floc.
2245	July 15	6 57	7		+35°	1.0	110	Wu	f tree-shaped
2255	July 16	7 8	2	−52		1.0	143		d
			4		−31	27.0	77	Wu	n
2264	July 17	6 46	2	−41		2.4	220	Su	form an arch. 2_1 may be the extremity of prom. 2. dark floc. between 2 and 2_1 and prom. is bent in toward it
			2_1	−73		0.5	242	Eu	
			3	−85		1.5	120	Eu	d
			4		−72	1.8	121	Su	d d
			6		−35	1.5	176	Su	n small bright floc.
2284	July 19	6 56	2	−84		0.8	132	E ||	f
			5		+25	—	176		large cloud
			7	+10		20.0	48		bright floc.
2297	July 20	7 9	2	−76		1.0	132	Su	d long thin prom.
2303	July 21	7 2	1	−15		10.0	55		bright floc.
			4	−84		0.6	132		f tree-shaped
2312	July 22	7 13	3		−85	2.0	132	Su	f
2318	July 23	9 46	2		−84	2.0	121	Su	d
			6	+48		3.0	110	Eu	
2329	July 24	7 32	3		−30	—	143		small clouds d d
2345	July 25	6 53	1		−83	1.0	120	Su	
			2		−30	2.0	121	Su	bright floc.
			4		− 7	10.0	132	Nu	f f
2354	July 26	6 45	1	−12		12.0	66		
			5		−21	0.4	154	Wu	d n bright floc.
2373	July 28	6 54	2		−81	1.3	121	Su	
			10	+19		10.0	55	Eu	f
2383	July 29	6 49	4		−44	6.0	101	Wu	d
			7		+33	10.0	55	Wu	
2393	July 30	7 19	7		+30	18.0	37		
2403	July 31	7 32	2		−80	1.0	121	Su	d one filament
			4		+23	11.0	42		
			6	− 2		12.0	37		bright floc.
2420	Aug. 2	7 11	6		+11	10.0	59		v
2430	Aug. 3	6 47	2	−44		1.3	132		f
2439	Aug. 4	7 6	1	−44		1.0	132	Su	d f
			3		−11	13.0	66		
2457	Aug. 6	6 52	2	−29		1.0	121	Eu	
			13	+ 1		12.0	48		
2476	Aug. 8	7 8	4		−52	4.0	121	Su	
			7	+56		0.6	110		
2495	Aug. 10	6 54	7	−71		2.0	253	Su	d d f; near base
2507	Aug. 12	6 53	1	−43		11.0	60		
			5	+19		25.0	77		bright floc. on Aug. 11

TABLE I—*Continued*

Plate No.	Date	P.S.T.	No. Prom.	Heliographic Lat. Quadrant East	West	Base	Height	Direction	Remarks
2516	1907 Aug. 13	6h50m	6	+59°		15°0	67″		
			7	+26		40.0	99		v v very large prom. small bright floc.
2525	Aug. 14	6 58	2	−43		10.0	48	Eu	
			10	+61		2.5	143		d
			11	+20		17.0	77		v small bright floc.
2534	Aug. 15	6 48	8	+11		20.0	52		
2543	Aug. 16	6 44	1	−14		10.0	44		bright and dark floc.
			5	+38		10.0	77	Eu	
2554	Aug. 17	17 9	1	−11		23.0	60		
			5		−57°	2.5	242	Wu	f } form an arch
			6		−47	3.0	220	Su	
2560	Aug. 18	6 55	4		−73	1.5	132		d small cloud above prom.
2569	Aug. 19	6 53	1	−36		15.0	34		
			9		+43	—	198		cloud high above chromosphere
2578	Aug. 20	6 45	9		+16	12.0	55		
			10		+58	10.0	88	Wu	
			12	+47		3.0	132	S ||	f
2587	Aug. 21	6 52	3	−54		13.0	48	Eu	
			9		+52	1.5	165		
2597	Aug. 22	6 52	4	−58		13.0	77	Eu	
2606	Aug. 23	7 0	1	−21		13.0	143		
2619	Aug. 24	6 59	1	−21		—	253	Eu	d cloud high above chromosphere
			2	−59		7.0	120		bright floc.
2629	Aug. 25	7 15	1	−26		12.0	132		bright floc.
2657	Aug. 28	7 10	5		+ 4	20.0	92		bright floc.
2667	Aug. 29	7 24	2	−84		21.0	88		
			5		+10	18.0	81		bright floc.
2685	Aug. 31	7 6	4		−55	37.0	121		d f
2699	Sept. 3	7 2	1	−44		15.0	72	Eu	
2708	Sept. 4	6 54	1	−44		12.0	66		
			3		−19	15 0	59		bright floc.
2718	Sept. 5	7 11	2		−20	4.0	132		d bright floc.
2736	Sept. 7	7 4	4		−14	—	132		d d small cloud above chromosphere. bright floc.
2752	Sept. 9	7 27	6		− 8	1.5	132		
2759	Sept. 10	7 15	4		− 4	0.3	132		f
			7		+57	10.0	103	Wu	
2780	Sept. 13	7 15	5		−47	5.0	132	Wu	
2789	Sept. 14	7 13	2		−39	20.0	126	Su	f
			3		− 6	—	120		d small cloud
2798	Sept. 15	7 20	6	+28		17.0	70		f
2806	Sept. 16	7 16	3		+42	1.3	120		d
2815	Sept. 17	7 7	10	+24		16.0	120		f d

TABLE I—*Continued*

Plate No.	Date	P.S.T.	No. Prom.	Heliographic Lat. Quadrant East	Heliographic Lat. Quadrant West	Base	Height	Direction	Remarks
	1907								
2824	Sept. 18	7^h16^m	3		−49°	2°0	132″		d
2851	Sept. 21	7 30	1	−47°		—	121		d d small cloud
2897	Sept. 27	7 11	6		−65	6.0	120		d d
2906	Sept. 28	7 16	6		−59	1.4	121	Su	
2918	Sept. 30	7 19	4		+19	5.0	165	Wu	
2927	Oct. 1	7 15	7		+24	7.0	120	Wu	f
2979	Oct. 10	8 9	1	− 2		14.0	120		d d
			9		−30	20.0	103		
2988	Oct. 11	7 50	1	−40		13.0	41		v
			7		−37	4.4	176	Su	
			8		−30	1.3	100	Su	
3043	Oct. 20	7 44	7	+67		6.0	120		
3059	Oct. 29	8 5	1	+ 4		8.0	121		
			9		+33	11.0	87		d
3084	Nov. 2	8 17	4	−82		1.0	120	Eu	f
			6		+23	11.4	35		
3102	Nov. 4	8 59	5	+35		—	120		d cloud above chromosphere
3208	Nov. 22	8 7	5		+ 7	0.4	110		f
			6		+ 9	0.4	110	Wu	f bright floc. spot
3258	Nov. 30	8 27	2	−34		14.0	44		bright floc.
			7		−28	13.0	60		
3386	Dec. 27	10 26	1	−43		3.0	143	S dw	f
	1908								
3504	Feb. 4	8 40	2		−46	—	132		cloud
3519	Feb. 7	8 41	1		−52	11.0	100		
3646	Mar. 11	8 12	6	−15		3.4	110		f
3745	Mar. 24	8 3	2		+46	1.0	136	Wu	
3763	Mar. 28	10 8	2		+ 4	10.0	66		f

TABLE II

HYDROGEN ($H\alpha$) PROMINENCES

Height $\geqq$ 100″, or Base $\geqq$ 10°

Plate No.	Date	P.S.T.	No. Prom.	Heliographic Lat. Quadrant East	West	Base	Height	Direction	Remarks
	1908								
4019	May 5	8^h16^m	1		−31°	11°.0	55″		
4069	May 16	8 11	4	−20°		18.0	76		
4084	May 17	17 29	2	−18		12.0	77		f f dark floc. on $H\alpha$ plate
4292	June 12	17 58	2		+ 7	10.0	42		bright floc.
4485	July 1	6 55	1		−41	11.0	66	Su	
4534	July 5	17 43	2		+ 1	10.0	66		spot. bright floc.
4702	July 26	7 12	3		+34	5.0	108		
4925	Aug. 27	6 40	2		− 4	10.0	30		bright floc.
4944	Aug. 29	6 44	5		− 2	16.0	77		bright floc.
5009	Sept. 4	7 12	6		−15	—	132		d d cloud above chromosphere. spot
5021	Sept. 5	7 35	1	−9		13.0	44		
5036	Sept. 8	7 17	3		+33	10.0	59	W ‖	n
5115	Sept. 18	7 26	1	−21		2.8	110		d d n
5238	Oct. 7	7 19	3	−56		6.4	110		
5267	Oct. 10	7 45	3		−38	1.8	102		cloud detached from upper part of chromosphere
5358	Oct. 28	7 26	7		+14	10.0	42		bright floc.
5380	Oct. 31	7 15	3		−34	10.0	57	W ‖	cloud detached from upper part of prom.
5420	Nov. 9	7 49	5	+13		24.0	88		
5426	Nov. 10	7 26	1	+ 5		25.0	88		
5456	Nov. 17	7 43	3		−32	10.0	57		
5461	Nov. 18	7 43	5		+14	1.0	136		f bright floc.
5477	Nov. 23	7 46	1	−11		—	100		large detached cloud above chromosphere
5491	Nov. 30	8 2	5		+35	2.6	154		
5494	Dec. 7	7 58	2		−20	4.0	143		small bright floc.
	1909								
5657	Feb. 26	8 37	1	−10		15.0	56		d
5661	Feb. 27	8 19	5	− 7		20.0	62		f
5666	Feb. 28	8 39	4	− 3		16.0	54		
5703	Mar. 14	8 40	2		+11	15.0	60		small bright floc.
5722	April 1	9 0	2		+17	5.0	37		bright floc.

TABLE III

COMPARISON OF PROMINENCES OBSERVED AT MOUNT WILSON AND WILLIAMS BAY

Mount Wilson							Williams Bay*						
Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction	Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
Calcium (H) Prominences							Calcium (H) Prominences						
1906							1906						
	1	+71°		1°	67″	Eu		1	+72°		2°	85″	N dw
	2	+ 5		1	56								
500	3	−22		6	34		2038						
June 30							June 30	2	−69		4	53	
14^h50^m							17^h44^m	3		−78°	—	64	
	4		−53°	2	22			4		−54	1	32	
	5		+ 8	6	22								
	1	−34		2	67	Su							
	2	−54		3	90	Eu		1	−54		7	42	
	3		−60	2	22	Wu		2		−60	1	32	Wu
575	4		−63	1	34		2047	3		−62	1	21	Su
July 11	5		−19	3	56		July 11						
14^h52^m	6		+ 5	7	56		21^h53^m						
								4		+20	6	32	
	7		+50	6	56			5		+50	2	32	
	8	+ 4		1	90	N ‖							
	1	−55		5	45								
	3		−62	1	34								
578	31		—60	1	34								
July 12	4		−11	2	22								
1^h48^m	5		− 3	2	45								
	6		+ 3	4	56								
	7		+18	3	34								
	8		+50	3	67								
	1	−48		1	168	f							
1907	2	−67		2	67	Eu	1907	1	−71		10.0	244	Eu
1827							2281	2	−77		2	74	
May 21	3		−86	3	34		May 21	3		−84	2	42	
14^h58^m	4		−43	3	45		16^h52^m	4		−44	4	53	S dw
	5	−30		1	45								
	1	−64		3	45			1	−62		1	32	
								2	−66		3	53	
								3	−72		1	42	
								4		−46	5	42	
	2		−27	1	34	Wu		5		−25	2	42	
1879	3		−20	2	34		2288	6		−18	2	42	S ‖
May 28	4		−13	6	78		May 28						
15^h24^m	5		− 6	2	34		16^h12^m	7		− 5	2	74	
	6		− 1	4	34								
								8		+31	4	42	
	7		+39	3	56								
	8		+41	2	34								
	9	+34						9	+33		3	32	

* It should be stated that these plates taken at Williams Bay are not suitable for a statistical comparison of this sort, because the diameter of the solar image is so nearly equal to the width of the plate that prominences near the east and west limbs generally do not make a sufficient impression. To cover these portions of the sun two exposures are necessary, with the limbs successively set nearer the center of the plate. Unfortunately, in the plates compared here, this procedure had not been followed, as no such comparison was contemplated.

TABLE III—*Continued*

MOUNT WILSON

Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
		Calcium (H) Prominences				
1907						
2264 July 17 14^h47^m	1	−11°		8°	45″	
	2	−41		2	112	S ‖
	3	−70		2	157	E ‖
	4	−84		1	22	
	5		−60°	1	34	
	6		−37	2	168	Su
	7		+ 1	8	34	
	8	+38		2	34	
2495 Aug. 10 14^h54^m	1	−13		1	67	Eu
	2	−20		4	67	Eu
	3	−34		4	22	
	4	−42		6	90	
	5	−51		5	56	
	6	−72		—	235	d
	7	−88		2	45	
	8		+53	4	34	
	9	+53		3	34	
2525 Aug. 14 14^h58^m	1	−23		3	22	
	2	−43		6	45	Eu
	3	−79		1	55	
	4		−32	5	45	
	5		+ 7	0.5	90	Wu
	6	+19		14	67	
	7	+61		2	143	
2606 Aug. 23 15^h0^m	1	−20		15	146	E ‖
	2	−55		12	90	
	3	−77		6	67	
	4		−74	3	45	
	5	+56		2	45	
2780 Sept. 13 15^h15^m	1	−38		5	56	
	2	−51		4	34	
	3	−78		3	67	d
	4		−46	5	123	W ‖
	5		−38	7	67	W ‖
	6		+11	0.5	33	
	7		+21	3	34	N ‖
	8		+27	2	34	N ‖
		Hydrogen (*Hα*) Prominences				
1908						
3925 April 20 15^h57^m	1	+62		2	44	
	2	−17		2	34	
	3	−25		6	34	

WILLIAMS BAY

Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
		Calcium (H) Prominences				
1907						
2390 July 17 17^h56^m	1	− 7°		3°	32″	S ‖
	2	−82		4	106	
	3		−73°	1	64	
	4		−61	14	201	W |
	5	+40		1	21	
2443 Aug. 10 15^h1^m	1	−14		2	42	Eu
	2	−23		4	64	Eu
	3	−37		5	32	
	4	−44		6	85	Su f
	5	−54		4	53	
	6	−74		—	170	d
	7	−84		3	95	d d
	8		+53	4	32	
	9	+51		3	32	
2463 Aug. 14 17^h36^m	1	−31		3	85	E dw
	2	−45		8	42	f
	3	−81		2	42	
	4	+59		—	127	
2479 Aug. 23 15^h54^m	1	−22		9	64	E ‖
	2	−56		11	74	S dw
	3	−80		6	64	
	4	−74		1	32	
2507 Sept. 13 16^h54^m	1	−33		5	64	
	2	−43		6	32	
	3	−56		2	32	
	4	−77		1	53	
	5	−81		2	21	
	6		+ 8	7	53	
	7		+21	2	42	N ‖
	8		+28	2	32	N ‖
	9	− 5		4	21	
		Calcium (H) Prominences				
1908						
2684 April 20 16^h34^m	1	+60		8	42	
	2	+22		1	32	
	3	−85		—	64	d d

TABLE III—*Continued*

Mount Wilson — Hydrogen ($H\alpha$) Prominences

Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
1908						
4233 June 6 16^h5^m	1		+ 4°	2°	56″	
	2		+ 8	3	45	d d
	3		+11	1	22	
	4		+20	2	56	
	5		+27	1	22	
4292 June 13 1^h58^m	1	−51°		3	34	
	2		+ 6	7	34	E dw
	3	+56		2	22	
	4	−22		4	45	Nu
4437 June 26 15^h11^m	1		−26	1	22	
	2		+27	2	45	
	3		+51	2	22	
4594 July 14 14^h46^m	1	−25		2	34	
	2	−40		2	56	
	3	−58		3	34	
	4		−71	1	34	
	5		−21	3	22	
	6		+17	3	34	
	7	+53		2	34	
	8	+ 1		5	56	Nu
	9	− 4		4	—	
4601 July 15 14^h35^m	1	−27		2	34	
	2	+ 1		1	45	
	3	−11		1	22	
4667 July 22 14^h45^m	1	−20		2	22	
	2		−67	1	45	
	3		−60	1	34	
	4		−35	2	78	
4719 July 28 14^h54^m	1		−59	1	22	
	2		−31	3	45	
	3		+38	2	22	
	4		+40	2	34	
	5		+53	1	34	
	6		+56	2	45	
	7	+21		4	22	

Williams Bay — Calcium (H) Prominences

Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
1908						
2736 June 6 16^h44^m	1	−43°		3°	32″	
	2		−45°	2	21	
	3		−34	1	21	
2741 June 12 15^h55^m	1	−55		4	42	
	2	+55		3	21	
2783 June 26 15^h43^m	1		−22	5	32	
	2		+27	3	42	
	3		+52	3	21	
	4		−55	—	117	
2827 July 14 16^h36^m	1	−28		2	21	
	2	−45		5	74	E ‖
	3	−61		5	95	E ‖
	4		−65		85	
	5		+19	4	53	
	6	+52		2	21	
	7	+ 1		2	32	
	8	−14		5	32	
2838 July 15	1	−29		2	42	S ‖
	2	−52		1	53	Eu
	3		−39	1	64	
	4		−35	2	64	
	5	−13		2	42	
2847 July 22 15^h56^m	1		−66	6	85	W ‖
	2		−59	1	64	S ‖
	3		−31	2	95	Wu
	4		+59	1	32	
	5	−48		—	64	
	6		−76	3	42	W ‖
2875 July 28 15^h22^m	1		−61	3	95	
	2		−57	2	148	
	3		−31	4	53	
	4		+38	4	53	
	5		+42	2	42	
	6		+53	5	42	

TABLE III—*Continued*

Mount Wilson

Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
Hydrogen ($H\alpha$) Prominences						
1908						
4729 July 29 15^h42^m	1		−31°	5°	34″	
	2		−25	2	56	
	3		−16	3	22	
	4		+39	3	22	
	5		+54	7	45	
	6	+32°		1	22	
	7	+28		1	34	
	8	+17		1	22	
	9	+11		1	56	
4747 Aug. 5 2^h3^m	1	−50		8	101	
	2		+34	2	34	
	3	+45		2	34	
4776 Aug. 8 14^h40^m	1	−30		3	22	
	2		− 5	2	45	
	3		− 2	2	22	
	4		+25	3	34	
	5		+29	2	45	
	6		+34	3	56	
	7	+53		2	45	
4811 Aug. 13 14^h51^m	1	−48		2	67	
	2	−53		2	67	
	3	−57		1	45	
	4		−41	1	34	
	5		−25	2	22	
	6		+34	2	22	
	7		+39	3	34	
	8	+56		3	45	
	9	+52		1	34	
	10	+18		2	34	Eu
4819 Aug. 14 14^h54^m	1	−52		4	34	
	2	−56		0.5	45	
	3	−60		0.5	22	
	4		−38	1	45	Su
	5		−35	1	34	S ||
	6		−21	2	34	
	7		+25	1	34	
	8		+36	3	45	

Williams Bay

Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
Calcium (H) Prominences						
1908						
2879 July 28 15^h58^m	1		−61°	3°	95″	
	2		−56	2	148	
	3		−30	4	53	
	4		−38	5	42	
	5		+42	2	42	d d
	6		+54	5	42	
2883 July 29 17^h14^m	1		−29	4	32	
	2		−22	2	74	W ||
	3		−15	2	32	
	4		+40	4	42	
	5		+54	7	42	
2914 Aug. 4 20^h56^m	1	−53°		8	95	
	2		+34	4	21	
	3	+45		3	32	
	4	−35		1	32	
2938 Aug. 8 16^h8^m	1	−32		3	32	
	2	−57		2	32	
			− 4	2	32	
	3		+26	5	42	
	4		+34	2	53	W ||
	5	+55		7	42	
2947 Aug. 13 15^h42^m	1	−48		5	95	E ||
	2	−54		3	95	E ||
	3	−59		2	53	
	4		−40	1	42	
	5		−33	5	32	
	6		+24	5	32	
	7		+35	5	117	N dw
	8	+55		3	32	
	9	−36		1	32	
2956 Aug. 14 17^h56^m	1	−56		7	64	
	2	−62		1	53	
	3		−35	3	42	d d

TABLE III—*Continued*

Mount Wilson — Hydrogen ($H\alpha$) Prominences

Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
1908						
4881 Aug. 21 15^h0^m	1		−57°	2°	34″	
	2		−31	2	78	Su
	3		+12	—	45	
	4		+46	4	56	
	5	+22°		1	34	
4890 Aug. 22 15^h1^m	1	−10		3	34	
	2	−19		2	34	
	3		−63	2	22	
	4		+48	6	67	
	5	+17		6	34	
	6	+10		2	34	
4925 Aug. 27 14^h40^m	1	−29		8	34	
	2		− 4	9	34	
	3		+28	7	22	
	4	+12		4	22	
	5	+ 4		2	22	
4944 Aug. 29 14^h44^m	1	−19		1	22	d
	2	−32		0.5	45	d
	3	−39		2	67	d
	4		−55	2	34	
	5		− 1	11	67	
	6		+28	2	56	
	7		+44	1	22	
4985 Sept. 2 15^h16^m	1	− 3		2	22	
	2	−12		3	22	
	3	−25		2	34	Su
	4		− 7	9	101	
	5		− 3	1	45	
	6		+13	1	45	
	7		+17	1	34	
	8		+42	1	45	
	9	+48		1	34	
4998 Sept. 3 15^h22^m	1	−27		1	34	
	2		− 5	—	56	
	3		+18	9	74	
	4	+63		2	22	
	5	+52		3	56	E ‖
	6	+46		1	56	N ‖
5009 Sept. 4 15^h12^m	1	− 9		4	34	
	2	−14		1	34	
	3	−20		2	22	
	4	−29		0.5	11	
	5		+ 6	2	22	
	6		+10	1	34	
	7		+17	4	34	

Williams Bay — Calcium (H) Prominences

Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
1908						
2970 Aug. 21 17^h14^m	1		−54°	2°	32″	
	2		−50	3	32	
	3		+48	4	64	
2974 Aug. 22 15^h44^m	1	−11°		2	21	
	2	−20		1	21	
	3		−61	2	21	
	4		+36	1	53	W ‖
	5		+48	6	74	
2990 Aug. 27 16^h5^m	1	−31		8	21	
	2		− 4	10	42	d
	3		+20	10	85	d d
3006 Aug. 29 15^h36^m	1	−20		1	21	
	2	−33		1	53	
	3	−39		2	85	Su
	4		−54	2	32	
	5		+28	2	64	
3036 Sept. 2 16^h37^m	1	− 2		4	21	
	2	−25		1	32	S ‖
	3		+15	1	42	
	4		+42	1	53	W ‖
	5	+47		2	32	
3042 Sept. 3 17^h37^m	1	−29		1	32	
	2		+14	1	78	N dw
	3		+29	2	42	
	4	+61		0.5	21	
	5	+52		1	74	Eu
	6	+45		1	32	
3047 Sept. 4 17^h44^m	1	− 9		7	21	
	2		+ 6	4	32	
	3		+16	4	32	

TABLE III—*Continued*

Mount Wilson — Hydrogen ($H\alpha$) Prominences

Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
1908						
5036 Sept. 8 15^h17^m	1	−53°		1°	22″	
	2		−29°	6	42	
	3		+34	7	64	W ||
5199 Oct. 1 15^h14^m	1	−50		2	42	
	2		−14	2	42	W ||
	3		+ 4	1	21	N ||
	4		+17	5	21	
	5		+38	3	42	
5238 Oct. 7 15^h19^m	1	+ 1		3	42	
	2	−33		4	64	E ||
	3	−56		7	95	Su f
	4		−26	8	32	
	5		+29	3	64	Nu
	6		+34	2	42	
	7	+38		1	21	
5267 Oct. 10 15^h45^m	1	−29		1	32	
	2	−71		3	21	
	3		−39	2	22	Su
	4		−14	2	22	S ||
	5		− 6	1	21	
5420 Nov. 9 15^h49^m	1	− 4		3	32	
	2	−17		4	32	
	3		−49	2	85	Su
	4		−44	1	42	
	5	+11		>12	74	

Williams Bay — Calcium (H) Prominences

Plate No., Date, and G.M.T.	No. of Prom.	Hel. Lat. and Quad. East	Hel. Lat. and Quad. West	Base	Height	Direction
1908						
3060 Sept. 8 17^h9^m	1	−53°		4°	32″	d d
	2		+32°	3	42	W ||
	3		+37	4	74	W ||
	4	−33		2	21	
3138 Oct. 1 15^h43^m	1	−51		3	42	
	2		+ 6	2	42	
	3		+19	7	32	
	4		+40	5	42	
3160 Oct. 7 17^h55^m	1	− 1		2	42	
	2	−36		4	64	E dw
	3	−57		8	>148	Su
	4		+30	1	95	
	5		+35	2	42	
3172 Oct. 10 16^h10^m	1	−29		1	32	
	2	−70		5	32	
	3		−53	3	21	
	4		− 7	1	42	
	5	+ 3		1	32	
3205 Nov. 9 16^h49^m	1	− 5		3	53	
	2	−18		3	42	
	3		+40	2	21	

REMARKS

Mount Wilson

1879 (2)–(6) big prominence covering almost all the space between −27 W and +1 W.

2264 between (2) and (3) big arch. (6) big cloud almost detached from chromosphere.

Williams Bay

2038 (3) small cloud detached from chromosphere.

2281 (1) big ring. *Astroph. Journ.*, **26**, 155, 1907

2288 (5)–(7) big prominence covering all the space between −25 W and −4 W.

2390 (4) beautiful prom. extending very far toward W.

REMARKS—*Continued*

Mount Wilson		Williams Bay	
2495	(6) big cloud floating above chromosphere.	2443	(6) cloud detached from chromosphere: from a later plate the cloud seems to fall on the chromosphere. (7) high cloud detached from chromosphere.
2525	(5) d. thin filament.	2463	(1) f. prom. falling on the chromosphere over a low prom. (4) thin filament floating above chromosphere.
2606	(1) big mass compact.	2479	(1) very d toward E.
		2507	from (1) to (3) in connection.
4233	(1) thin at base like a detached cloud floating above chromosphere.	2684	(3) detached from chromosphere.
4594	(4) a small detached cloud.	2783	(4) arrow-shaped prom. detached from chromosphere.
		2827	(2) and (3) d. cloud hanging over the chromosphere between prom. (2) and (3). (4) small prom. hanging over the chromosphere.
4881	(3) small cloud detached from chromosphere.	2827	(7) and (8) one hour later on plate 2828. prom. (7) is 117″ high and (8) is 42″ high.
4985	(4) big cloud floating above chromosphere. (8) detached from chromosphere.	2847	connection between (1), (2), and (6). (6) toward (1); (1) toward (2); (2) toward (1).
4998	(2) detached from chromosphere. (5) and (6): (5) is bent toward (6) and (6) toward (5).	2883	long thin stream going W.
5009	(6) in connection with (7).	2947	(1) almost going to touch (9). (7) big mass going up and falling on the chromosphere at +45 W.
5199	(2) small cloud suspended toward W.	3006	(4) small arch.
5238	(2) extending to −29 E toward East.	3160	(2) long arm extending downward to −25°. (4) almost detached from chromosphere.
5267	(3) very faint cloud going toward S.		

We desire to express our appreciation of the kindness of Mr. Ellerman and of Mr. Fox, which enabled us to make the above measures.

Mount Wilson Solar Observatory
December 1910

PHOTOGRAPHIC DETERMINATIONS OF STELLAR PARALLAX MADE WITH THE YERKES REFRACTOR. III

By FRANK SCHLESINGER

THE METHOD OF REDUCTION

Before we can compare one plate with another, and so derive the shifts due to parallax and proper motion, it is necessary to clear the measures of certain other causes of difference. These may be classified into two groups, the first of which comprises *orientation*, *scale*, and *zero* corrections; the second, *refraction*, *aberration*, *precession*, and *nutation*. Those in the first group are to be determined from the measures themselves, and, under these circumstances, as is well known, we need pay no attention to those in the second group. The reason for this is that the correction for scale (a) of the co-ordinate X, is of the form $a \cdot X$; that for orientation (b) is proportionate to the other co-ordinate Y, and takes the form $b \cdot Y$; while the zero correction (c) is applied as a constant to all the measures. The sum of the three is $a \cdot X + b \cdot Y + c$. It is obvious that this is also the form assumed by any correction whatever that we may take to be linear over the narrow area covered by a plate; consequently, as Turner first pointed out, any such correction is automatically applied when we determine the scale, etc., from the measures themselves. The more rigorous expressions for the corrections in the second group contain terms in X^2, Y^2, and $X \cdot Y$, not to mention those of still higher order; but all of these are inappreciable on these plates, the distance from the parallax star to any comparison star never exceeding half a degree.

For those cases in which both co-ordinates have been measured, the two have been reduced without reference to each other in order that the derived parallaxes may be entirely independent. Had we combined the two sets into one solution, it would have been necessary to apply corrections for refraction, and to employ Jacoby's method. This additional work is justified if the parallax star is far removed from the mean position of the comparison stars.

In reducing all the plates to the same scale, etc., it makes no practical difference which of them is assumed to be the standard. The truth of this statement is tolerably obvious and a rigorous proof is easily adduced. Having then chosen one of the plates for this purpose, or the mean of several, the usual method is to deduce a, b, and c from a least-squares solution. The observation equations, of which there will be one for each of the n comparison stars, take this form:[1]

$$\left.\begin{aligned} a\cdot X_1+b\cdot Y_1+c+(X'_1-X_1)&=v_1 \\ a\cdot X_2+b\cdot Y_2+c+(X'_2-X_2)&=v_2 \\ \text{etc.}& \end{aligned}\right\} \quad (1)$$

Here X_1, X_2 refer to the standard plate and X'_1, X'_2 to the plate that we are reducing. The analytical expressions will be simplified if we suppose that a certain constant has been subtracted from the co-ordinates on the standard plate, so as to make $[X]$ and $[Y]$ each equal to zero. The normal equations are then as follows:

$$\left.\begin{aligned} [X^2]a+[X\cdot Y]b \qquad &= -[X(X'-X)] \\ +[Y^2]b \qquad &= -[Y(X'-X)] \\ +n\cdot c &= -[X'-X]=-[X'] \end{aligned}\right\} \quad (2)$$

Whence

$$\left.\begin{aligned} a&=\frac{[Y(X'-X)][X\cdot Y]-[X(X'-X)][Y^2]}{[X^2][Y^2]-[X\cdot Y]^2}= \\ &\qquad\qquad 1-\frac{[X\cdot X'][Y^2]-[X'\cdot Y][X\cdot Y]}{[X^2][Y^2]-[X\cdot Y]^2} \\ b&=\frac{[X(X'-X)][X\cdot Y]-[Y(X'-X)][X^2]}{[X^2][Y^2]-[X\cdot Y]^2}= \\ &\qquad\qquad \frac{[X\cdot X'][X\cdot Y]-[X'\cdot Y][X^2]}{[X^2][Y^2]-[X\cdot Y]^2} \\ c&=-\frac{1}{n}[X'] \end{aligned}\right\} \quad (3)$$

Finally we compute for the parallax star

$$X'_\pi+a\cdot X_\pi+b\cdot Y_\pi+c. \quad (4)$$

This quantity, which we shall designate by m', may be called the solution of this plate. A comparison of the values of m', m'', from various plates enables us to determine the parallax.

[1] We confine our attention for the present to the case in which co-ordinates in only one direction have been measured.

The labor of computing the plate-constants by this method, for as large a number of plates as are included in the present work, is very considerable. The following method is much shorter and leads to precisely the same results.

The plate-constants are of no interest in themselves in the present connection. Their computation may be altogether avoided, and the numerical work much abridged, by expressing m' directly in terms of the observed quantities X'_1, X'_2, etc. To effect this we substitute in (4) the values of a, b, and c as given in equations (3), and then collect the terms in X'_1, those in X'_2, etc.

$$\left.\begin{aligned}
m' &= X'_\pi + X_\pi \\
&\quad - X'_1\left\{\frac{X_1(X_\pi[Y^2]) - Y_1(X_\pi[XY]) + Y_1(Y_\pi[X^2]) - X_1(Y_\pi[XY])}{[X^2][Y^2]-[XY]^2} + \frac{1}{n}\right\} \\
&\quad - X'_2\left\{\frac{X_2(X_\pi[Y^2]) - Y_2(X_\pi[XY]) + Y_2(Y_\pi[X^2]) - X_2(Y_\pi[XY])}{[X^2][Y^2]-[XY]^2} + \frac{1}{n}\right\} \\
&\quad - \text{etc.}
\end{aligned}\right\} \quad (5)$$

In this notation the comparison stars are distinguished from each other by subscripts. The co-ordinates on the standard plate are distinguished from those upon others by the omission of the primes. It will be noticed that the quantities in parentheses are the same for all the comparison stars, and those in the curved brackets the same for all the plates. It will be convenient to have a name for these coefficients in curved brackets; as they show the dependence of the solution (m) upon the comparison stars, we shall call them *dependences* and shall designate them by D_1, D_2, etc. In comparing the values of m from different plates we are at liberty to add a constant to them all, and we may accordingly omit the term X_π that appears outside the brackets in equation (5). This leaves the very simple expression,

$$m' = X'_\pi - [D \cdot X'] \quad (6)$$

Similarly for any other plate $m'' = X''_\pi - [D \cdot X'']$.

The following is a simple numerical example. It relates to the double star *Positiones Mediae 2164* ($18^h\ 42^m, +59°\ 29'$) which is one of those that were measured in right ascension and declination.

Standard Plate		Plate 664 1st Exposure	Plate 664 2d Exposure
$X_1 = -378$	$Y_1 = +62$	$X'_1 = 198.766$	$X''_1 = 175.999$
$X_2 = -8$	$Y_2 = +200$	$X'_2 = 569.351$	$X''_2 = 546.657$
$X_3 = +84$	$Y_3 = -130$	$X'_3 = 660.934$	$X''_3 = 638.083$
$X_4 = +302$	$Y_4 = -132$	$X'_4 = 879.485$	$X''_4 = 856.622$
$X_\pi = -40.3$	$Y_\pi = +75.5$	$X'_\pi = 535.133$	$X''_\pi = 512.404$

From the co-ordinates on the standard plate we have

$$[X^2] = 241{,}208, \qquad [Y^2] = 78{,}168, \qquad [XY] = -75{,}820.$$

Substituting in (5), we obtain

$$D_1 = +.000196X_1 + .00116Y_1 + 0.250 = 0.248$$
$$D_2 = +.000196X_2 + .00116Y_2 + 0.250 = 0.480$$
$$D_3 = +.000196X_3 + .00116Y_3 + 0.250 = 0.115$$
$$D_4 = +.000196X_4 + .00116Y_4 + 0.250 = 0.156$$

These results may be controlled by the equations

$$[D] = 1, \qquad [D \cdot X] = X_\pi, \qquad [D \cdot Y] = Y_\pi \tag{7}$$

We have computed these dependences to three decimal places. This is more than ample, and we may if we wish round them off to two decimals. In that case the second and the third control equations will not be so closely satisfied, but this is of no consequence *so long as the orientation and the scale of all the plates to be compared are approximately the same.* The first control should be exactly satisfied by whatever dependences are adopted. In the present example we may take

$$D_1 = 0.25, \quad D_2 = 0.48, \quad D_3 = 0.11, \quad D_4 = 0.16.$$

The work up to this point is done once for all. We now proceed to reduce the separate plates, or rather the separate exposures; for the numerical work that this method involves is so slight, that it requires hardly more time to treat the exposures separately than to take the means for the three sets of measures on each plate, and then to reduce these means as a whole. The advantages of separate reduction are obvious.

Applying the dependences to Plate 664, for which the measurements of the first and the second exposures are given above, we obtain,

$$m' = -1.267, \qquad m'' = -1.239, \qquad m''' = -1.243.$$

It is the mean of these three that appears under the column *m* in the tables of results to be given later.

The dependences not only abridge the computations but they have several other useful applications. They furnish an exact criterion for the *importance* of a comparison star. In the present example we see that as the dependence for the second comparison star is from two to four times that for the others, any inaccuracy in the measurement of this star has a correspondingly greater effect upon *m;* for this reason we measured it twice and the other comparison stars only once. An example of the opposite kind is presented by *60 Krüger* ($22^h\ 24^m$, $+57°\ 12'$), details for which will appear later. The five stars most suitable for the purpose gave dependences of +0.299, +0.057, +0.286, +0.017, and +0.340. The fourth of these is so small that any error in the measurement of the corresponding comparison star would affect *m* very little indeed. It is therefore not worth while to measure this star at all, and it was accordingly dropped. The other dependences then become +0.289, +0.076, +0.289, and +0.345.

It has often been stated that a differential method like the present yields a relative parallax for the parallax star that is less than its absolute value by the *mean* of the parallaxes of the comparison stars. But this is at best a rough statement that applies only if the parallax star is close to the mean position of the comparison stars. The true difference would always be obtained by multiplying the parallax of each of the comparison stars by its dependence and taking the sum. In certain rather extreme cases one of the dependences may come out negative; and then, if the parallax of the corresponding comparison star were unusually large, we should have the curious result that the deduced parallax is *greater* than its absolute value. An example of this case is afforded by *Lalande 23917* ($12^h\ 45^m$, $+1°\ 45'$), one of the stars in the present list. The adopted dependences for the four comparison stars are +0.33, +0.52, −0.30, and +0.45. If the parallax of the third star were greater than 0.″022, while those of the others were all 0.″005, then the parallax here deduced for *Lalande 23917* is greater than the absolute parallax. But in this case, as in every other, the *most probable* value of the absolute parallax is obtained

by adding the theoretical parallax corresponding to the mean magnitude of the comparison stars.

Earlier in these papers I stated that guiding error would be largely eliminated if the magnitude of the parallax star were reduced to the mean of the comparison stars. The more exact condition is equality with the sum of the products formed by multiplying the dependence of each star by its magnitude.

If we let E represent the purely accidental error in the measurement of a star (bisection error, distortion of the film, uncertainties in instrumental corrections, etc.), and E_m a similar quantity for m, then we have from equation (6),

$$E^2_m = E^2 + [D^2]E^2 \qquad (9)$$

This expression should be borne in mind in selecting comparison stars; other things being equal, preference should be given to that set for which the sum of the squares of the dependences is the smallest. But too much stress should not be laid upon this condition to the neglect of others. Stars near the edges of the plate should be avoided, and all the comparison stars should be of nearly the same brightness, for then the length of exposure can be adjusted to give the best intensity to all, and further, the guiding error will probably be more perfectly eliminated under such conditions.

Equation (9) also indicates that to guard against accidental errors only a few comparison stars need be used. From this point of view, four well-distributed stars are practically as good as forty, especially when we consider that other than purely accidental effects tend to swell the total plate error. From another point of view, a somewhat greater number is desirable: to obtain the absolute parallax we add the mean parallax of stars as bright as the comparison stars. In the present work this correction averages 0.″005. The more comparison stars employed, the more confident may we be that this correction is close to the truth in any particular case. I therefore aimed to get five or six good comparison stars in each field; but in seven cases only four were available, and in five others I had to be content with three, the minimum that will suffice. The area of these plates corresponds to less than half a square degree, or only about one-ninth of that

for the Astrographic Catalogue plates. As a consequence it is not always practicable without prolonging the exposures unduly, to obtain as many comparison stars as one could wish. For some objects even three comparison stars are not at hand, and their parallaxes cannot be determined to advantage with this instrument. The field of good definition of the 40-inch objective would probably cover a considerably greater area than that of the plates employed, and this disadvantage might therefore be obviated by the use of larger plates, with of course a larger plate-carrier at the telescope and a larger measuring engine.

Under some conditions the computer may wish to derive the residuals for the individual measurements of the comparison stars. The method of effecting the solutions by means of dependences cannot then be employed, as the plate-constants must be known. The computation of the individual residuals enables the observer to detect discrepancies in the measures and the reductions; but much of the same advantage is retained by the separate reduction of each exposure. Those upon the same plate should differ from each other only on account of purely accidental causes. Even when the computer decides not to employ the dependences to effect the solutions, they should nevertheless be computed, since a knowledge of them is indispensable for an intelligent discussion of the measures. The additional labor involved in deriving the dependences is a matter of perhaps fifteen minutes for each region.

Another method for reducing a plate was earlier employed in this work for preliminary purposes. It is only a trifle shorter than the dependence method and is not based upon the principle of least squares; I therefore abandoned it for even preliminary reductions. But the general process may be of use in applications totally different from the present, and in some of these it may save considerable time; so that I shall take space to describe it briefly.

In the very special case presented by a set of comparison stars whose mean position coincides with that of the parallax star, it is obviously unnecessary to compute the plate-constants. X_π and Y_π are then each equal to zero and we have from (4),

$$m' = X'_\pi - \frac{1}{n}(X'_1 + X'_2 + \text{etc.}).$$

This simplification may be artificially brought about, with any distribution of comparison stars, by *underweighting* some of them so as to make the means of both co-ordinates respectively equal to those for the parallax star. These artificial weights W_1, W_2, etc., may easily be deduced by a graphical process that it would be superfluous to describe here. Having obtained them once for all, the plates are now reduced by simply subtracting the weighted mean of the X-measures upon the comparison stars, from the X-measure upon the parallax star.

This procedure strikes one at first sight as being unduly arbitrary, but it is really no more so than the selection of comparison stars. We are at liberty to use or reject any particular star for this purpose—there can therefore be no objection to our pursuing a middle course and assigning to it half-weight. The only objection to the method is that it is not carried out in accordance with the principle of least squares. This suggests the idea that as the artificial weights may be assigned in an infinite number of ways that conform to the conditions,[1] one of these must correspond to the method of least squares; namely, that one for which

$$\frac{[W^2]}{[W]^2} \text{ is a minimum.}$$

It can be shown that then

$$\frac{W_1}{[W]}=D_1, \quad \frac{W_2}{[W]}=D_2, \text{ etc.}$$

Or, otherwise expressed, the method of artificial weighting becomes identical with the rigorous dependence method. This is in fact the train of thought by which I was led up to the latter, and it was only afterward that I saw that the dependences could be derived in the simpler way already indicated.

Thus far we have referred only to the case in which the displacements in one direction have been measured. If both co-ordinates have been measured, *and if they are to be treated separately*, the same dependences apply to both. That is, letting l' represent the solution in the Y direction,

$$l'=Y'_\pi-(D_1\cdot Y'_1+D_2\cdot Y'_2+\text{ etc.}) \tag{10}$$

[1] Providing of course that there are more than three comparison stars.

There remains the case in which the two co-ordinates are to be combined into one solution after the application of refraction corrections. We shall set down the formulae that are then appropriate, although there is no example of this case in the present work. Let A_1, A_2, etc., be the dependences of the solution in the X-direction upon the X-measures; and B_1, B_2, etc., the dependences of the same solution upon the Y's. Then

$$\left.\begin{aligned} A_1 &= \frac{X_\pi \cdot X_1 + Y_\pi \cdot Y_1}{[X^2+Y^2]} + \frac{1}{n} \\ A_2 &= \frac{X_\pi \cdot X_2 + Y_\pi \cdot Y_2}{[X^2+Y^2]} + \frac{1}{n} \\ &\text{etc.} \end{aligned}\right\} \qquad (11)$$

$$\left.\begin{aligned} B_1 &= \frac{X_\pi \cdot Y_1 - Y_\pi \cdot X_1}{[X^2+Y^2]} \\ B_2 &= \frac{X_\pi \cdot Y_2 - Y_\pi \cdot X_2}{[X^2+Y^2]} \\ &\text{etc.} \end{aligned}\right\} \qquad (12)$$

$$m' = X_\pi' - (A_1 \cdot X'_1 + A_2 \cdot X'_2 + \text{etc.}) - (B_1 \cdot Y_1 + B_2 \cdot Y_2 + \text{etc.}) \qquad (13)$$

If the parallax is to be derived from the Y-displacements as well, the same dependences apply except for some changes of sign:

$$l' = Y_\pi' - (A_1 \cdot Y'_1 + A_2 \cdot Y'_2 + \text{etc.}) + (B_1 \cdot X'_1 + B_2 \cdot X'_2 + \text{etc.}) \qquad (14)$$

It has already been mentioned that owing to variations in the atmospheric conditions, plates taken with a telescope of so great a focal length are of very different worth. It was therefore necessary to frame a table from which the weight of each plate could be assigned. In this connection other factors than the appearance of the images had to be considered. A plate that had been measured by two observers is entitled to greater weight than one that had been measured only once. Again, the weight cannot be assumed to be proportionate to the number of exposures that a plate contains, since all the exposures upon the same plate have a tendency (doubtless chiefly because of guiding error) to residuals of the same sign. Our table of weights should therefore be one of triple entry: the quality of the images, the number of exposures, and the number of measurements, whether one or two.

From preliminary discussions of several regions, carried out

with the use of approximate weights, information as to these various points was gathered, and the results are embodied in Table III. This table was used in the definitive reductions of all

TABLE III

Plate Weights Used in the Definitive Reductions

	Two Measurements			One Measurement		
	Good	Fair	Poor	Good	Fair	Poor
One exposure......	0.5	0.4	0.3	0.4	0.3	0.2
Two exposures.....	0.8	0.6	0.4	0.6	0.5	0.3
Three exposures....	1.0	0.8	0.5	0.9	0.7	0.4

the regions here discussed. The "quality of the images" was assigned while the plate was being measured; and not to split hairs on a matter like this, only three designations were employed: "good," "fair," and "poor." If the images showed marked triangularity or if one of the comparison stars could not be measured, the plate was given smaller weight than appears in the table. In the tables of observations that follow, note is made of any circumstance that affects the value of a plate or of one of the exposures. In the case of a few poor plates the measurements were rejected before the parallaxes had been derived, and therefore without any reference to accordance or non-accordance with other plates of the same region. But no plate was rejected, or its weight reduced, because it showed a large residual. The temptation to break this rule is sometimes very strong, but I have thought it better, particularly for the sake of gathering statistical information that will aid in a continuation of this work, to retain every plate whose rejection was not clearly allowable on a priori grounds.

Table III is of considerable general interest, but I shall not discuss it from this point of view; for it is to be considered only as a second approximation. A third and better approximation may be based upon the definitive residuals, and a discussion of them will be found at the end of these papers.

One other preliminary investigation should be described before proceeding to the detailed results: it relates to the effect of a mal-

adjustment of the plate to the focus of the 40-inch objective. The scale of the plate is obviously affected, but this is of no consequence in reductions like the present, where the scale-correction is determined from the measures themselves. There is, however, another and more serious effect that depends upon the position of a star and upon its magnitude. With an objective as excellent as this one, an image in the optical axis is practically perfect as to symmetry, and there is therefore no apparent change in its position for different positions of the plate with regard to the focal plane. The matter is otherwise for an image that is removed from the optical axis by a considerable arc. The distribution of light is not symmetrical and varies moreover for different sections; that is, for different positions of the plate with reference to the focus. The apparent place of a star will therefore depend to some extent upon the adjustment to focus. It will also depend upon the magnitude of the star; for with a faint object only the nucleus appears upon the developed plate, while with a bright one we get in addition the outlying unsymmetrical aberrations.[1]

To ascertain whether this effect is of appreciable size in the present case we measured (in May 1904) the positions of a number of stars upon plates taken as much as 5 mm out of focus. We found that bright stars were always apparently shifted toward the optical axis, with reference to faint stars in their neighborhood. It made no difference whether the plate had been placed within or without the focal plane—the shift was always in this sense. Numerically, it is roughly equal to $0\rlap{.}''01$ for each millimeter that the plate is removed from the focal plane, for a difference of one magnitude, and at a distance of $11'$ from the optical axis. There is therefore little to fear from this source of error in the present case, but we have here a warning that the plates must be adjusted to focus with some accuracy, that the comparison stars should not differ in brightness too much among themselves, and that we should try to avoid using a set of comparison stars such that all to the east

[1] This is only one among several reasons why the photographic image of a bright star is larger than that of a fainter; others are: imperfect following, poor seeing, chromatic aberration, atmospheric dispersion, and the purely photographic effect known as "creeping," probably due to reflections within the sensitive film.

of the parallax star are bright, and all to the west faint, or vice versa.

THE RESULTS

In the detailed results that now follow, the designation employed for each object is that adopted by Kapteyn and Weersma in their recent compilation.[1] The right ascension and declination that follow the name are for 1900. The co-ordinates of the comparison stars (and their diameters) are expressed in quarter-millimeters; the positive sign indicates that the longitude or the latitude[2] of the corresponding star is *greater* than the mean for all the comparison stars. From these co-ordinates the "computed dependences" result, and these are then rounded off into the "adopted dependences."

For each object a table of particulars concerning the plates is given. The hour angle appears in hours and tenths, and the sign indicates whether the telescope was east or west of the meridian (at the middle of the exposures), the latter being indicated by the positive sign. The number of usable exposures upon the plate is equal to the number of observers in the next column. Here F denotes Fox; J, Jordan; Su, Sullivan; and S, Schlesinger. Any circumstance that affects the weight to be assigned to a plate is noted under the "Remarks."

There then follows a table giving the results of the measurements and reductions. In the column marked t the time is given in days, the zero being chosen somewhere near the middle date so as to simplify the numerical work. In the last column the residuals have been transformed into arc and multiplied by the square root of the weight.

In some cases (all of which are noted in the tables), one or more of the images of a comparison star could not be measured, either because they were missing altogether or because of defects in the film. If only one of the three images was missing, its place could usually be interpolated with sufficient accuracy from the other two, by applying differences ascertained from the other stars upon

[1] *Publications of the Astronomical Laboratory at Groningen*, No. 24.

[2] In some cases these co-ordinates are referred to the equator instead of the ecliptic.

the plate. If this was done, due regard was always had for the change in orientation in going from one exposure to the next, a change that is especially marked in high declinations. Where two or more of the images of a comparison star were missing it was necessary to go to more trouble; approximate dependences were obtained graphically, the comparison star in question being left out of consideration. The plate was now reduced with these dependences; but the solution thus obtained is not comparable with that from other plates, as a constant difference is involved. To obtain this difference and to apply it to the solution of our defective plate, a few exposures upon other plates were reduced with both sets of dependences. A plate or an exposure that had been treated in this way was given reduced weight. In two or three cases, where the missing comparison star had a large dependence, the plate was not used at all.

Throughout these papers *probable errors* are used exclusively.

Groombridge 34 ($0^h\ 13^m$, $+43°\ 27'$)

This is an 8th-magnitude star with a proper motion of nearly $3''$ per annum. Twelve plates were secured under the circumstances shown in Table 1.

TABLE 1

PLATES OF *Groombridge 34*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
61......	1903 Aug. 2	−1.1	Su	Fair	
185......	Dec. 20	−0.7	S, S	Good	Star (1) lacking
433......	1904 Aug. 25	−0.4	S, Su, S	Good	
449......	Sept. 4	−0.3	S, Su, S	Good	
538......	Nov. 20	−0.2	S, Su, S	Good	
558......	Dec. 17	−0.5	S, Su, S	Good	
725......	1905 July 25	0.0	F, Su, F	Poor	
735......	Aug. 12	−0.6	F, Su, F	Poor	
740......	Aug. 19	−1.0	F, F, F	Fair	
760......	Sept. 10	0.0	Su, Su, Su	Good	
853......	Nov. 12	−0.6	Su, J, Su	Fair	
867......	Dec. 10	−0.2	Su, J, Su	Poor	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
1..........	0.50	−344	−218	+.311	+.315
2..........	0.75	−153	−322	+.100	+.10
6..........	0.91	+ 29	+149	+.277	+.275
7..........	0.82	+309	− 13	−.039	−.04
9..........	0.88	+159	+404	+.351	+.35
Parallax star.	1.55	− 70.3	+ 83.6		

The first six plates were measured by Miss Ware and the writer, the last six by Miss Ware alone. If there is any systematic difference between the two observers for this region, it will be almost entirely eliminated from the deduced parallax, as three of the plates measured by both have negative parallax factors, while the other three have positive factors.

TABLE 2

Reductions for *Groombridge 34*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
61 ..	0.055	0.3	+0.974	−471	−.018	−″03
185 ..	0.244	0.6	−0.889	−331	− 34	−.07
433 ..	1.142	1.0	+0.783	− 82	+ 18	+.05
449 ..	1.156	0.5	+0.661	− 72	+ 13	+.02
538 ..	1.286	1.0	−0.568	+ 5	+ 52	+.14
558 ..	1.261	1.0	−0.872	+ 32	− 18	−.05
725 ..	2.036	0.4	+1.002	+252	− 28	−.05
735 ..	2.116	0.4	+0.906	+270	+ 11	+.02
740 ..	2.114	0.7	+0.845	+277	− 4	−.01
760 ..	2.135	0.9	+0.583	+299	− 19	−.05
853 ..	2.227	0.7	−0.446	+362	− 1	.00
867 ..	2.267	0.4	−0.808	+390	− 3	.00

Each plate furnishes an equation of the form

$$P\cdot\pi+T\cdot\mu+c=m,\ \text{weight}\ p$$

where π is the annual parallax, μ is the proper motion in longitude *in 100 days*, and T is the time factor, equal to one one-hundredth

of the corresponding number of days in the column (t). The resulting normal equations are

$$
\begin{aligned}
+4.609\pi+\ 2.011\mu+0.677c&=+\ 1.860\\
+48.394\ \ +6.603\ &=+21.918\\
+7.900\ &=+11.956
\end{aligned}
$$

These yield

$$
\begin{aligned}
c&=+1.275\\
\mu&=+0.2750=+0''.732\\
\pi&=+0.0962=+0.256\pm0''.019
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0151=\pm 0''.040$.

The residuals resulting from this solution appear in next to the last column of Table 2.[1] They were checked by means of the control equations,

$$[p\cdot v]=0,\qquad [p\cdot P\cdot v]=0,\qquad \text{and } [p\cdot T\cdot v]=0.$$

Groombridge 34 is accompanied at a distance of 39″ by a faint star (magnitude 10.5) that shares its large proper motion. This star is measurable upon most of the plates and a determination of its parallax with considerable weight is possible. Its distance from *Groombridge 34* being so large, the dependences are appreciably different for the two objects. In the same system of co-ordinates as are given above in connection with the bright star, the position of the companion is

$$X=-55.8,\qquad Y=+83.2.$$

The resulting dependences are

Star (1), +.295; (2), +.10; (6), +.275; (7), −.02; (9), +.35.

Solutions made on this basis appear under column (m) in Table 3. Where the weights in this table differ from the corresponding ones in Table 1, the companion is either too weak for accurate measurement or has not been measured at all for one or two of the exposures.

[1] They are transformed to arc by applying the factor 2.66.

TABLE 3

REDUCTIONS FOR THE COMPANION TO *Groombridge 34*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p} \cdot v$ in Arc
61 . .	0.052	0.3	+0.974	−471	−.004	−″.01
185 . .	0.259	0.6	−0.889	−331	+ 3	+.01
433 . .	1.129	1.0	+0.783	− 82	+ 14	+.04
449 . .	1.147	0.3	+0.661	− 72	+ 16	+.02
538 . .	1.206	0.7	−0.568	+ 5	− 14	−.03
558 . .	1.256	0.7	−0.872	+ 32	− 8	−.02
725 . .	2.036	0.4	+1.002	+252	− 26	−.04
740 . .	2.102	0.7	+0.845	+277	− 13	−.03
760 . .	2.152	0.9	+0.583	+299	+ 2	+.01
853 . .	2.239	0.7	−0.446	+362	+ 18	+.04

The normal equations are

$$
\begin{aligned}
+3.607\pi + 2.479\mu + 0.938c &= +\ 2.234\\
+39.261\ +3.996\ &= +16.151\\
+6.300\ &= +\ 9.156
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +1.263\\
\mu &= +0.2764 = +0''.734\\
\pi &= +0.1012 = +0''.269 \pm 0''.011
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0073 = \pm 0''.019$.

The weight of the parallax from this solution is somewhat less than for *Groombridge 34* itself, but the measurements are so much more accordant that the computed probable error comes out only little more than half that in the former case. Most of the discordance among the residuals for the brighter star is due to Plate 538. A least-squares solution for the brighter star without this plate yields 0″.022 as the probable error corresponding to unit weight.[1] This is only about half that in the definitive solution first given. But there is no a priori reason for rejecting this plate; the images are good and the six separate values of *m* (three exposures, each measured by two observers) are accordant. The discrepancy cannot be due to inaccuracies in the measurement of the comparison stars, since these same measurements were used in

[1] This solution gives +0″.281 for the parallax.

the solution for the companion, and in this solution Plate 538 not only yields a small residual, but it has the opposite sign from the residual in the solution for *Groombridge 34*. Nor can the trouble be due to distortion of the film, for the images of the companion are close to those of the bright star. The most plausible explanation for this large residual is guiding error: *Groombridge 34* is nearly two magnitudes brighter than the mean of the comparison stars.

The rotating disk might have been used to reduce the light of *Groombridge 34*, but in that case it would not have been possible to measure the companion. The parallax here deduced for the latter is the first that has been published; its close agreement with that for *Groombridge 34* must be regarded as conclusive evidence that the two form a true binary. This might also have been inferred from the community of proper motion, but authorities in double-star astronomy do not always take this point of view. For example, the present pair is not included in Burnham's *General Catalogue of Double Stars*, although it is a system of unusual interest. So far as I am aware, the separation of this pair (39″) is greater than for any other known with certainty to be a binary. There are, however, a few of wider separation whose binary character may be inferred from considerations of proper motion.

Combining in accordance with their probable errors the two values that we have derived for *Groombridge 34* and its companion, we have as the definitive parallax of the system,

$$\pi = +0.''266 \pm 0.''010.$$

Other determinations of the parallax of the brighter component are as follows:

Auwers (equatorial transits)	+0.″29	±0.024
Flint (transit circle)	+0.31	.034
Russell (photography)	+0.25	.012
Chase (heliometer)	+0.31	.010

μ Cassiopeiae ($1^h\ 2^m$, $+54°\ 26'$)

This is a 5th-magnitude star of very large proper motion—nearly 4″ per annum. Nine plates were secured as described in Table 1.

TABLE 1

PLATES OF μ *Cassiopeiae*

No.	Date	Hour Angle	Observers	Quality of Images
418..........	1904 Aug. 7	-0^h5	S, Su, S	Good
421..........	Aug. 13	-0.6	S, S	Good
426..........	Aug. 14	-0.6	S, Su,	Good
539..........	Nov. 20	-0.3	S, Su, S	Good
559..........	Dec. 17	-0.4	Su, Su, Su	Good
741..........	1905 Aug. 19	-1.1	Su, Su, Su	Good
770..........	Sept. 12	-0.6	Su, Su	Fair
778..........	Sept. 19	-1.8	Su, Su, Su	Fair
886..........	Dec. 31	-0.2	Su, J, Su	Good

COMPARISON STARS

No.	DIAMETER	X (longitude)	Y (latitude)	DEPENDENCE	
				Computed	Adopted
3..........	0.55	-266	$+160$	$+.197$	$+.20$
4..........	0.73	-140	-235	$+.145$	$+.14$
5..........	0.76	$+42$	-325	$+.152$	$+.15$
8..........	0.75	$+43$	$+241$	$+.244$	$+.24$
11..........	0.59	$+321$	$+159$	$+.266$	$+.27$
Parallax star.	0.70	$+30.7$	$+48.4$		

The rotating disk was used with this region, and the diameter given for the parallax star is as it actually appears upon the plates. Each of the first five plates was measured by Miss Ware and the writer, the remaining four by Miss Ware alone.

TABLE 2

REDUCTION FOR μ *Cassiopeiae*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
418 ..	0.070	1.0	$+1.009$	-200	$+11$	$+0''03$
421 ..	0.078	0.8	$+0.794$	-194	$+6$	$+.01$
426 ..	0.051	0.8	$+0.792$	-193	-23	$-.05$
539 ..	0.247	1.0	-0.315	-95	$+8$	$+.02$
559 ..	0.281	1.0	-0.707	-68	-1	$.00$
741 ..	0.891	0.9	$+0.871$	$+177$	$+3$	$+.01$
770 ..	0.928	0.5	$+0.385$	$+201$	-5	$-.01$
778 ..	0.949	0.7	$+0.481$	$+208$	$+3$	$+.01$
886 ..	1.106	0.8	-0.681	$+311$	-7	$-.02$

The normal equations are:

$$
\begin{aligned}
+5.238\pi - 3.109\mu + 2.629c &= +0.732\\
+26.960\ \ -0.184\ &= +5.721\\
+7.500\ &= +3.516
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.460\\
\mu &= +0.2203 = +0''.586\\
\pi &= +0.0393 = +0''.105 \pm 0''.010
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0073 = \pm 0''.019$.

In spite of the small number of plates the parallax has been determined with a small accidental error. The largest residual for any plate is 0''.06, and all the others are under 0''.03.

Other reliable determinations of this parallax are:

Peter (heliometer)	+0''.13	±0''.013
Flint (transit circle)	+0.07	.026

Lalande 5761 ($3^h\ 3^m$, $+25°\ 58'$)

This is an 8th-magnitude star with a proper motion of a little less than 1'' per annum. Ten plates were secured as shown in Table 1.

TABLE 1

PLATES OF *Lalande 5761*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
81 ...	1903 Aug. 16	$-1^h.7$	S, Su, S	Fair	Images triangular and elongated
99 ...	Sept. 20	−0.1	S, S	Fair	Stars (9) and (20) lacking
203 ...	1904 Jan. 31	+1.9	S, Su, S	Fair	Telescope east
209 ...	Feb. 4	+0.3	S, S	Good	
437 ...	Aug. 25	−0.7	S, Su, S	Good	
451 ...	Sept. 4	−1.1	S, Su, S	Fair	Images triangular
585 ...	1905 Jan. 29	0.0	S, S, Su	Fair	
763 ...	Sept. 10	−1.2	Su, Su, Su	Fair	
784 ...	Sept. 23	−1.2	Su, Su	Fair	
789 ...	Sept. 24	−1.8	Su, Su	Fair	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence Computed	Dependence Adopted
4..........	1.07	−321	−330	+.227	+.225
9..........	0.56	−160	+ 22	+.138	+.14
10..........	0.59	−109	+ 28	+.143	+.14
13..........	0.72	+101	+116	+.145	+.15
14..........	0.68	+145	+ 7	+.185	+.18
20..........	1.23	+344	+157	+.165	+.165
Parallax star.	1.56	− 12.7	− 24.0		

The last three plates were measured by Miss Ware alone, the other eight by both Miss Ware and myself.

TABLE 2

Reductions for *Lalande 5761*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
81 ..	0.460	0.4	+1.011	−357	+38	+″06
99 ..	0.398	0.6	+0.806	−322	− 8	−.02
203 ..	0.368	0.8	−0.972	−189	+28	+.06
209 ..	0.290	0.8	−0.981	−185	−48	−.11
437 ..	0.258	1.0	+0.988	+ 18	+ 1	.00
451 ..	0.242	0.5	+0.936	+ 28	−11	−.02
585 ..	0.203	0.8	−0.967	+175	+23	+.05
763 ..	0.068	0.7	+0.894	+399	−21	−.05
784 ..	0.066	0.5	+0.768	+412	−17	−.03
789 ..	0.118	0.5	+0.757	+413	+35	+.07

The normal equations are:

$$
\begin{aligned}
+5.628\pi + 4.518\mu + 1.395c &= +0.188 \\
+47.595 \quad +2.286 &= -1.477 \\
+6.600 &= +1.630
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.261 \\
\mu &= -0.0440 = -0.''117 \\
\pi &= +0.0038 = +0.''010 \pm 0.''020
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0172 = \pm 0.''046$.

This result agrees well with that of Chase, who derived $-0''.001$, with a probable error of $\pm 0''.033$. From five plates taken by Rambaut at the Radcliffe Observatory, Kapteyn obtains a much larger value, $+0''.085 \pm 0''.022$.

Lalande 7443 ($3^h\ 56^m$, $+35°\ 3'$)

This is a star between the 8th and the 9th magnitude with a proper motion of over 2″ per annum. Eleven plates were secured as shown in Table 1.

TABLE 1

PLATES OF *Lalande 7443*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
104...	1903 Sept. 22	$-1^h.4$	S, Su, S	Poor	Stars (27) and (28) lacking
201 ...	1904 Jan. 31	−0.3	S, Su, S	Good	
218 ...	Feb. 9	+1.3	S, Su, S	Fair	Telescope east
453 ...	Sept. 4	−0.8	S, Su, S	Fair	Images slightly triangular
464 ...	Sept. 11	−1.0	S, Su, S	Poor	
477 ...	Sept. 25	−0.1	S, S, S	Good	
554 ...	Dec. 15	−0.4	S, Su, S	Good	
562 ...	1905 Jan. 3	−0.5	S, Su, S	Good	
765 ...	Sept. 10	−1.1	Su, Su, Su	Good	Star (28) lacking on first exposure
786 ...	Sept. 23	−1.0	Su, J, Su,	Fair	
791 ..	Sept. 24	−1.0	Su, J, Su	Fair	Third exposure poor

COMPARISON STARS

No.	DIAMETER	*X* (longitude)	*Y* (latitude)	DEPENDENCE	
				Computed	Adopted
10..........	1.33	−209	−124	+.422	+.417
11*a*.........	1.21	−252	+188	+.389	+.40
27..........	0.59	+202	− 25	+.113	+.10
28	0.67	+259	− 39	+.076	+.083
Parallax star.	1.32	−144.0	+ 14.4		

For the first plate, which lacks (27) and (28), two other comparison stars, numbered (5) and (6), were used instead. The last

three plates were measured by Miss Ware alone, the others by the writer as well.

TABLE 2

REDUCTIONS FOR *Lalande 7443*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
104 . .	0.072	0.2	+0.911	−320	+ 5	+″01
201 . .	0.224	1.0	−0.902	−189	−10	−.03
218 . .	0.266	0.8	−0.955	−180	+20	+.05
453 . .	0.570	0.8	+1.000	+ 28	−11	−.03
464 . .	0.606	0.5	+0.973	+ 35	+14	+.03
477 . .	0.610	1.0	+0.882	+ 49	− 1	.00
554 . .	0.689	0.9	−0.327	+130	+24	−.06
562 . .	0.747	1.0	−0.614	+149	+10	+.03
765 . .	1.147	0.5	+0.979	+399	+18	+.03
786 . .	1.133	0.7	+0.900	+412	−14	−.03
791 . .	1.160	0.6	+0.892	+413	+12	+.02

The normal equations are:

$$
\begin{aligned}
+5.758\pi + 8.784\mu + 1.431c &= +\ 2.132\\
+42.393\ \ +6.936 &= +10.032\\
+8.000 &= +\ 5.250
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.526\\
\mu &= +0.1476 = +0''.393\\
\pi &= +0.0145 = +0''.039 \pm 0''.013
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0095 = \pm 0''.025$.

The parallax of this star has recently been determined by Chase and Russell. By means of the heliometer the former gets $+0''.04 \pm 0''.040$; and from photographic plates Russell obtains $-0''.011 \pm 0''.006$.

c Persei ($4^h\ 1^m$, $+47°\ 27'$)

A number of helium stars that were under observation for radial velocity with the Bruce spectograph of the Yerkes observatory, were put upon the observing program in order to ascertain by direct observation whether stars of this type are as distant as their small proper motions would imply. The present list contains

four of these stars, of which *c Persei* is the first. The magnitude is 4.0 and the proper motion (according to Boss) 0".044 per annum. The parallax rests upon the eight plates described in Table 1. The rotating disk was used to reduce the light of the parallax star.

TABLE 1

PLATES OF *c Persei*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
517 ...	1904 Oct. 30	$-0^h.2$	S, Su, S	Good	Second exposure poor
524 ...	Nov. 17	−0.9	S, Su, S	Poor	Images triangular
583 ...	1905 Jan. 22	−0.6	S, Su, S	Fair	
586 ...	Jan. 29	−0.4	S, Su, S	Fair	First exposure good
787 ...	Sept. 23	−0.6	Su, J, Su	Fair	Second exposure poor
799 ...	Oct. 3	−0.3	Su, J, Su	Fair	
826 ...	Oct. 10	−0.4	Su, J, Su	Fair	
901 ...	1906 Jan. 28	−0.5	Su, J, Su	Fair	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
1	0.73	−124	−329	+.211	+.20
2	0.56	−300	− 92	+.398	+.40
4	0.69	+288	+ 95	+.110	+.125
5	1.00	+136	+326	+.281	+.275
Parallax star .	0.78	− 74	− 3		

TABLE 2

REDUCTIONS FOR *c Persei*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
517 ..	0.042	0.9	+0.508	−216	− 1	".00
524 ..	0.058	0.5	+0.217	−198	+ 8	+.01
583 ..	0.026	0.8	−0.799	−132	−12	−.03
586 ..	0.045	0.9	−0.864	−125	+ 4	+.01
787 ..	0.044	0.6	+0.927	+112	− 5	−.01
799 ..	0.050	0.7	+0.845	+122	− 1	.00
826 ..	0.053	0.7	+0.774	+129	+ 1	.00
901 ..	0.055	0.7	−0.854	+239	+ 4	+.01

There is another good comparison star, numbered (3), at $X = +64$, $Y = -290$; but the dependence comes out small (+.053) and this star was accordingly not used. Each of the first four plates was measured by both Miss Ware and the writer, the other four by Miss Ware alone.

The normal equations are:

$$
\begin{aligned}
+3.384\pi + 1.229\mu + 0.240c &= +0.024 \\
+15.898 \quad -1.013 &= -0.005 \\
+5.800 &= +0.264
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.046 \\
\mu &= +0.0024 = +0.''006 \\
\pi &= +0.0031 = +0.''008 \pm 0.''009
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0060 = \pm 0.''016$.

No other determination of this parallax has been published.

ALLEGHENY OBSERVATORY
February 1911

[*To be continued*]

PLATE IX

THE JANUARY COMET OF 1910

Stockholm, January 28, 1910, at G.M.T. $5^h\ 10^m$. Exposure 33^m. Enlargement fourfold. Scale: 1 cm = $1^{\circ}.25$

Minor Contributions and Notes

THE JANUARY COMET, 1910 *a*

The accompanying reproduction, Plate IX, of a photograph of the remarkable January comet of 1910 is from a negative made on January 28, 1910, at the Stockholm Observatory with a Voigtländer "Dynar" objective. A reproduction of this negative was first given in *Astronomische Nachrichten*, No. 4433 (**185**, 261, 1910). The scale of the picture is 1 cm = $1^\circ.25$.

In addition to the principal tail, there issues from the head of the comet a secondary tail 2° long. The principal tail is slightly curved and extends to a length of 18°, passing between θ and ϵ *Pegasi*. At its extremity the tail is bifurcated. The head forms a very slender pencil. At the time of this exposure the head of the comet was very close to the equator.

Karl Bohlin

Stockholm Observatory
December 28, 1910

Reviews

Geodäsie, eine Anleitung zu geodätischen Messungen für Anfänger, mit Grundzügen der Hydrometrie und der directen (astronomischen) Zeit- und Ortsbestimmung. Von H. Hohenner. Leipzig und Berlin: B. G. Teubner, 1910. 8vo, pp. 347, with 216 figures. Bound, M. 12.

This is primarily a textbook for engineering students, written by a Doctor of Engineering, not a Doctor of Philosophy; it should therefore be a "practical" book, free from the defects that always abound in textbooks if the authors have formed the bad habit of making researches in science—researches intended to widen the bounds of human knowledge.

We are often told that the americanization of Germany is about to be completed. Doubtless there, as well as here, the demand made by the engineering student upon his school is that he be transformed, with a minimum of trouble to himself, and in a minimum of time, into a money-earning machine. He has an avid desire to acquire saleable knowledge and a persistent disinclination for "mere theory." Let the force of gravitation be what it will: he desires to be lectured on the horse-power required to overcome that force in the case of a 50-passenger 40-story express elevator; and he prefers to have the lectures presented in the form adopted in the printed documents distributed by elevator manufacturers, and which they are accustomed to call their "literature."

But German thoroughness dies hard. If we are to credit the preface, this book contains only an extract from the author's lectures at the technical schools of Munich, Stuttgart, and Braunschweig: if surveying students there receive instruction in the entire contents of the volume, they certainly have the benefit of a very thorough course. Especially noteworthy is the constant use made of the method of least squares. A brief explanation of it is given on pp. 8–11; later, on p. 143, a very complete treatment begins; and problems are at all times adjusted in strict accord with rigorous principles.

The famous triangulation between Göschenen and Airolo (the two ends of the St. Gothard tunnel) is given as a numerical example on p. 213. This was perhaps the only case in which extremely precise geodetic

methods have been found essential in ordinary railroad engineering practice until the construction of the Hudson River tunnel was undertaken at New York.

There are valuable chapters on trigonometric and barometric leveling, as well as current measurement in hydrographic work; it is to be regretted that photographic surveying has been omitted. The book should certainly receive attention from American teachers of civil engineering.

H. J.

Boletin Mensual del Observatorio del Ebro. Vol. I, No. 1. Enero de 1910.

We welcome the appearance of this initial number of what promises to be an interesting and valuable series of bulletins.

The object of the Observatory del Ebro[1] is to study the relation between solar activity and the magnetic and electric phenomena of the earth. To this end three lines of research are being pursued, viz., astrophysics, meteorology, and geophysics. The first includes the study of sun-spots and flocculi; the second, in addition to the ordinary meteorological observations, includes investigations in atmospheric electricity, the ionization of the air, atmospheric potential, etc.; the last embraces terrestrial magnetism, earth currents, and seismology.

The text of the bulletin is given in Spanish and French in parallel columns. The observations for the month are presented first in tabular form, then graphically. The latter is exceedingly interesting and instructive. All the curves are placed on the same page with the times of observation as abscissas. A comparison of the simultaneous variations of the twenty-four different phenomena is thus rendered easy.

FREDERICK SLOCUM

Newcomb-Engelmanns Populäre Astronomie. Vierte Auflage. Edited by P. KEMPF. Leipzig: Wilhelm Engelmann, 1911. Large 8vo, pp. xvi+772, with 213 illustrations in the text and on 21 plates. M. 14; bound, M. 15.60.

The second and third German editions of this work were edited by the late H. C. Vogel, both on account of his regard for his deceased friend R. Engelmann, translator and editor of the first edition, and on

[1] Situated at Roquetas, Tortosa, Spain; connected with the Colegio Máximo de la Compañia de Jesús.

account of his high appreciation of Newcomb's book. It was a natural desire of the publishers that the fourth edition also should be edited at the Potsdam Observatory, and the three astronomers who were associated with Vogel in preparing the third edition, Messrs. Kempf, Eberhard, and Ludendorff, have likewise participated in the latest edition, as has also Professor Schwarzschild.

It is a fine testimonial to the appreciation of the German-reading public for a solid and comprehensive work on a special science that a new edition should be called for within five years. We say "German-reading" instead of German, because of our conviction that a large call for the work has come from other European countries. The book has never been, nor was it intended to be, popular in the sense of giving the reader a pleasant entertainment. In Newcomb's own words in the preface of the original edition in English:

> The present work is not designed either to instruct the professional investigator or to train the special student of astronomy. Its main object is to present the general reading public with a condensed view of the history, methods, and results of astronomical research, especially in those fields which are of most popular and philosophic interest at the present day, couched in such language as to be intelligible without mathematical study.

In its present form the work is of decidedly composite origin: many eminent authorities have contributed to one or another of the German editions, either in special statements of their views on particular points, or in suggestions to the editor after a careful reading of an earlier edition with a view to revision. Among them may be mentioned Young, Dunér, Küstner, Seeliger, Kapteyn, Kobold, besides the gentlemen named above as editors. Newcomb's own views, as expressed in some of his last popular books in English, are also quoted under some topics. Coming from such sources the book ought to be a good one, and it is undoubtedly the best of its kind in any language.

The principal changes from the third German edition have been these: the section on the determination of orbits (*Bahnbestimmung*) has been rewritten by Schwarzschild; the sections on stellar parallaxes and stellar motions have been revised by Ludendorff; several sections on the sun, by Kempf and Ludendorff; those on the physical constitution of the stars, and on "new stars," by Eberhard; on variable stars, by Kempf; and some sections on the structure of the universe and cosmogony, by Schwarzschild. Many new illustrations have been added, including nine plates.

In accordance with the purpose of the book, no references are given

to the original papers upon which statements are based. This is a relief to the general reader, as such references only disturb the continuity of his thought. They are also unnecessary for the teacher, to whom we especially recommend the book, as he will find in it authoritative opinions on all points of interest, both new and old, in the whole range of astronomy. The many statistical tables, all revised to represent the latest information, are also exceedingly valuable for reference. We wish that an equally good, and up-to-date, edition in English of this standard work could be prepared for those who do not easily read German.

F.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

Where an unusual number of illustrations may be required for an article, special arrangements are made whereby the expense is shared by the author or by the institution he represents.

Authors will please carefully follow the style of this JOURNAL in regard to footnotes and references to journals and society publications.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

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The ASTROPHYSICAL JOURNAL is published monthly except in February and August. The annual subscription price is $5.00; postage on foreign subscriptions 62 cents additional. Business communications should be addressed to *The University of Chicago Press, Chicago, Ill.*

All papers for publication and correspondence relating to contributions should be addressed to *Editors of the* ASTROPHYSICAL JOURNAL, *Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.*

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXXIII APRIL 1911 NUMBER 3

THE VALUE OF THE SOLAR CONSTANT OF RADIATION

By C. G. ABBOT and F. E. FOWLE, JR.[1]

In Vol. II of the *Annals of the Astrophysical Observatory of the Smithsonian Institution* we published results of a large number of observations of the solar constant of radiation, and we stated that these were not expressed on the true scale of heat units (calories) but on a provisionally adopted scale whose unit was less than a calory by an unknown amount. In a paper published by us in 1909,[2] we stated that further attempts had been made by us to fix the true pyrheliometric scale, with the result that we believed that the values in our *Annals* should be *decreased* on account of pyrheliometry by 7.6 per cent, but we still called our proposed new scale "provisional." In the same article we called attention to the need of carrying energy-spectrum observations further in the ultra-violet in solar-constant work than we had done, and expressed the view that improvements in this respect then contemplated might tend to *increase* the values in our *Annals* by perhaps 10 per cent. We expressed the hope of assuring ourselves on these two points.

We are now in a position to state definite conclusions in both respects. As shown by Abbot and Aldrich in this *Journal* (**33**, 125, 1911), satisfactory agreement has been at length arrived at in the use of two Standard Water-flow Pyrheliometers, and the constant of Secondary Pyrheliometer No. IV, used on Mount Wilson (1800 meters = 5800 ft. altitude) since 1906, has been found to be

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] *Astrophysical Journal*, **29**, 281, 1909.

0.8581±0.0018. We provisionally adopted in the *Annals* the value 0.9020 for this constant, so that the correction is 5 per cent.

Messrs. Abbot, Ingersoll, and Fowle have observed on Mount Wilson in 1909 and 1910 with several different forms of spectro-bolometer, using for a time a flint glass prism and silvered mirrors, as in 1905–8; for a time a quartz prism and magnalium mirrors; generally an ultra-violet crown glass prism and silvered mirrors. Mr. Abbot has observed in 1909 and 1910 on Mount Whitney (4420 meters = 14,502 ft. altitude) with a quartz prism and two magnalium mirrors in a spectro-bolometer pointed directly at the sun, and without any other optical surfaces whatever altering the quality or intensity of the rays. The bolographic observations with the ultra-violet glass prism now generally cover the region of spectrum from $\lambda=0.34\,\mu$ to $\lambda=2.48\,\mu$. On Mount Whitney in 1909 the observations extended from $\lambda=0.29\,\mu$ to $\lambda=3.0\,\mu$ and in 1910 from $\lambda=0.30\,\mu$ to $\lambda=3.8\,\mu$. In the Mount Whitney work no appreciable energy could be discovered in the solar spectrum beyond $\lambda=0.29\,\mu$, although the quartz and magnalium spectro-bolometer was transparent to rays of still shorter wave-length.

We now find that we neglected to observe, in 1905 and 1906, a region of spectrum in the ultra-violet containing considerable energy. Hence we have asked Miss Graves to reduce the observations of several days of 1909 by two processes: first, including all the spectrum observed with the ultra-violet glass prism apparatus, and applying a correction determined from Mount Whitney observations for rays still further in the ultra-violet; second, employing only the spectrum region included in 1905–6, with the same corrections for the ultra-violet used in computing for Vol. II of the *Annals*. Her results follow. They give the ratio of the value of the solar constant derived by the first method to that derived by the second.

Date	June 20	July 11	July 13	July 14	August 11	September 2	Mean
Ratio...	1.005	0.992	1.020	1.004	1.007	0.996	1.004

From these results it appears that, although an important part of the spectrum was neglected in 1905 and 1906, the neglect caused

almost no change in the results. The cause of this paradox is not very obscure. As stated at pp. 55–56, 77–78, of *Annals*, Vol. II, we corrected the bolographic data for an assumed change of sensitiveness of the apparatus, by comparing the areas of the bolographic curves with the readings of the pyrheliometer. We still do this, but now that all the ultra-violet spectrum that counts for anything is observed, the range of the correcting factors has fallen to only one or two per cent, whereas it was formerly nearly ten per cent. For the ultra-violet rays increase in their intensity so rapidly as the sun mounts toward the zenith that the areas of the old bolographic curves, in which the ultra-violet was absent, grew less rapidly toward noon than the pyrheliometer readings. But we attributed this in 1905–6 to a change of bolometric sensitiveness. Our old correcting factors for bolometric sensitiveness, by decreasing the computed values of the atmospheric transmission coefficients, compensated almost exactly for the omission of the ultra-violet.

There is therefore no occasion to raise the values of the solar constant of 1905–6 on account of the omission of the ultra-violet spectrum, but the *coefficients of atmospheric transmission* of 1905–6 are all about 1.4 per cent too low. We shall not correct them more definitely until Vol. III of the *Annals* is issued.

We showed in the *Annals*, Vol. II, p. 102, that determinations of the solar constant made on the same days at Washington (sea-level) and Mount Wilson (1800 meters) agreed within the error caused by changes of the sky transparency during observation at Washington. In fact, the average value of the solar constant from 44 Washington observations agreed within 2 per cent with the average value from 121 Mount Wilson observations. Mr. Abbot's observations on Mount Whitney (4420 meters = 14,500 ft.) in 1909 and 1910 were made simultaneously with observations on Mount Wilson. Rejecting one day when the observations on Mount Whitney were unsatisfactory, there remain the following four days for comparison:

Date	1909, September 3	1910, August 12	1910, August 13	1910, August 14
Mount Wilson......	1.943	1.943	1.924	1.904
Mount Whitney.....	1.959	1.979	1.933	1.956

The Mount Whitney values average higher by 1.4 per cent, but we do not think this difference is large enough to be regarded as significant.

We conclude that the values of the solar constant computed from the method of high and low sun observations do not depend on the altitude of the observing station up to altitudes of 4420 meters, provided the sky conditions are satisfactorily clear and uniform.

Reducing values published in Vol. II of the *Annals* to standard calories at 15° C., and including the mean values obtained in later years,[1] we have:

MEAN VALUES OF THE SOLAR CONSTANT

Place	Washington	Mount Wilson					Mount Whitney	
Date	1902–7	1905	1906	1908	1909	1910	1909	1910
Times observed	44	59	62	113	95	28	1	3
Mean value	1.960	1.925	1.921	1.929	1.896	1.914	1.959	1.956

The Mount Wilson values 1905–9 are platted in the accompanying illustration. They indicate on their face a solar variability within a range of 8 per cent. The march of the results for successive days in 1908 and 1909 does not look as if the variation is accidental. The agreement of results at different altitudes does not indicate an atmospheric origin for the fluctuation.

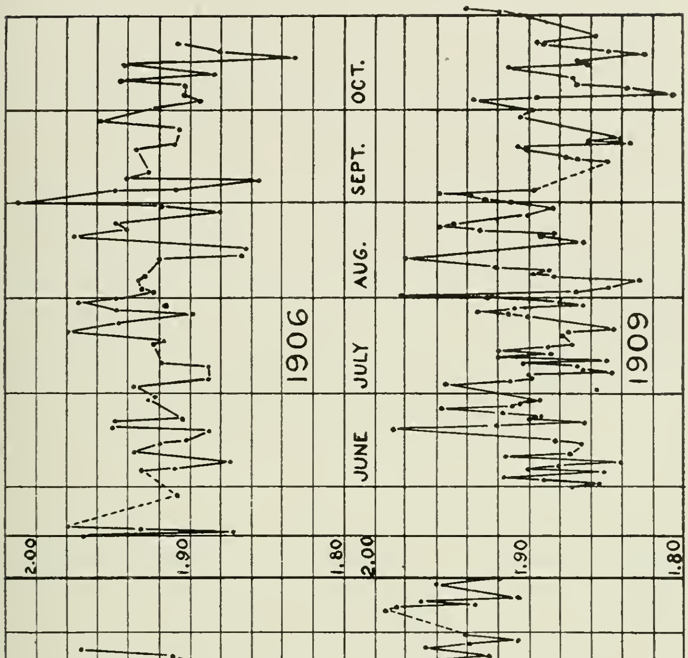
Our observations indicate as the mean value of the solaı constant of radiation

1.922 calories (15° C.) per square centimeter per minute.

The observations having been obtained mainly near the time of sun-spot maximum we think it probable that their mean is a little below the truly representative value.

We cannot tell now, and perhaps can never know, whether the sun emits a noteworthy amount of radiation of shorter wavelength than 0.29 μ. It is possible that if observations could be made beyond the atmosphere, the extreme ultra-violet rays, not transmitted by air, might appreciably increase the above value of the solar constant. We doubt if the increase would be so much as one per cent, for the reason that the sun itself seems to contain matter which restricts its ultra-violet output.

[1] Most of the values of 1910 are not yet reduced.



Determinations of the solar constant of radiation, Mt. Wilson

Mr. Very read a paper last summer in which he criticized our methods and argued for a higher value of the solar constant. We had not the advantage of hearing him, but the guidance of the published abstract of his paper has not sufficed to convince us that any increase should be made over the value we have given. We await the full publication of his work.

CONCLUSION

We conclude that, according to our experiments, and subject to no uncertainties that we can now discern, except the one mentioned above, the mean value of the solar constant of radiation for the epoch 1905–9 was 1.922 calories (15° C.) per square centimeter per minute. We believe that this value represents the intensity of solar radiation as it would have been obtained if observed with a standard pyrheliometer situated in free space at the earth's mean distance from the sun. We see no probability that the error of this value is greater than 1 per cent.

Our results indicate on their face solar variability within a range of 8 per cent. The step-by-step march of the fluctuations observed from day to day seems to indicate that they are not accidental. If they were due to faulty estimates of atmospheric transmission, it would be surprising that simultaneous observations at Washington, Mount Wilson, and Mount Whitney should so well agree. We think, therefore, that the most probable explanation is that there are really variations of 0.03 stellar magnitude in the solar radiation outside our atmosphere. This indication we hope to confirm by a long series of simultaneous spectro-bolometric observations at two excellent, widely separated stations.

Astrophysical Observatory
Smithsonian Institution
Washington, D.C.
January 30, 1911

ON THE VARIATION OF *S ARAE*

By ALEX. W. ROBERTS

1. The light-curve of *S Arae*, and of stars of this definite type of variation, of which the salient characteristics are a distinct stationary period, a very rapid rise to maximum, and a slow fall to minimum phase, exhibits features that apparently arise from a combination of two other definite types, an eclipse-curve superimposed upon an ordinary short-period curve. The variation of *S Arae* is, therefore, on this supposition, of a compound character, a part being due to intrinsic light-change in one of the component stars, a part due to the position of the stars with respect to the sun.

This exposition of the variation of *S Arae* was impressed upon me two years ago, after I had made a special series of observations of the star at and near the stationary phase. The similarity between the light-curves of *RR Puppis*, *S Velorum*, and *S Arae* at this critical stage of their light-variation seemed to urge this contention, that the same cause or causes which produce a constant phase as *RR Puppis* and *S Velorum* pass through minimum, also operate in producing an almost similar constant phase in the case of *S Arae*.

In Figs. 1, 2, and 3 are given the portion of the light-curves of *RR Puppis*, *S Velorum*, and *S Arae* which includes the termination of the stationary phase and the beginning of the ascending phase. The curves are determined from observations made at Lovedale during the past ten years.

Now the only efficient and sufficient explanation of the stationary period at minimum phase of *RR Puppis* and *S Velorum* is that of eclipse. No other reasonable theory has, as yet, been suggested. In the case of these two stars, and of stars of a similar type, a small but bright companion passes behind its primary once every revolution, and is then for a space eclipsed. It may therefore reasonably be contended that at the stationary phase

of *S Arae*, a small but bright satellite has passed behind its central and darker sun.

It is evident, also, that if the two components are not exceedingly disproportionate in brightness then there will be a slight

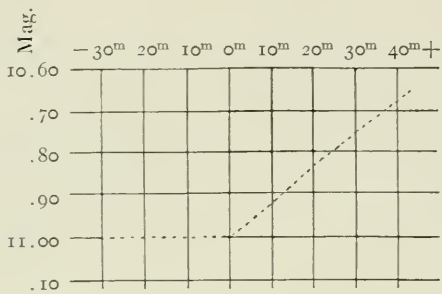


FIG. 1.—*RR Puppis* at end of stationary period.

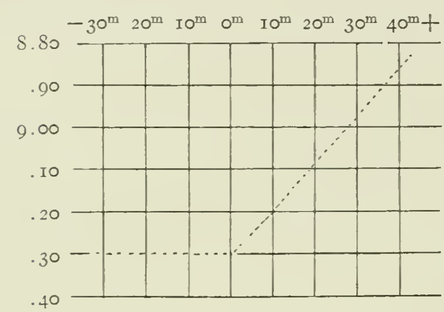


FIG. 2.—*S Velorum* at end of stationary period.

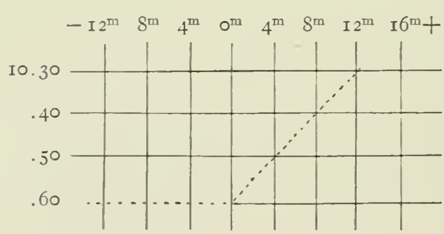


FIG. 3.—*S Arae* at end of stationary period.

diminution in brightness when S_2 passes in front of S_1: the middle dates of these two phases would be at the instant when S_2 crosses the line of sight. But while the stationary period of *S Arae* at minimum associates it intimately with stars of the *S Velorum* type, during the rest of its light-curve, and especially during its decreasing phase, the character of its light-changes allies it to stars of a different type. The variation of *S Arae* is then of the

well-known short-period type. So that we have in the variation of *S Arae* the revelation of the following circumstances.

The star is evidently a binary system, the smaller but brighter component of which varies continuously as a Cepheid or Geminid variable would: while the nearness of the component stars and the inclination of their orbit are such that with every revolution we have the phenomena of a primary and secondary eclipse.

Albrecht in the *Lick Observatory Bulletin*, 4, 138, has pointed out that the maximum light of all short-period stars, in which the relation of orbital movement to light variation can be established, synchronizes with the maximum velocity of approach, occurring a little before minimum velocity, and the minimum brightness with the maximum recession, being usually a little after maximum velocity. If *S Arae* varies conformably to short-period type, then its maximum brightness should occur a little before it reaches approaching quadrature, and minimum brightness after it has passed receding quadrature. The conditions of eclipse give the instant when the component stars are in line with the sun, and consequently the position in time of quadrature passage.

The present paper is to consider how far this theory is borne out by the facts of observation. The investigation is urged with some diffidence. Campbell has pertinently and timely said that "facts brought out concerning the eccentricities of orbits, relative size and brightness of the component stars, the average density, the distance between components . . . . based exclusively upon photometric data, without the illuminating assistance of radial velocity measurements for orbital proportions, and scale values, must be considered as roughly approximate." But while there is reluctance to enter into avenues of argument that are at all times ill-defined, it is also true that there is no other direction open for progress, with respect to such stars as *S Arae*, until a complete revolution has been effected in spectroscopic research. And thus, in the meantime, it is more reasonable to go forward, even uncertainly, than to wait indefinitely for the illumination that spectroscopic data afford.

The special series of observations discussed in this paper was made in 1910. *S Arae* has, however, been under regular obser-

vation at Lovedale for ten years. The star was discovered by Innes in 1898. A most valuable discussion of the star's variation by this observer may be found in the *Annals of the Cape Observatory*, **9**, 126B ff. Innes there derives a period of

$$10^{h}\ 50^{m}\ 43^{s}.5$$

for the star's variation.

The Lovedale observations yield a period of

$$10^{h}\ 50^{m}\ 43^{s}.45,$$

a value which Innes confirms in a private letter. The Lovedale observations also indicate that this period is at present decreasing at the rate of

$$0^{s}.000049$$

per revolution.

A word may be allowed as to the method of determining the period. With very rapid variation the method that has been found most accurate is to determine instants of time when the variable passes through a certain definite magnitude in its increasing phase. In the case of *S Arae* the magnitude adopted was $10^{m}.0$.

Table I sets forth the observed instants, as determined during 1910.

The computed dates are obtained from the elements:

$$\text{J.D. } 2418854\text{–}5^{h}\ 17^{m}.0\ (\text{G.M.T.})+(10^{h}\ 50^{m}\ 43^{s}.45)E.$$

The mean departure of a single observation is $2^{m}.5$. In this space of time *S Arae*, in its ascending phase, moves through 0.075 magnitude.

3. The observations made during 1910, and indeed all observations of *S Arae*, were secured in the usual manner which obtains at Lovedale with stars fainter than 9.0 magnitudes; each observation is the mean of two determinations of magnitude, one with the field direct, the other with the field reversed.

The star is related to a group of comparison stars—as far as possible stars in the same field—and by a series of sequences the magnitude of the variable and of its comparison stars is determined. Each observation is then reduced for scale value and standard values. Standard values are determined as far as pos-

TABLE I

Date of Passage through $10^m.0$	Julian Day of Passage	Computed Date	O. − C.
1910	241		
April 1 $9^h 24^m.7$	8763 $9^h 24^m.7$	$9^h 21^m.5$	$+3^m.2$
10 10 20.0	8772 10 20.0	10 16.0	+4.0
11 7 59.6	8773 7 59.6	7 57.4	+2.2
15 9 31.0	8777 9 31.0	9 33.9	−2.9
30 7 26.0	8792 7 26.0	7 27.8	−1.8
May 3 11 25.5	8795 11 25.5	11 22.9	+2.6
4 8 59.5	8796 8 59.5	9 4.3	−4.8
7 12 59.5	8799 12 59.5	12 59.4	+0.1
8 10 38.0	8800 10 38.0	10 40.8	−2.8
9 8 18.5	8801 8 18.5	8 22.3	−3.8
10 6 1.0	8802 6 1.0	6 3.7	−2.7
14 7 36.5	8806 7 36.5	7 40.3	−3.8
June 1 9 31.5	8824 9 31.5	9 29.2	+2.3
2 7 7.5	8825 7 7.5	7 10.6	−3.1
3 4 49.6	8826 4 49.6	4 52.0	−2.4
3 15 46.2	15 46.2	15 42.8	+3.4
6 8 49.5	8829 8 49.5	8 47.2	+2.3
11 8 4.5	8834 8 4.5	8 5.1	−0.6
15 9 42.2	8838 9 42.2	9 41.7	+0.5
Aug. 3 5 5.2	8887 5 5.2	4 59.9	+5.3
5 11 14.6	8889 11 14.6	11 13.5	+1.1
6 8 54.2	8890 8 54.2	8 54.9	−0.7
10 10 29.0	8894 10 29.0	10 31.4	−2.4
11 8 12.0	8895 8 12.0	8 12.9	−0.9
Sept. 13 7 57.0	8928 7 57.0	7 55.8	+1.2
22 8 54.0	8937 8 54.0	8 50.2	+3.8

sible by limiting apertures, but too much confidence should not be placed in this method of securing absolute magnitudes.

Table II shows the observations secured.

In column 1 is given the heliocentric Greenwich mean time of observation; in column 2, this date reduced to the mean curve of July 1, 1910, by the period $10^h\ 50^m\ 43^s.45$; in column 3 is found the magnitude of the variable at the time of observation; and in column 4 is given the mean magnitude, for the same date, as indicated by the mean light-curve for July 1, 1910 (Fig. 4, and paragraph 4).

The residuals are given in the last column. From these it appears that the mean error of a single observation is $0^m.070$. This is a somewhat higher value than obtains in purely eclipse stars, and leads to the conclusion that the variation of *S Arae* fluctuates to the extent of about $0^m.04$. An examination of the article by Innes already referred to points to errancy of this char-

TABLE II

Date of Observation	Reduced Date	Observed Magnitude	Computed Magnitude	O.−C.
April 1 $8^h 32^m$	July 1 $4^h 28^m$	10.60	10.61	−0.01
8 37	4 33	10.65	10.61	+0.04
8 42	4 38	10.61	10.61	0.00
8 52	4 48	10.60	10.61	−0.01
8 57	4 53	10.61	10.60	+0.01
9 2	4 58	10.55	10.52	+0.03
9 9	5 5	10.30	10.35	−0.05
9 15	5 11	10.15	10.19	−0.04
9 21	5 17	10.08	10.01	+0.07
9 26	5 22	9.98	9.85	+0.13
9 32	5 28	9.87	9.67	+0.20
9 38	5 34	9.59	9.51	+0.08
9 47	5 43	9.39	9.36	+0.03
9 54	5 50	9.37	9.35	+0.02
10 9 4	4 5	10.65	10.61	+0.04
9 26	4 27	10.65	10.61	+0.04
9 40	4 41	10.65	10.61	+0.04
9 54	4 55	10.59	10.58	+0.01
10 6	5 7	10.43	10.31	+0.12
10 16	5 17	9.99	10.01	−0.02
10 22	5 23	9.88	9.82	+0.06
10 29	5 30	9.57	9.61	−0.04
10 37	5 38	9.36	9.42	−0.06
11 7 54	5 14	10.23	10.10	+0.13
7 59	5 19	10.05	9.94	+0.11
8 3	5 23	9.85	9.82	+0.03
8 8	5 28	9.62	9.67	−0.05
8 13	5 33	9.48	9.53	−0.05
8 19	5 39	9.40	9.40	0.00
15 8 46	4 29	10.64	10.61	+0.03
9 4	4 47	10.64	10.61	+0.03
9 40	5 23	9.69	9.82	−0.13
9 46	5 29	9.54	9.64	−0.10
9 59	5 42	9.39	9.37	+0.02
30 6 37	4 26	10.61	10.61	+0.00
6 51	4 40	10.59	10.61	−0.02
7 12	5 1	10.57	10.46	+0.11
7 18	5 7	10.24	10.31	−0.07
7 23	5 12	10.11	10.16	−0.05
7 27	5 16	9.95	10.04	−0.09
7 33	5 22	9.91	9.85	+0.06
7 39	5 28	9.61	9.67	−0.06
7 46	5 35	9.35	9.49	−0.14
May 3 10 11	4 5	10.60	10.61	−0.01
10 33	4 27	10.60	10.61	−0.01
10 45	4 39	10.60	10.61	−0.01
10 57	4 51	10.55	10.61	−0.06
11 7	5 1	10.44	10.46	−0.02
11 16	5 10	10.20	10.22	−0.02
11 20	5 14	10.10	10.10	−0.00
11 26	5 20	10.02	9.91	+0.11
11 33	5 27	9.80	9.70	+0.10
11 42	5 36	9.43	9.46	−0.03

TABLE II—*Continued*

Date of Observation		Reduced Date	Observed Magnitude	Computed Magnitude	O.−C.
May 4	7^h40^m	July 1 3^h53^m	10.64	10.61	+0.03
	7 47	4 0	10.65	10.61	+0.04
	8 2	4 15	10.65	10.61	+0.04
	8 12	4 25	10.64	10.61	+0.03
	8 22	4 35	10.60	10.61	−0.01
	8 25	4 38	10.61	10.61	−0.00
	8 35	4 48	10.61	10.61	−0.00
	8 43	4 56	10.45	10.55	−0.10
	8 51	5 4	10.11	10.38	−0.27
	8 55	5 8	10.08	10.28	−0.20
	9 2	5 15	9.97	10.07	−0.10
	9 11	5 24	9.61	9.79	−0.18
	9 20	5 33	9.44	9.54	−0.10
	10 5	6 18	9.44	9.43	+0.01
7	12 36	4 54	10.60	10.58	+0.02
	12 44	5 2	10.49	10.43	+0.06
	12 53	5 11	10.24	10.19	+0.05
	12 57	5 15	10.04	10.07	−0.03
	13 0	5 18	10.00	9.97	+0.03
	13 5	5 23	9.69	9.82	−0.13
	13 9	5 27	9.51	9.70	−0.19
	13 16	5 34	9.34	9.51	−0.17
8	9 51	4 27	10.59	10.61	−0.02
	10 2	4 38	10.59	10.61	−0.02
	10 13	4 49	10.59	10.61	−0.02
	10 26	5 2	10.54	10.43	+0.11
	10 31	5 7	10.25	10.31	−0.06
	10 34	5 10	10.17	10.22	−0.05
	10 37	5 13	10.08	10.13	−0.05
	10 44	5 20	9.69	9.91	−0.22
	10 50	5 26	9.51	9.73	−0.22
	11 1	5 37	9.26	9.44	−0.18
9	7 29	4 23	10.65	10.61	+0.04
	7 48	4 42	10.65	10.61	+0.04
	7 57	4 51	10.65	10.61	+0.04
	8 3	4 57	10.50	10.53	−0.03
	8 8	5 2	10.40	10.43	−0.03
	8 10	5 4	10.35	10.38	−0.03
	8 14	5 8	10.25	10.28	−0.03
	8 20	5 14	9.90	10.10	−0.20
	8 24	5 18	9.76	9.97	−0.21
	8 29	5 23	(9.47)	9.82	......
	8 40	5 34	9.35	9.51	−0.16
10	5 28	4 41	10.64	10.61	+0.03
	5 33	4 46	10.64	10.61	+0.03
	5 43	4 56	10.54	10.55	−0.01
	5 49	5 2	10.29	10.43	−0.14
	5 54	5 7	10.13	10.31	−0.18
	5 58	5 11	10.10	10.19	−0.09
	6 3	5 16	9.99	10.04	−0.05
	6 8	5 21	9.81	9.88	−0.07
	6 15	5 28	9.41	9.67	−0.26
	6 26	5 39	9.32	9.41	−0.09

TABLE II—*Continued*

Date of Observation	Reduced Date	Observed Magnitude	Computed Magnitude	O.−C.
May 14 $6^h 44^m$	July 1 $4^h 21^m$	10.60	10.61	−0.01
7 9	4 46	10.60	10.61	−0.01
7 19	4 56	10.59	10.55	+0.04
7 29	5 6	10.30	10.33	−0.03
7 34	5 11	10.20	10.19	+0.01
7 39	5 16	(9.59)	.....	......
7 41	5 18	(9.56)	.....	......
7 44	5 21	(9.56)	.....	......
7 50	5 27	9.45	9.70	−0.25
June 1 8 33	4 21	10.65	10.61	+0.04
9 0	4 48	10.59	10.61	−0.02
9 10	4 58	10.60	10.52	+0.08
9 16	5 4	10.53	10.38	+0.15
9 24	5 12	10.30	10.16	+0.14
9 29	5 17	9.95	10.01	−0.06
9 36	5 24	9.92	9.79	+0.13
9 44	5 32	9.64	9.56	+0.08
9 52	5 40	9.54	9.39	+0.15
2 6 32	4 38	10.64	10.61	+0.03
6 45	4 51	10.60	10.61	−0.01
6 53	4 59	10.50	10.50	+0.00
6 58	5 4	10.30	10.38	−0.08
7 1	5 7	10.20	10.31	−0.11
7 6	5 12	10.12	10.16	−0.04
7 11	5 17	10.05	10.01	+0.04
7 13	5 19	9.80	9.94	−0.14
7 19	5 25	9.60	9.76	−0.16
7 34	5 40	9.30	9.39	−0.09
3 4 34	4 59	10.55	10.50	+0.05
4 39	5 4	10.35	10.38	−0.03
4 48	5 13	10.07	10.13	−0.06
4 54	5 19	9.82	9.94	−0.12
5 2	5 27	9.50	9.70	−0.20
5 7	5 32	9.30	9.56	−0.26
15 15	4 49	10.64	10.61	+0.03
15 31	5 5	10.64	10.36	+0.28
15 37	5 11	10.36	10.19	+0.17
15 42	5 16	10.15	10.04	+0.11
15 45	5 19	10.02	9.94	+0.08
15 47	5 21	10.03	9.88	+0.15
15 49	5 23	9.95	9.82	+0.13
15 53	5 27	9.75	9.70	+0.05
15 56	5 30	9.53	9.61	−0.08
16 2	5 36	9.46	9.46	+0.00
16 5	5 39	9.26	9.41	−0.15
5 4 42	9 43	10.40	10.34	+0.06
4 56	9 57	10.37	10.37	+0.00
5 36	10 37	10.40	10.44	−0.04
6 1	0 12	10.43	10.47	−0.04
6 41	0 52	10.54	10.54	+0.00
7 26	1 37	10.55	10.57	−0.02
8 2	2 13	10.57	10.57	+0.00
8 49	3 0	10.63	10.61	+0.02

TABLE II—*Continued*

Date of Observation	Reduced Date	Observed Magnitude	Computed Magnitude	O.−C.
June 5 $9^h 31^m$	July 1 $3^h 42^m$	10.63	10.61	+0.02
6 4 36	1 6	10.54	10.56	−0.02
5 21	1 51	10.62	10.57	+0.05
5 41	2 11	10.60	10.57	+0.03
5 57	2 27	10.60	10.59	+0.01
6 40	3 10	10.62	10.61	+0.01
7 5	3 35	10.60	10.61	−0.01
7 30	4 0	10.57	10.61	−0.04
7 51	4 21	10.64	10.61	+0.03
8 9	4 39	10.61	10.61	+0.00
8 25	4 55	10.60	10.57	+0.03
8 35	5 5	10.54	10.36	+0.18
8 45	5 15	10.06	10.07	−0.01
8 48	5 18	10.05	9.97	+0.08
8 51	5 21	9.95	9.88	+0.07
8 54	5 24	9.90	9.79	+0.11
8 57	5 27	9.85	9.70	+0.15
9 1	5 31	9.76	9.59	+0.17
9 7	5 37	9.45	9.44	+0.01
9 24	5 54	9.30	9.34	−0.04
9 8 7	0 42	10.44	10.52	−0.08
9 12	1 47	10.54	10.57	−0.03
9 52	2 27	10.60	10.58	+0.02
10 6 37	1 30	10.60	10.57	+0.03
11 4 47	1 59	10.64	10.58	+0.06
6 7	3 19	10.65	10.61	+0.04
7 2	4 14	10.65	10.61	+0.04
7 42	4 54	10.43	10.58	−0.15
7 49	5 1	10.33	10.46	−0.13
7 54	5 6	10.28	10.33	−0.05
7 59	5 11	10.13	10.19	−0.06
8 17	5 29	9.70	9.64	+0.06
8 22	5 34	9.49	9.51	−0.02
8 31	5 43	9.35	9.36	−0.01
8 42	5 54	9.25	9.34	−0.09
13 6 27	8 16	10.10	10.08	+0.02
14		10.21		
15 5 42	1 17	10.59	10.56	+0.03
8 24	3 59	10.54	10.61	−0.07
8 32	4 7	10.60	10.61	−0.01
8 41	4 16	10.59	10.61	−0.02
8 49	4 24	10.57	10.61	−0.04
8 57	4 32	10.59	10.61	−0.02
9 7	4 42	10.57	10.61	−0.04
9 13	4 48	10.57	10.61	−0.04
9 21	4 56	10.52	10.55	−0.03
9 28	5 3	10.33	10.41	−0.08
9 32	5 7	10.28	10.31	−0.03
9 37	5 12	10.10	10.16	−0.06
9 42	5 17	10.02	10.01	+0.01
9 48	5 23	9.89	9.82	+0.07
9 52	5 27	9.45	9.70	−0.25
10 7	5 42	9.25	9.37	−0.12

TABLE II—*Continued*

Date of Observation	Reduced Date	Observed Magnitude	Computed Magnitude	O.−C.
June 19 $15^h 35^m$	July 1 $9^h 34^m$	10.35	10.31	+0.04
16 7	10 6	10.49	10.38	+0.11
July 30 10 31	1 34	10.60	10.57	+0.03
Aug. 2 5 11	3 10	10.64	10.61	+0.03
5 50	3 49	10.59	10.61	−0.02
3 4 48	5 5	10.42	10.36	+0.06
4 53	5 10	10.31	10.22	+0.09
4 57	5 14	10.21	10.10	+0.11
5 3	5 20	10.07	9.91	+0.16
5 7	5 24	10.05	9.79	+0.26
5 11	5 28	9.82	9.67	+0.15
5 17	5 34	9.41	9.51	−0.10
5 25	5 42	9.34	9.37	−0.03
5 40	5 57	9.29	9.33	−0.04
6 7	6 24	9.29	9.47	−0.18
6 48	7 5	9.55	9.71	−0.16
8 25	8 42	10.05	10.17	−0.12
9 23	9 40	10.15	10.33	−0.18
10 1	10 18	(10.09)	10.40	(−0.31)
10 17	10 34	10.30	10.43	−0.13
10 35	0 2	10.42	10.45	−0.03
4 4 55	7 31	9.90	9.85	+0.05
5 10	7 46	9.95	9.94	+0.01
5 42	8 18	10.14	10.09	+0.05
5 4 56	9 50	10.42	10.35	+0.07
5 45	10 39	10.44	10.44	+0.00
8 46	2 50	10.55	10.60	−0.05
9 46	3 50	10.57	10.61	−0.04
10 1	4 5	10.59	10.61	−0.02
10 25	4 29	10.60	10.61	−0.01
10 44	4 48	10.60	10.61	−0.01
10 58	5 2	10.54	10.43	+0.11
11 5	5 9	10.24	10.25	−0.01
11 8	5 12	10.15	10.16	−0.01
11 11	5 15	10.11	10.07	+0.04
11 19	5 23	9.90	9.82	+0.08
11 23	5 27	9.76	9.70	+0.06
11 28	5 32	9.66	9.56	+0.10
11 35	5 39	9.56	9.41	+0.15
11 45	5 49	9.41	9.34	+0.07
11 58	6 2	9.56	9.36	+0.20
12 31	6 35	9.59	9.53	+0.06
6 8 20	4 42	10.55	10.61	−0.06
8 41	5 3	10.42	10.41	+0.01
8 45	5 7	10.42	10.31	+0.11
8 52	5 14	10.05	10.10	−0.05
8 58	5 20	9.86	9.91	−0.05
9 4	5 26	9.50	9.73	−0.23
9 10	5 32	9.45	9.56	−0.11
9 25	5 47	9.24	9.34	−0.10
9 43	6 5	9.34	9.37	−0.03
10 13	6 35	9.50	9.53	−0.03
7 4 45	3 26	10.65	10.61	+0.04

TABLE II—*Continued*

Date of Observation	Reduced Date	Observed Magnitude	Computed Magnitude	O.−C.
Aug. 10 $5^h 20^m$	July 1 $0^h 6^m$	10.54	10.46	+0.08
6 10	0 56	10.60	10.55	+0.05
9 45	4 31	10.54	10.61	−0.07
10 5	4 51	10.55	10.61	−0.06
10 12	4 58	10.59	10.52	+0.07
10 19	5 5	10.55	10.36	+0.19
10 25	5 11	10.18	10.19	−0.01
10 28	5 14	10.10	10.10	+0.00
10 30	5 16	9.92	10.04	−0.12
10 34	5 20	9.81	9.91	−0.10
10 40	5 26	9.56	9.73	−0.17
11 5 25	2 29	10.57	10.58	−0.01
7 27	4 31	10.60	10.61	−0.01
7 48	4 52	10.65	10.61	+0.04
7 59	5 3	10.42	10.41	+0.01
8 10	5 14	10.09	10.10	−0.01
8 14	5 18	9.92	9.97	−0.05
8 19	5 23	9.66	9.82	−0.16
8 30	5 34	9.44	9.51	−0.07
8 43	5 47	9.34	9.34	+0.00
8 55	5 59	9.41	9.35	+0.06
9 30	6 34	9.46	9.53	−0.07
10 1	7 5	9.73	9.71	+0.02
10 25	7 29	9.98	9.85	+0.13
11 15	8 19	10.15	10.10	+0.05
Sept. 13 7 14	4 35	10.59	10.61	−0.02
7 36	4 57	10.57	10.54	+0.03
7 47	5 8	10.17	10.28	−0.11
7 52	5 13	10.10	10.13	−0.03
7 58	5 19	10.00	9.94	+0.06
8 1	5 22	9.85	9.85	+0.00
8 5	5 26	9.89	9.73	+0.16
8 18	5 39	9.50	9.41	+0.09
22 7 5	3 32	10.62	10.61	+0.01
7 30	3 57	10.64	10.61	+0.03
7 55	4 22	10.62	10.61	+0.01
8 10	4 37	10.60	10.61	−0.01
8 24	4 51	10.59	10.61	−0.02
8 30	4 57	10.49	10.54	−0.05
8 40	5 7	10.42	10.31	+0.11
8 45	5 12	10.25	10.16	+0.09
8 50	5 17	10.08	10.01	+0.07
8 55	5 22	10.05	9.85	+0.20
9 0	5 27	9.90	9.70	+0.20
9 5	5 32	9.79	9.56	+0.23

acter. The light-curve determined by this refined and accurate observer (*Annals Cape Observatory*, 4, 129B) shows residuals greater than his accustomed errors of observations.

4. Combining the preceding observations in sets, we obtain the following data (Table III) for the mean curve of *S Arae*. This

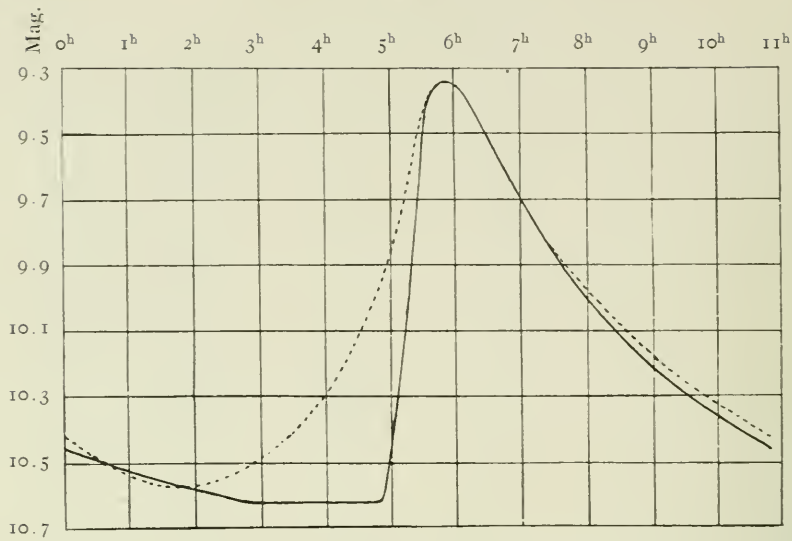


FIG. 4.—Light-curve of *S Arae*, reduced to mean curve of July 1, 1910. Unbroken line, observed line. Broken line, theoretical curve.

curve is set forth in Fig. 4, unbroken line. Its peculiar character —a stationary period at minimum, a rapid rise to maximum, and a slow decline to constant phase—will at once be manifest.

TABLE III

MEAN MAGNITUDES OF *S Arae*, JULY 1, 1910

Date	Mean Magnitude	Observations Used	Date	Mean Magnitude	Observations Used
1910			1910		
July 1 $0^h 33^m.6$	$10^m.51$	5	July 1 $5^h 13^m.7$	$10^m.09$	10
1 21.8	10.58	4	15.7	10.03	9
48.5	10.59	4	17.5	9.98	9
2 26.2	10.58	6	19.5	9.93	10
3 19.0	10.63	7	21.8	9.90	9
52.0	10.60	6	23.3	9.82	10
4 5.1	10.61	7	26.4	9.72	12
20.4	10.62	8	28.0	9.67	10
27.3	10.62	8	31.7	9.56	10
34.4	10.60	9	34.1	9.44	8
40.2	10.61	10	37.9	9.40	10
47.7	10.61	10	42.9	9.36	9
52.6	10.59	11	53.8	9.34	6
57.3	10.54	12	6 21.9	9.45	7
5 1.9	10.44	10	7 23.2	9.82	5
4.3	10.39	10	8 23.8	10.11	4
6.8	10.30	10	9 45.2	10.34	5
9.6	10.22	9	10 27.8	10.41	6
11.6	10.16	10			

5. As already stated (paragraph 1) the probable explanation of this curve, and of all curves of this type, is that we have *Algol*- or eclipse-variation superimposed upon ordinary short-period variation.

The stationary phase of *S Arae* lasts from $2^h\ 52^m$ to $4^h\ 52^m$, that is for 2^h out of a period of $10^h\ 50^m\ 43^s\!.45$.

It is suggested that during this period one of the components, the smaller and brighter, is eclipsed totally by the primary component of the binary system, *S Arae*. It is not possible to determine definitely from the light-curve when the eclipse begins or ends, but in order to arrive, in the first instance approximately, at these dates, an arbitrary short-period light-curve was drawn, such a light-curve as is shown by a dotted line in Fig. 4. If *S Arae* were simply a short-period variable this would probably be the form of its light-curve.

The arbitrary uneclipsed light-curve, first drawn from experience of other short-period curves, was modified to meet the argument adopted and thus finally the harmonic curve indicated by a dotted line in Fig. 4 was arrived at.

The harmonic elements of this regular curve are:

Magnitude at any time T

$$
\begin{aligned}
&= 10^m\!.151 \\
&+ 0.491\ (\cos\ \theta - 38°58') \\
&- 0.170\ (\cos 2\theta - 37°36') \\
&+ 0.081\ (\cos 3\theta - 58°40') \\
&- 0.041\ (\cos 4\theta - 53°58') \\
&+ 0.025\ (\cos 5\theta - 69°33') \\
&- 0.014\ (\cos 6\theta - 68°58')
\end{aligned}
$$

where

$$\theta = (0°\!.5532)T$$

T being time from July 1, $0^h\ 0^m$, expressed in minutes.

This fundamental or primary light-curve of *S Arae* would indicate that eclipse is ended by $5^h\ 48^m$.

6. Grouping together and amplifying these statements we have:

Duration of stationary period = 2 hours

Magnitude of system at stationary period, that is of S_1 on the theory of eclipse $= 10^m\!.61$

Middle of total eclipse of S_2	=	$3^h 52^m$
End of total eclipse of S_2	=	4 52
End of eclipse of S_2	=	5 48
S_2 in approaching quadrature	=	6 35

Taking

R = radius of S_1
r = radius of S_2
1 = radius of orbit, considered as circular,

Then, on the supposition that the eclipse is central,

$$R+r=\cos\left\{\left(\frac{47^m}{10^h 50^m 43^s}\right)360^\circ\right\}$$
$$=0.899$$
$$R-r=\cos\left\{\left(\frac{1^h 43^m}{10^h 50^m 43^s}\right)360^\circ\right\}$$
$$=0.545$$

Therefore

$$R \quad =0.722$$
$$r \quad =0.177,$$

and distance between circumference of components

$$=0.101$$

7. Fig. 5 is a diagrammatic view of the binary system *S Arae*, a system comprising a central star, S_1, of radius 0.722, of constant brightness $10^m.61$, and a satellite star, S_2, of radius 0.177, revolving round the central star in $10^h\ 50^m\ 43^s.45$, and constantly varying as it revolves.

Where the constantly varying *comes*, S_2, is eclipsed by S_1, we have a stationary period when only S_1 is visible. This phase lasts two hours. We shall also have a similar eclipse, but less pronounced, when S_2 passes before S_1.

It remains now to discover how far this theory meets the facts of variation as determined by observation. In Table IV is given the computed magnitudes of *S Arae* at intervals of ten minutes.

In column 1 we find the date, the day being, as before, July 1, 1910. Column 2 gives the uneclipsed magnitude of the system as computed from the harmonic elements stated in paragraph 5, and charted down (dotted line) in Fig. 4. Column 3 gives the eclipsed light of S_1 at any instant, its uneclipsed light is taken as

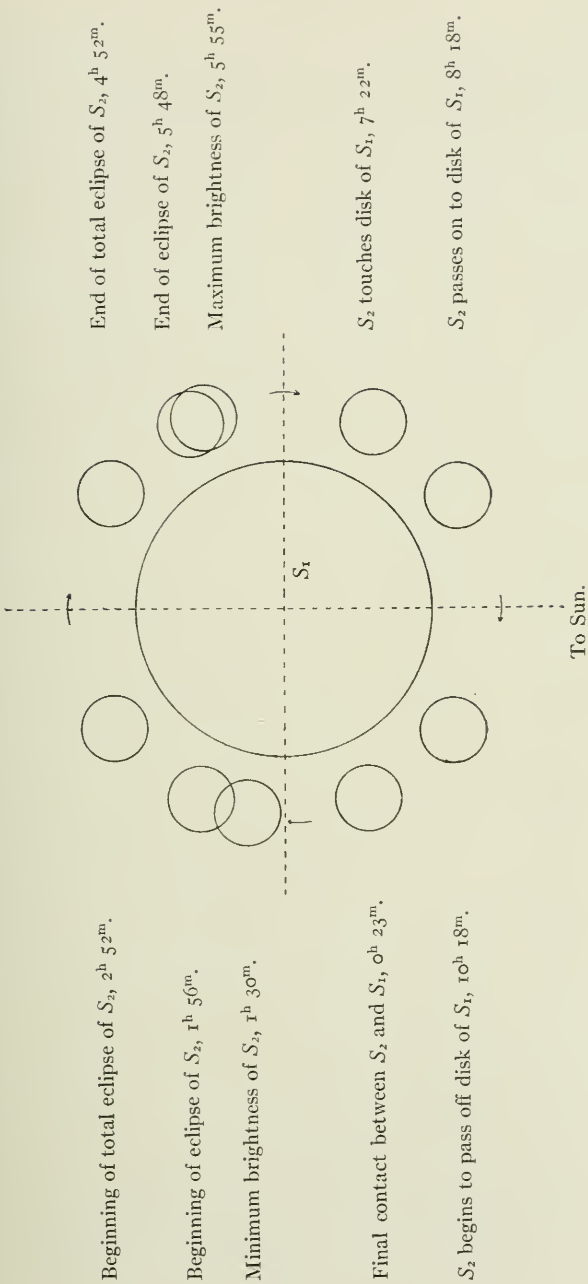


FIG. 5.—System of *S Arae:* Dates from mean curve of July 1, 1910

TABLE IV

Date	Theoretical Magnitude of System (Uneclipsed)	Light of S_1 (Eclipsed)	Light of S_2 (Uneclipsed)	Light of S_2 (Eclipsed)	Light of System (Eclipsed)	Theoretical Magnitude of System (Eclipsed)
$0^h\ 0^m$	$10^m.42$	0.97	0.19	0.19	1.16	10.45
10	10.45	0.98	0.16	0.16	1.14	10.47
20	10.47	0.99	0.14	0.14	1.13	10.49
30	10.50	1.00	0.11	0.11	1.11	10.50
40	10.52	1.00	0.09	0.09	1.09	10.52
50	10.54	1.00	0.07	0.07	1.07	10.54
1 0	10.55	1.00	0.06	0.06	1.06	10.55
10	10.56	1.00	0.05	0.05	1.05	10.56
20	10.56	1.00	0.05	0.05	1.05	10.56
30	10.57	1.00	0.04	0.04	1.04	10.57
40	10.57	1.00	0.04	0.04	1.04	10.57
50	10.57	1.00	0.04	0.04	1.04	10.57
2 0	10.56	1.00	0.05	0.04	1.04	10.57
10	10.56	1.00	0.05	0.04	1.04	10.58
20	10.55	1.00	0.06	0.04	1.04	10.58
30	10.54	1.00	0.07	0.03	1.03	10.58
40	10.53	1.00	0.08	0.02	1.02	10.59
50	10.51	1.00	0.10	0.01	1.01	10.60
3 0	10.49	1.00	0.12	0.00	1.00	10.61
10	10.47	1.00	0.14	0.00	1.00	10.61
20	10.44	1.00	0.17	0.00	1.00	10.61
30	10.41	1.00	0.20	0.00	1.00	10.61
40	10.38	1.00	0.24	0.00	1.00	10.61
50	10.34	1.00	0.28	0.00	1.00	10.61
4 0	10.30	1.00	0.33	0.00	1.00	10.61
10	10.25	1.00	0.39	0.00	1.00	10.61
20	10.19	1.00	0.47	0.00	1.00	10.61
30	10.12	1.00	0.57	0.00	1.00	10.61
40	10.05	1.00	0.68	0.00	1.00	10.61
50	9.98	1.00	0.79	0.00	1.00	10.61
5 0	9.89	1.00	0.94	0.13	1.13	10.48
10	9.79	1.00	0.14	0.43	1.43	10.22
20	9.64	1.00	1.44	0.90	1.90	9.91
30	9.48	1.00	1.83	1.49	2.49	9.61
40	9.36	1.00	2.16	2.06	3.06	9.39
50	9.34	1.00	2.22	2.22	3.22	9.34
6 0	9.35	1.00	2.19	2.19	3.19	9.35
10	9.39	1.00	2.08	2.08	3.08	9.39
20	9.44	1.00	1.94	1.94	2.94	9.44
30	9.50	1.00	1.78	1.78	2.78	9.50
40	9.56	1.00	1.63	1.63	2.63	9.56
50	9.62	1.00	1.49	1.49	2.49	9.62
7 0	9.68	1.00	1.36	1.36	2.36	9.68
10	9.74	1.00	1.23	1.23	2.23	9.74
20	9.80	1.00	1.11	1.11	2.11	9.80
30	9.85	0.99	1.01	1.01	2.00	9.85
40	9.90	0.98	0.92	0.92	1.90	9.91
50	9.94	0.96	0.85	0.85	1.82	9.96
8 0	9.98	0.95	0.79	0.79	1.74	10.01
10	10.02	0.94	0.72	0.72	1.67	10.06
20	10.06	0.94	0.66	0.66	1.60	10.10
30	10.09	0.94	0.61	0.61	1.55	10.13

TABLE IV—*Continued*

Date	Theoretical Magnitude of System (Uneclipsed)	Light of S_1 (Eclipsed)	Light of S_2 (Uneclipsed)	Light of S_2 (Eclipsed)	Light of System (Eclipsed)	Theoretical Magnitude of System (Eclipsed)
8^h 40^m	10^m13	0.94	0.56	0.56	1.50	10.17
50	10.16	0.94	0.51	0.51	1.45	10.20
9 0	10.19	0.94	0.47	0.47	1.41	10.23
10	10.21	0.94	0.45	0.45	1.39	10.26
20	10.23	0.94	0.42	0.42	1.36	10.28
30	10.25	0.94	0.39	0.39	1.33	10.30
40	10.28	0.94	0.36	0.36	1.30	10.33
50	10.30	0.94	0.33	0.33	1.27	10.35
10 0	10.32	0.94	0.31	0.31	1.25	10.37
10	10.34	0.94	0.28	0.28	1.22	10.39
20	10.36	0.94	0.26	0.26	1.20	10.41
30	10.38	0.95	0.24	0.24	1.18	10.43
40	10.40	0.96	0.21	0.21	1.17	10.44

unity. In column 4 we find the uneclipsed light of S_2, and in column 5 the eclipsed light. In column 6 is given the total light of the system, which in column 7 is reduced to magnitudes.

8. From Table IV we are able to place computed values against the observed magnitudes given in paragraph 4, from which the mean curve of *S Arae* has been drawn.

Table V sets forth the agreement between observation and theory. The discordance amounts to 0^m010.

9. If we inquire what measure of confidence we may place in the theory of variation of stars of the *S Arae* and *V Lyrae* type, thus preferred, it may be urged,

a) There is a remarkable accordance between the computed and observed light-curves.

b) A constant phase at minimum can only reasonably be explained by the theory of eclipse.

c) Albrecht and others have drawn attention to the fact that maximum brightness takes place a little before maximum approach, and minimum brightness a little after maximum recession, in the case of such short-period stars as are bright enough to yield these facts.

In the case of *S Arae* the theory urged places maximum 40 minutes before maximum approach and minimum 20 minutes after maximum recession.

d) In the *Lick Observatory Bulletin*, No. 151, Duncan suggests a theory of Cepheid variables that has much to commend it. It certainly explains the variation of *S Arae*. The distance of S_2 from S_1 is only one-tenth of the radius of their orbit. It is evident

TABLE V

Date	Observed Magnitude	Computed Magnitude	O.−C.
July 1 $0^h 33^m.6$	10.51	10.51	−0.00
1 21.8	10.58	10.56	+0.02
48.5	10.59	10.57	+0.02
2 26.2	10.58	10.58	+0.00
3 19.0	10.63	10.61	+0.02
52.0	10.60	10.61	−0.01
4 5.1	10.61	10.61	+0.00
20.4	10.62	10.61	+0.01
27.3	10.62	10.61	+0.01
34.4	10.60	10.61	−0.01
40.2	10.61	10.61	+0.00
47.7	10.61	10.61	+0.00
52.6	10.59	10.60	−0.01
57.3	10.54	10.54	+0.00
5 1.9	10.44	10.44	+0.00
4.3	10.39	10.37	+0.02
6.8	10.30	10.31	−0.01
9.6	10.22	10.23	−0.01
11.6	10.16	10.17	−0.01
13.7	10.09	10.11	−0.02
15.7	10.03	10.05	−0.02
17.5	9.98	9.98	+0.00
19.5	9.93	9.93	+0.00
21.8	9.90	9.86	+0.04
23.3	9.82	9.81	+0.01
26.4	9.72	9.72	+0.00
28.0	9.67	9.67	+0.00
31.7	9.56	9.57	−0.01
34.1	9.44	9.51	−0.07
37.9	9.40	9.43	−0.03
42.9	9.36	9.37	−0.01
53.8	9.34	9.34	+0.00
6 21.9	9.45	9.45	+0.00
7 23.2	9.82	9.82	+0.00
8 23.8	10.11	10.11	+0.00
9 45.2	10.34	10.34	+0.00
10 27.8	10.41	10.42	−0.01

that at this distance from S_1, S_2 must revolve through the very rarefied outer envelope of the primary. Thus the advancing hemisphere of S_2 will be greatly more luminous than the following surface, the difference in brightness depending on the amount of atmosphere which has been driven off from the advancing hemi-

sphere. And as the tenuous atmosphere surrounding S_1 is denser, the deeper it is, maximum brightness of S_2 will occur before the star reaches quadrature. This is an essential condition of the theory suggested by Duncan.

e) At its minimum the light of S_2, surface for surface, is only a little fainter than that of S_1.

f) The period of the system is decreasing.

10. I have not been unmindful, while preparing this paper, of the many and weighty arguments which tell against the validity of the theory proposed. Indeed I was so conscious of the difficulties that arose, hydra-headed, that for some time I set the theory aside. Campbell's pregnant remark that "the future will probably establish that the cluster variables are binaries" gave an appearance of reasonableness to the hypothesis I have tried to prove.

Against the hypothesis of combined eclipse and Cepheid variation as an explanation of cluster variables, of which *S Arae* may be taken as the type, there are:

a) The extraordinary range of magnitude that must be admitted for S_1. At maximum this companion is *fifty* times brighter than it is at minimum. It is difficult to premise an atmosphere dense enough to absorb forty-nine fiftieths of the light of the nucleus. Of course it is possible that friction adds to the luminosity of the advancing hemisphere, but this would mean that the revolution and rotation of S_2 were synchronous.

b) S_2 must revolve faster than S_1 rotates, a difficult assumption, yet not one out of accord with recent views on cosmic evolution.

c) Roche's criterion demands a high density for S_2. Here again the density is in agreement with the luminosity, at least with the luminosity of the nucleus.

There are other difficulties that will arise in the minds of those who deal with such investigations as I have endeavored to bring to some issue in this paper.

11. In conclusion it remains to be said that not the least value of the theory suggested is that it co-ordinates variation of several

types, bringing them all under the fundamental conception of binary movement.

That stars of the *S Arae* type should be found mainly in clusters need not surprise us. It is in such aggregations that we might expect to find stars that are apparently at the starting-point of their history.

LOVEDALE, SOUTH AFRICA
November 25, 1910

THE ZEEMAN EFFECT FOR CHROMIUM[1]

By HAROLD D. BABCOCK

The following work was undertaken some months ago as part of a comparative study of the Zeeman effect in sun-spots and in the laboratory. It was begun with the apparatus at hand, but on account of prospective additions to our equipment the greater part of it has been deferred and will be taken up, it is hoped, in the near future. The results already obtained for chromium, however, appear of sufficient interest to warrant a description of them at present.

The spectra were produced with the vertical Littrow spectrograph described elsewhere.[2] The earlier photographs were taken with the 13-cm objective of 4 m focal length, in conjunction with a 13-cm Rowland plane grating having 568 lines to the millimeter. The third-order spectrum was used with this grating, giving a scale of about 1.37 Ångström units per millimeter. All the later photographs, including a majority of the plates measured, were taken with a 20-cm objective of 9.1 m focal length, used with a 20-cm Michelson plane grating of 500 lines to the millimeter. The spectrum of second order was generally used in this case, giving a scale of about 0.95 Ångström unit per millimeter. Under these conditions an interval of about 400 Ångström units can be obtained in good focus upon one plate.

A Du Bois half-ring electromagnet supplied the field. Pole distances of about 6 mm were customarily used, the pole-pieces being chisel-shaped with end faces 3 mm×12 mm. The magnet was fed with currents of 8 to 15 amperes supplied by a 110-volt generator. Occasional adjustment of a resistance in series with the magnet sufficed for maintaining the field constant during an exposure. During the progress of the work the magnet developed

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 52.

[2] *Ibid.*, No. 27; *Astrophysical Journal*, **28**, 244, 1908.

a defect and required rewinding. Advantage was taken of this opportunity to provide for it a water-cooling system by winding a layer of thin copper tubing around the bare iron cores. With this improvement it was found that a current of 14 amperes could be maintained in the magnet coils for several hours without noticeable heating of the cores. The field-intensities varied for different plates between 15,000 and 23,400 gausses. Measurements of intensity of field were made with a bismuth spiral calibrated at the National Bureau of Standards. It is probable that the measurements are reliable within 1 or 2 per cent.

The spark terminals consisted of lumps of chromium held in brass clamps supported by a frame of fiber. The length of the spark was ordinarily about 3 mm. A second spark-gap of about the same length was inserted in series with it. Energy for the sparks was supplied from a 5-kw transformer fed with current at 104 or 52 volts and regulated by a resistance in series with the low-tension side. The high-tension side was connected either for 16,000 or 32,000 volts, although the actual voltages delivered were doubtless considerably less than these figures on account of the drop in voltage on the low-tension regulating resistance. With the transformer set for 16,000 volts the spark is much brighter than for 32,000 volts, other things being the same, but the heat developed at the terminals is sometimes so great as to cause excessive oxidation of the metal. Two condensers consisting of sheet copper between plate glass were used. Both are entirely immersed in oil. Each consists of numerous sections which can be connected together in various ways so as to obtain a large range of values of the capacity. The capacities used were 0.068 mf and 0.136 mf, generally the former. These figures are the values obtained by ballistic comparisons with mica condensers, and accordingly represent the actual working capacities only approximately. Inductance was used in the spark circuit occasionally, when a plate was taken for strong lines only.

The Zeeman components for a given line were always photographed in two groups. A nicol prism mounted directly above the slit of the spectograph was turned so as to transmit light in which the vibrations were normal to the magnetic field and a

photograph taken showing the *n*-components. The nicol was then rotated 90° and a second photograph taken showing the *p*-components, with vibrations parallel to the field. Comparison spectra were placed on each plate by turning off the field and using an occulting bar over the slit. The complete aspect of a line as observed at right angles to the magnetic field is then to be had by superposing the two photographs.

Table I presents the results of the measures obtained on these plates. The wave-lengths are Rowland's except for those cases in which less than three decimal places are given. In these cases the values are those of Hasselberg[1] or of Exner and Haschek.[2]

The second column gives the order of separation, i.e., the total number of components into which the line appears separated when viewed at right angles to the magnetic field. A number of lines are classified in this column as complex. These lines as a rule have more than four components, but higher resolving power is required to determine completely their character. For some complex lines measurements are given which show merely the average separation of the groups of components. The third and fourth columns give for the *n*-components and the *p*-components respectively the value of $\Delta\lambda \cdot 10^8/\lambda^2$ for a field of 20,000 gausses. For all but the weakest lines measured, the values are the means of several determinations on different photographs at different field-strengths. Under the column headed "Remarks" will now and then be found two or more numbers which express relative intensities, as one proceeds from violet to red, of the components indicated. Thus *n* 1:3 means that of the two *n*-components observed for a certain line, the one on the red is three times as intense as the other. These numbers have no relation to the relative intensities of *n*- and *p*-components, nor to the relative intensities of components of different lines. A few abbreviations are employed in the remarks, viz., −r = not resolved; E = in Lockyer's list of enhanced lines.

The table shows a number of lines of remarkable behavior in

[1] B. Hasselberg, "Die Spectra der Metalle im electrischen Flammenbogen," *Kongl. Svenska Vetenskaps-Akademiens Handlingar*, Bandet 26, No. 5, 1894.

[2] *Wellenlangen Tabellen der Funkenspectren der Elemente*, Leipzig and Vienna, 1902.

TABLE I

SEPARATION OF CHROMIUM LINES IN MAGNETIC FIELD

λ	Total Number Components	$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	
3744.029	3	2.53		
3749.110	4	3.96	1.39	*p* 2:1
3757.824	3	1.93		*n* 4:1
3768.358	3	0.67		*n* 2:1
3768.871	3?			
3788.999	3?	2.73		*n* 2:1
3790.629	3?	1.84		Red component not distinguishable
3791.517	3?	2.92		*n* 1:2
3792.294	3?	3.80		Uncertain
3793.429	3?	2.93		*n* 3:1
3794.02	3?	2.99		
3794.753	3?	3.03		
3797.283	3	2.63		*n* 1:2
3797.860	3	2.85		*n* 2:3
3804.934	3	2.80		
3806.97	3?			Probably two *n*-components
3808.076	3?			Probably two *n*-components
3812.37	3?	0.86		
3821.71	3?	1.64		
3823.653	3?	2.71		
3841.420	Complex	1.83	1.14	
3849.15	4	2.85	1.17	
3849.48	3	2.39		
3849.66	3	2.75		
3850.13	Complex	1.22		
3852.33	3	2.75		
3854.36	4?	2.95	1.54	
3855.41	3	4.55		
3856.40	3	3.99		
3857.74	6	{ 4.61 } { 2.26 }	2.22	Red components decrease in intensity from inside to outside
3865.674	3	1.77		E
3883.41	3	2.74		
3883.778	Unaffected			
3885.364	3	2.73		
3886.942	3	2.80		
3892.07	3?	1.70		Components not sharp
3894.165	3	2.75		
3897.83	3?	1.71		*n* 1:2 not sharp
3902.22	3?	1.18		*n* 1:3 not sharp
3903.090	3	2.76		
3903.30	3	2.74		
3905.660	3	1.74		E
3907.91	3?	1.64		Doubtful
3908.900	3	2.71		
3914.45	Unaffected			
3916.383	3	2.76		
3917.731	3?	1.42		

TABLE I—*Continued*

λ	Total Number Components	$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	p	
3919.309	3	2.72		
3921.188	3	2.71		
3926.779	3	2.09		
3928.357	3			
3928.783	3	2.71		
3941.637	3	2.74		
3951.219	Complex		1.59	n-components uncertain
3952.549	4?	1.56		p-components wide, diffuse
3960.902	3	3.06		
3963.831	3	2.10		
3969.287	4?	2.36	1.12	n 4:1
3969.899	8	3.96	1.61	Probably four inner n-components −r
3971.402	7	2.25	1.08 0.00	Outer n-components −r; n 1:4:4:1; p 1:2:1
3972.830	3?	4.29		
3976.839	4?	3.36	1.88	Difficult
3978.809	3	2.70		
3979.664	3	1.95		E
3979.936	3?			n uncertain, one p-component observed
3981.376	7	.... 2.04	1.22 0.00	Outer n-components too weak to measure
3984.059	3	1.50		
3984.479	Complex			See description in text
3990.129	3	2.11		
3991.333	3	1.22		
3991.830	Complex			See description in text
3992.971	3	3.09		
3994.092	3	2.96		
3999.818	3	1.83		
4001.595	3	2.00		
4003.48	3	1.73		E
4004.11	4?			
4012.631	3	1.69		E
4014.85	5?		1.32	Probably three n-components
4016.955	3	1.44		
4022.401	3	1.49		n 2:1
4023.90	4?		1.62	n difficult
4025.158	3	1.15		
4026.318	Complex	3.36		
4027.253	6	....	1.86	Inner n-components −r; p 2:1
4030.82	4?	2.23		p-components wide −r
4033.44	3	1.49		Measured normal and red component
4037.449	Unaffected			
4038.19	3	1.92		E
4039.244	3	2.04		
4042.40	3	2.31		
4043.956	3	1.97		

TABLE I—*Continued*

λ	Total Number Components	$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	p	
4044.24	3			
4046.612	3	1.65		
4050.18	3?			
4051.491	3	1.87		
4052.15	3	2.06		E
4056.221	3	1.45		
4056.93	3	1.86		
4058.915	3	1.50		E
4060.77	3	1.80		
4065.861	3	1.95		
4067.05	Complex		1.97	
4067.94	6	{ 4.59 { 1.82	1.54	*p*-components doubtful
4071.13	3?			E
4075.055	3	1.96		
4076.201	3	1.79		
4077.221	6	{ . . . . { 1.33	1.64	Outer *n*-components too weak to to measure
4077.81	3	1.56		
4081.88	Unaffected			Shifted to red
4082.53	4	1.97	0.68	E
4085.15	Complex			
4090.474	3	1.56		*n* 2:1
4099.16	3	2.68		Difficult
4104.90	4	1.55	0.75	
4108.54	4?		1.53	*n*-components uncertain
4109.734	Complex	1.04		
4111.19	3	1.90		E
4120.78	Complex		1.67	
4121.477	3	1.81		
4121.963	3	1.88		
4123.55	3	2.07		
4126.673	3	2.42		
4127.426	4?	{ 2.16 { 0.00		Peculiar line, one central *n*-component, one *p*-component; *n* 1:2:1
4127.77	4	2.59	2.20	
4128.53	3	2.04		
4131.507	3	1.84		
4142.330	3	1.89		
4142.629	6	{ . . . . { 1.16		*p*-components and outer *n*-components too weak to measure
4145.96	3	2.25		E
4152.927	3	1.79		
4153.222	3	2.60		
4153.971	5	{ 3.90 { 0.00		*p*-components −r
4161.571	3	1.92		
4163.818	3?	1.98		*p*-component shifted to violet
4165.676	3	1.82		
4169.94	3	2.05		

TABLE I—*Continued*

λ	Total Number Components	$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses n	p	Remarks
4170.372	3	1.88		
4172.96	3	1.93		
4174.973	4	3.01	1.42	
4176.09	3			
4179.408	3	1.75		
4185.058	4	2.24	0.60	
4186.496	3?			
4190.287	3	2.04		
4191.433	3	1.70		
4191.90	Complex			*p*-components very wide, fringed outside
4192.171	3	1.68		
4193.836	3	3.04		Measured violet component and normal
4195.006	Complex	3.61		
4197.390	4	3.15	1.95	Measurement of *p*-components doubtful
4198.65	3	2.69		
4200.261	4	1.92	1.06	Difficult
4203.730	3	0.68		Peculiar asymmetry, see remarks in text; *n* 2:1
4204.359	4?			
4204.622	3	1.72		
4207.059	3	1.85		
4208.514	3	1.15		*p*-component wide
4209.521	3	1.72		
4209.914	3	2.15		
4211.512	4	{2.03 / 0.00}		Three *n*-components, one *p*-component
4212.801	3	1.47		Violet component wider than red
4213.323	3	1.87		
4216.516	3?	1.81		Weak and diffuse
4217.720	3	2.09		
4221.737	3	1.19		
4222.890	3	1.97		*p*-component displaced to violet
4224.673	3	1.66		Red *n*-component blended with following line
4225.020	3	1.58		E
4230.61	3	1.60		*n* 2:1 not sharp
4232.35	3?			
4233.40	4	2.34	1.20	E
4234.64	3	2.11		
4239.107	3	2.55		
4240.872	3	2.08		
4242.535	3	2.10		E
4254.505	Complex	2.79		*n*-components fringed outside; *p*-component wide
4255.659	3	2.25		
4262.06	3	2.79		E
4263.290	3	1.97		
4268.90	3?			

TABLE I—*Continued*

λ	Total Number Components	$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	
4270.08	4?			
4274.958	3	3.56		
4280.556	3	1.72		
4284.382	3	1.10		E
4285.03	3	1.85		*p*-component slightly widened
4289.885	Complex	3.31		*n*-components fringed outside; *p*-component wide
4292.135	3	1.52		
4293.714	3			
4295.914	3	3.06		
4297.202	3	2.74		
4297.908	3	1.64		
4299.87	3?			
4300.68	3	1.86		
4301.33	3	2.02		
4302.95	3	2.17		
4305.614	3	3.90		
4307.1	3	1.62		E difficult
4312.65	3?			
4319.799	3?			
4320.75	3?			
4321.44	3?			
4323.70	3	1.98		
4325.306	3	1.62		
4332.75	3?			
4337.725	9	2.68 1.80 0.82	0.92 0.00	*n* 1:2:3:3:2:1; *p* 1:2:1
4338.56	3	1.82		
4338.95	3			
4339.617	Complex	1.83		
4339.882	Unaffected			
4340.297	3	2.81		
4344.670	3	2.10		
4346.987	5	2.58 0.00		*n* 1:1:3; *p* 1:3 *n*-components shifted to violet; *p*-components unsuitable for measurement
4351.216	5	2.76 0.00	2.83	
4351.930	3	2.21		
4357.70	3			
4359.784	12	3.61 2.69 1.78 0.90	1.78	*p* 3:1:1:3; inner *p*-components too weak for measurement
4363.267	3	1.51		
4371.442	4	2.49	1.19	
4373.415	8	5.61 2.79 0.00	2.85 0.00	*n* 3:2:1:2:3; *p* 2:3:2

TABLE I—*Continued*

λ	Total Number Components	$\frac{\Delta\lambda}{\lambda^2} \cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	
4374.331	6	{3.41, 1.46}	1.14	*n* 1:1*:3:1 *This component wide and diffuse
4375.493	3	1.68		
4376.942	3	2.16		
4377.73	3	2.03		
4381.274	7?	{3.12, 2.13}		*n* 1:3:3:1; *p*-component very wide, probably =3, -r
4385.144	4	2.60	0.97	
4387.54	3	1.94		*n*-components shifted to violet
4387.658	3	1.88		
4391.924	9?	{4.51, 3.64, ...}		Two very weak *n*-components inside those measured; *p*-component very wide
4395.58	3?			
4397.40	3			
4403.661	3	1.83		
4411.240	3?			
4414.011	3	2.20		
4423.430	Unaffected			
4424.20	3			
4424.457	Complex	2.22		
4428.711	4?			
4430.07	Complex			
4430.59	3?			
4432.330	3	2.74		
4432.93	3?			
4443.90	3			
4458.690	3	1.96		
4462.98	4?			
4465.08	3			
4465.519	Complex			
4473.91	3			
4475.470	3			
4480.40	3			Very close double
4481.57	3?			
4483.039	Complex	3.21	1.00	
4488.218	3	2.40		
4489.630	3	1.85		
4492.016	4?			
4492.475	4?	2.24		
4495.42	3			
4497.023	Complex	2.54	1.58	*n*-components fringed outside, centers measured
4498.897	3	2.75		
4500.451	4	1.52	0.51	
4501.264	5	{2.70, 1.37}		*n*-1:2:2:3; *p*-component shifted to violet
4501.92	3	2.69		
4507.00	3	1.78		
4512.063	3?	2.02		
4514.662	4?			

TABLE I—*Continued*

λ	Total Number Components	$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	
4515.60	3	2.66		
4521.304	3	1.72		
4522.18	3			
4525.01	3			
4526.269	3			
4526.632	3	2.44		
4527.53	Complex			
4527.65	3	2.25		Wide and diffuse
4530.020	3?	2.65		*n* 4:1
4530.910	7	3.96 0.61	1.95 0.00	*n* 1:4:4:1; *p* 1:2:1
4535.310	3?	3.54		Very doubtful
4535.879	5	3.95 0.00	1.40	*n* 3:1:2; *n*-components all shifted to red
4539.946	6?	1.94 1.60 0.94 0.51		*n* 4:1:2:2:4; *p*-component very wide; see description in text
4540.672	3?	1.77		
4540.880	3?	1.13		
4541.236	Complex			
4541.690	3	1.60		
4542.83	3	2.04		
4543.99	Complex			Very weak
4544.788	3	0.66		One *n*-component undisplaced, the other shifted to violet
4545.507	Complex			
4546.129	4	3.54	0.56	
4554.10	3			
4554.98	3			
4555.162	4?	2.38		E
4556.306	6?	2.31	1.05	*n*-components wide and diffuse, centers measured
4588.827	3	2.16		E
4563.43	3			
4563.82	3			
4564.36	3	1.87		
4565.688	3	3.46		
4569.76	4	3.26	1.32	
4571.27	3?			
4571.849	3	2.07		
4575.26	3			
4578.55	3			
4580.228	9	4.50 3.70 2.70	0.91 0.00	*n* 1:2:5:5:2:1; *p* 1:2:1
4584.02	3?			
4584.25	3?			
4585.08	3			
4585.23	3			

TABLE I—*Continued*

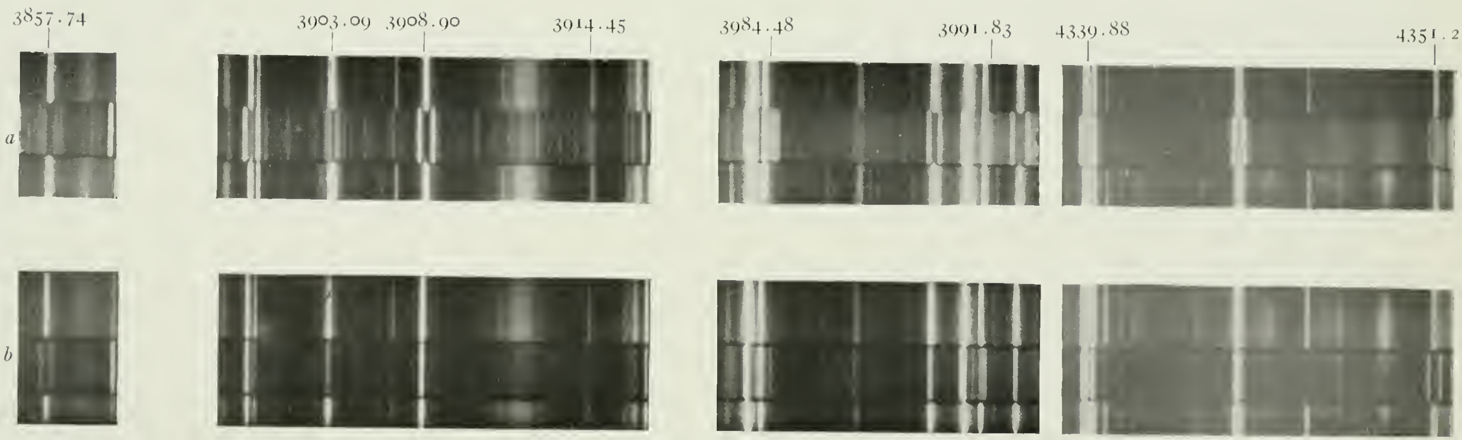
λ	Total Number Components	$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses n	p	Remarks
4586.31	3	2.33		
4588.381	3	1.96		E
4590.88	3			
4591.574	3	3.75		
4592.231	4	2.31	1.46	E
4594.57	3?			
4595.770	3	1.85		
4598.60	4?			
4600.279	4?	2.40	1.12	Perhaps higher type
4600.932	4	2.91	0.87	
4601.207	Complex			
4606.55	3			
4610.07	3			
4612.138	4			
4613.544	3	4.64		
4616.305	Complex	3.06	1.13	
4616.804	6?			E; not good for measurement
4618.971	3	1.59		E
4619.711	3	2.76		
4622.065	7?	{2.76, ...}	{2.00, 0.00}	Two wide diffuse n-components
4622.627	3	2.50		
4622.929	3	2.88		
4625.46	3			
4626.096	3?	2.31		
4626.358	6	{4.60, 2.74}	1.86	
4632.320	3			
4633.432	3			Very wide separation; too weak for measurement
4634.254	3	1.13		E
4637.352	3	1.86		n 2:1
4637.938	Complex			
4639.679	3?			
4639.85	3?			
4642.21	3?	2.09		
4646.347	3?	2.39		Components wide, better defined inside than outside
4648.297	3	1.31		
4649.613	3	2.18		
4651.461	9	{4.73, 2.80, 0.92}	{1.89, 0.00}	n 1:2:3:3:2:1; p 2:3:2
4652.343	Complex	2.14		
4654.912	3	5.51		
4656.61	3?			
4663.492	6	{5.40, 2.68}	3.01	
4663.999	Complex	1.64	1.94	
4664.965	4	2.98	1.28	
4666.076	3	1.09		

TABLE I—*Continued*

λ	Total Number Components	$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses *n*	*p*	Remarks
4666.387	3	0.70		
4666.655	4	3.14	0.91	
4667.339	3			
4669.504	4	2.80	0.86	
4681.037	3	1.20		
4689.540	Complex			
4694.125	3	1.95		
4695.331	3?			
4697.230	3	2.19		
4698.641	3	1.99		Components of this line and the following blended and measured together
4698.798	3?			
4699.76	3?			
4700.795	3			
4708.196	3	2.02		
4718.601	3	2.24		
4722.90	4			
4723.294	3			
4724.60	3			
4727.337	3	2.14		
4729.864	3	2.05		
4730.897	3	2.00		
4737.540	3	2.10		
4741.27	3			
4752.27	3	1.84		
4754.10	3			
4754.95	3			
4755.36	3			
4756.300	3	1.97		
4764.479	3	1.97		
4766.827	3			
4768.049	3			
4789.528	3	2.13		
4792.707	3	2.61		
4796.373	3			
4801.213	3	2.46		

the magnetic field. The first of these, λ 3914.45, is unaffected, but there are found in a space of 100 Ångström units or more to the violet of this line numerous lines which have nothing corresponding to them in the comparison spectrum. Their appearance is such as to suggest a fluting. λ 3984.479 shows instead of individual *n*-components a wide, sharp-edged band beginning a little to the violet of the normal and running out to the red. The band

PLATE X



SEPARATION OF CHROMIUM LINES BY MAGNETIC FIELD

a. *n*-Components *b*. *p*-Components

shows a maximum of intensity corresponding in width and position to the normal, and a slight minimum near the middle. In the center of this minimum there is a barely perceptible secondary maximum. In the *p*-component photograph this line shows three parts—a broad line corresponding in position to the normal and having a slight minimum of intensity at its center, and two sharp and narrow side-components, of which the one on the red side is about three times as intense as the other. This line is reproduced on Plate X, with the loss of some details in the enlargement. λ 3991.830 has as *n*-component a narrow band with violet edge sharp, fading off to the red. As *p*-components this line has a narrow band sharp on the red edge and diffuse on the violet, with a sharp line some distance to the violet. This line is also to be found on Plate X. λ 4081.88 is not separated in the field, but is shifted about 0.03 Ångström unit to the red. λ 4203.730 has two *n*-components close together, one being undisplaced from the normal position of the line and the other lying on the violet side, while the single *p*-component is also displaced to the violet 0.054 Ångström unit. λ 4211.512 is a quadruplet having three *n*-components and one *p*-component. Another of the same kind is λ 4127.426. The very intense lines $\lambda\lambda$ 4254.505, 4289.855 are complex types. The *n*-components are stronger at the inner edges and become weaker and more diffuse outside, while the *p*-components are very wide. Doubtless under higher resolving power the *n*-components would break up into many very narrow lines so arranged as to resemble flutings. The strong line at λ 4274.958 appears to be a normal triplet. λ 4337.725 is an interesting line showing nine components. λ 4339.882 is an unaffected line. The table contains six of this class. λ 4359.784 shows altogether twelve components, the largest number observed for one line. The eight *n*-components are of equal intensity, arranged in two symmetrical groups of four lines each. With slightly lower resolving power these groups would be seen as broad bands of uniform intensity similar to many actually observed. Of the four *p*-components the two stronger have the same separation as two of the *n*-components, while the inner pair are too faint to measure satisfactorily. They probably have the same separation as the

innermost n-components. λ 4373.415 is an octuplet, interesting on account of the large separation of its outermost n-components. Of the five n-components one is undisplaced, while the separations of the other two pairs are in the ratio 1 : 2. The separation of the outer p-components is that of the inner n-components. λ 4391.924 is a notable line, unfortunately weak on all of the plates. Its farthest separated n-components are the strongest and the intensities decrease rapidly as the normal is approached. This gives the effect of lines with fringes on the inner sides. Unresolved components showing fringes on the outside are much more common. λ 4501.264 deserves notice for having four n-components and only one p-component. The latter is shifted toward the violet. λ 4654.912 shows the largest separation of any triplet in the table, 5.51. The only line showing greater separation is the octuplet λ 4373.415, whose maximum is 5.61. λ 4539.946 is a peculiar line of complex type. Five n-components are shown, four of which can be measured. A sharp one to the violet of the normal has the greatest separation. The others all lie on the red side of the normal, the nearest being too weak to measure. The distances from the normal are in the ratios ?:1:2:3:4. Only one p-component is seen, but it is very wide and diffuse as though an unresolved multiple. This is one of the most remarkable cases of asymmetry found on the plates.

Of the enhanced lines found in the table, 19 are triple, 4 are quadruple, and one is probably sextuple. The mean value of $\Delta\lambda \cdot 10^8/\lambda^2$ for the triplets is 1.82 ± 0.060.

Excluding the enhanced lines, a list of 162 triplets has been selected which includes all on which good measurements could be obtained. This list does not contain those lines for which classification is doubtful. The smallest value of $\Delta\lambda \cdot 10^8/\lambda^2$ is 0.67, while the largest is 5.51. The mean of all is 2.17. Inspection of the values shows at once that they are not uniformly distributed over the whole range. If they are arranged in order of magnitude they are seen to divide themselves into two main groups, as shown in Table II. The first column contains a series of intervals in the values of $\Delta\lambda \cdot 10^8/\lambda^2$; while the second and third show the corresponding numbers of values which lie in those intervals.

The average of the 104 values which lie between 1.30 and 2.40 is 1.90±0.015. For the 39 values lying between 2.40 and 3.00 the mean is 2.70±0.014. From an inspection of the table and a comparison of the means it appears that the enhanced lines as a class are not different from the others in their separation and distribution.

TABLE II

DISTRIBUTION OF SEPARATIONS

Intervals in $\frac{\Delta\lambda}{\lambda^2} \cdot 10^8$	Number of Values	
	Not Enhanced	Enhanced
0.60–0.80	2	..
0.80–1.00	..	..
1.00–1.20	4	2
1.20–1.40	3	..
1.40–1.60	9	3
1.60–1.80	19	5
1.80–2.00	39	4
2.00–2.20	28	3
2.20–2.40	9	1
2.40–2.60	7	..
2.60–2.80	25	1
2.80–3.00	6	..
3.00–3.20	4	..
3.20–3.40	..	..
3.40–3.60	2	..
3.60–5.60	5	..

W. Hartmann[1] gives a comparison of his values of $\Delta\lambda \cdot 10^8/\lambda^2$ for six chromium lines with those of J. E. Purvis[2] and of W. Miller.[3] He has reduced all the determinations to a field strength of 23,850 gausses. The results of this comparison are given in Table III, the last column of which contains my values reduced to the same field. These show an average deviation of +3 per cent from Miller's and of +11 per cent from Hartmann's results.

Description of plates.—Plate X shows two photographs in the violet, each made with light taken at right angles to the field. In one the nicol prism was turned so as to transmit the n-components, while the other was taken after rotating the nicol through

[1] *Das Zeeman-Phaenomen im sichtbaren Spektrum von Kupfer, Eisen, Gold und Chrom*, Dissertation, Halle, 1907.

[2] *Proc. Camb. Phil. Soc.*, XIV, I, 41–84, 1906.

[3] *Annalen der Physik* (4), 24, 105–136, 1907.

90° so as to transmit the *p*-components. Comparison spectra are shown on each side. λ 3857.74 is a line showing six components. Next come the triplets $\lambda\lambda$ 3903.090 and 3908.900. λ 3914.45 is an unaffected line referred to above. Then follow the remarkable lines λ 3984.479 and λ 3991.830, to which attention has already been called. λ 4339.882 is a strong line apparently not affected by the field. λ 4351.216 is a good example of the quintuple class.

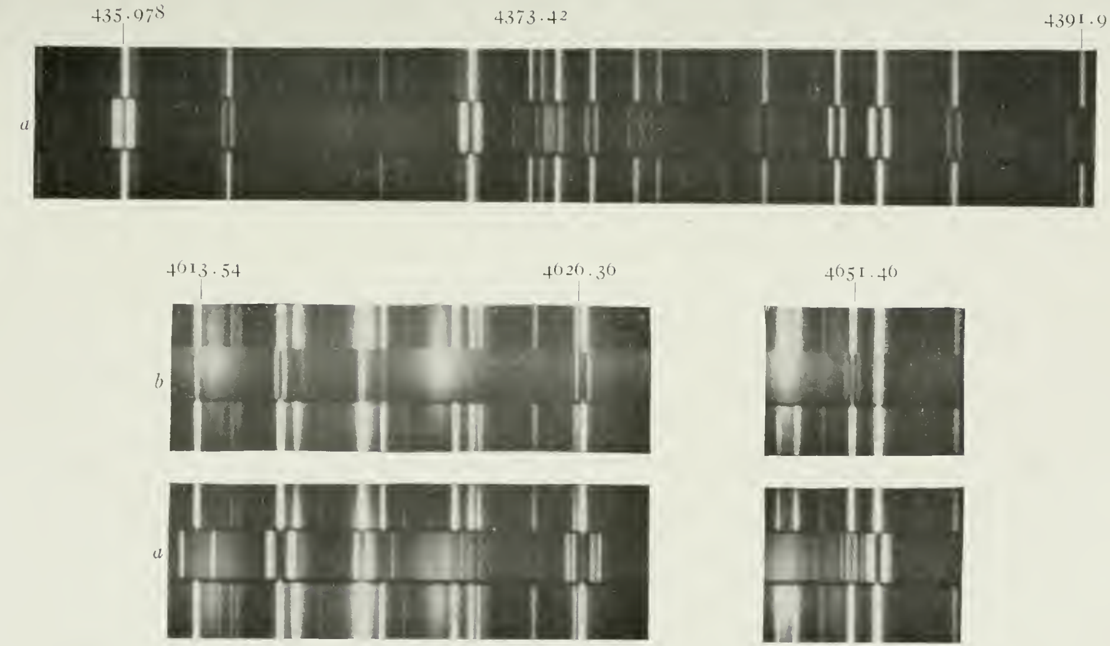
TABLE III

COMPARISON OF SEPARATIONS

	$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 23,850 Gausses				Remarks
	Purvis	Miller	Hartmann	Babcock	
4646.39	+0.93 0.00 −0.93	+1.31 0.00 −1.31	+1.21 0.00 −1.21	+1.42 0.00 −1.42	Components wide
4600.92		+1.70 0.00 −1.70	+1.58 +0.46 −0.46 −1.58	+1.73 +0.52 −0.52 −1.73	
4558.90	+0.92 0.00 −0.92		+1.19 0.00 −1.19	+1.29 0.00 −1.29	Enhanced—poor to measure
4526.65		+1.46 0.00 −1.46	+1.37 0.00 −1.37	+1.46 0.00 −1.46	
4351.97	+0.94 0.00 −0.94	+1.29 0.00 −1.29	+1.18 0.00 −1.18	+1.32 0.00 −1.32	
4344.7	+0.89 0.00 −0.89	+1.20 0.00 −1.20	+1.11 0.00 −1.11	+1.25 0.00 −1.25	

On Plate XI the upper photograph shows only the *n*-components for an interesting group of lines. The plates showing *p*-components for these lines are not well adapted for reproduction. λ 4359.784 shows eight closely packed components and has four more in which the vibrations are parallel to the field. λ 4373.415 shows five widely separated *n*-components, of which two are blended with those of a neighboring line. It has three *p*-components. λ 4391.924

PLATE XI



Separation of Chromium Lines by Magnetic Field

a. *n*-Components *b.* *p*-Components

shows two wide components fringed inside. On some of the original plates these fringes are partially resolved. The *p*-component is a wide diffuse line composed undoubtedly of several unresolved components. Passing to the lower part of the plate, λ4613.544 is a very wide triplet, λ4626.358 is a fine sextuple line, and λ4651.461 shows nine components, one of the outer *n*-components being blended with an adjoining line.

Table IV shows the ratios of the separations observed for lines having more than four components. When the total number of components is odd there is always one in the normal position of the line. These central components are indicated by ciphers in the table.

TABLE IV

RATIOS OF SEPARATION

Total Number of Components	5	6	7	8	9	12
	0:1:1	1:1:2	0:1:2:?	2:?:?:5	0:1:1:2:3	1:1:2:3:4
	0:1:2	2:?:3			0:1:3:4:5	
	0:1:3	3:4:?	0:1:2:?			
		?:1:2:3:4				
		2:3:5				
		1:1:2				

The writer is greatly indebted to Mr. King for many valuable suggestions and to Miss Griffin for assistance in measuring the plates and reducing the values.

Mount Wilson Solar Observatory
February 1911

PHOTOGRAPHIC DETERMINATIONS OF STELLAR PARALLAX MADE WITH THE YERKES REFRACTOR. IV

By FRANK SCHLESINGER

π^4 Orionis ($4^h\ 46^m$, $+5°26'$)

This is a spectroscopic binary of the helium type. Its binary character was discovered in 1903 by H. M. Reese, and independently by Frost and Adams. The orbit has recently been computed by R. H. Baker. The magnitude of the system is 3.8; accordingly the rotating disk was used to reduce its apparent brightness to the mean of the comparison stars. Eleven plates were secured as shown in Table 1.

TABLE 1

PLATES OF π^4 *Orionis*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
518 ...	1904 Oct. 30	$-0^h.1$	S, Su, S	Poor	First and third exposures only 1 mm apart
525 ...	Nov. 17	−0.2	S, Su, S	Fair	
563 ...	1905 Jan. 3	−0.2	Su, S	Fair	
584 ...	Jan. 22	−0.8	S, Su, S	Poor	
612 ...	Feb. 19	+0.5	S, Su, S	Good	First exposure only fair
792 ...	Sept. 26	−0.1	Su, Su	Fair	
827 ...	Oct. 10	−0.4	Su, J, Su	Poor	
834 ...	Oct. 15	−0.7	Su, Su, Su	Fair	
902 ...	1906 Jan. 28	−0.8	Su, J, Su	Good	
912 ...	Feb. 18	+0.4	Su, J, Su	Good	
920 ...	Feb. 25	+0.5	Su, J, Su	Fair	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence Computed	Dependence Adopted
1	0.52	−295	−192	+.163	+.17
2	0.91	+ 42	−312	+.256	+.25
3	0.80	− 65	+208	+.155	+.16
4	0.60	+123	+146	+.205	+.20
5	0.60	+195	+150	+.220	+.22
Parallax star.	0.76	+ 21	− 16		

Each of the first four plates was measured by Miss Ware and the writer, the others by Miss Ware alone.

TABLE 2

REDUCTIONS FOR π^4 *Orionis*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p} \cdot v$ in Arc
518........	0.073	0.5	+0.545	−216	−.006	−.″01
525........	0.080	0.8	+0.260	−198	0	.00
563........	0.082	0.6	−0.527	−151	− 1	.00
584........	0.078	0.5	−0.771	−132	− 7	−.01
612........	0.098	0.8	−0.974	−104	+ 12	+.03
792........	0.098	0.5	+0.923	+115	+ 20	+.04
827........	0.063	0.4	+0.802	+129	− 17	−.03
834........	0.080	0.7	+0.746	+134	0	.00
902........	0.084	0.9	−0.831	+239	− 3	−.01
912........	0.091	0.9	−0.969	+260	+ 4	+.01
920........	0.082	0.7	−0.987	+267	− 5	−.01

The normal equations are:

$$
\begin{aligned}
+4.647\pi - 3.459\mu - 2.007c &= -0.187 \\
+27.372 \quad +3.327 &= +0.299 \\
+7.300 &= +0.611
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.082 \\
\mu &= +0.0004 = +0.''001 \\
\pi &= -0.0044 = -0.''012 \pm 0.''007
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0054 = \pm 0.''014$.

No other determination of this parallax has been published.

ψ Orionis ($5^h\ 22^m$, $+3^\circ 1'$)

This is a spectroscopic binary of the helium type. Its binary character was discovered by Frost and Adams in 1903, and the orbit has recently been computed by Plaskett. Plates taken at the Allegheny Observatory show that both spectra are visible.

The magnitude of the system is 4.7, so that the rotating disk was used to reduce its brightness. Fourteen plates were secured as shown in Table 1.

TABLE 1

PLATES OF *ψ Orionis*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
526 . . .	1904 Nov. 17	$-0^h.2$	S, Su, S	Good	
555 . . .	Dec. 15	-1.0	S, Su, S	Good	
587 . . .	1905 Jan. 29	-1.2	S, Su, S	Good	
595 . . .	Feb. 14	0.0	S, S, S	Fair	
613 . . .	Feb. 19	$+0.7$	S, Su, S	Poor	Second exposure fair
802 . . .	Oct. 3	$+0.4$	Su	Poor	
829 . . .	Oct. 10	-0.3	Su	Fair	
836 . . .	Oct. 15	-0.1	Su	Poor	
845 . . .	Nov. 7	-0.7	Su	Good	
847 . . .	Nov. 11	-0.8	Su, J	Poor	
848 . . .	Nov. 11	-0.3	Su, J, Su	Poor	
904 . . .	1906 Jan. 28	-0.5	Su, J, Su	Good	
914 . . .	Feb. 18	$+0.6$	Su, J, Su	Good	*ψ Orionis* poor on third exposure
922 . . .	Feb. 25	$+1.4$	Su, J, Su	Good	

COMPARISON STARS

No.	DIAMETER	X (longitude)	Y (latitude)	DEPENDENCE	
				Computed	Adopted
1	0.90	-212	-51	$+.267$	$+.266$
2	1.40	-210	$+190$	$+.401$	$+.40$
4	1.08	$+422$	-139	$+.332$	$+.333$
Parallax star.	1.08	-0.4	$+16.4$		

Each of the first three plates was measured by Miss Ware and the writer, the others by Miss Ware alone.

TABLE 2

REDUCTIONS FOR ψ *Orionis*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
526........	0.104	1.0	+0.407	−298	−.006	−″02
555........	0.122	1.0	−0.069	−270	+ 9	+.02
587........	0.118	1.0	−0.751	−225	+ 1	.00
595........	0.109	0.7	−0.903	−209	− 9	−.02
613........	0.128	0.5	−0.935	−204	+ 9	+.02
802........	0.040	0.2	+0.935	+ 22	− 63	−.07
829........	0.136	0.3	+0.884	+ 29	+ 30	+.04
836........	0.080	0.2	+0.840	+ 34	− 26	−.03
845........	0.094	0.4	+0.563	+ 57	− 14	−.02
847........	0.123	0.3	+0.504	+ 61	+ 15	+.02
848........	0.149	0.3	+0.504	+ 61	+ 41	+.06
904........	0.135	0.9	−0.737	+139	+ 18	+.05
914........	0.089	0.7	−0.927	+160	− 30	−.07
922........	0.123	0.9	−0.964	+167	+ 4	+.01

The normal equations are:

$$\begin{aligned} +4.500\pi - 0.106\mu - 2.546c &= -0.318 \\ +32.824 \quad -5.746 &= -0.652 \\ +8.400 &= +0.963 \end{aligned}$$

These yield

$$\begin{aligned} c &= +0.112 \\ \mu &= -0.0002 = -0\overset{\prime\prime}{.}001 \\ \pi &= -0.0072 = -0\overset{\prime\prime}{.}019 \pm 0\overset{\prime\prime}{.}015 \end{aligned}$$

Probable error corresponding to unit weight, $\pm 0.0111 = \pm 0\overset{\prime\prime}{.}029$.

No other determination of this parallax has been published.

Weisse I, 5^h 592 ($5^h\ 26^m$, $-3^\circ\ 42'$)

This is a 9th-magnitude star with a proper motion of $2\overset{\prime\prime}{.}2$ per annum. Eighteen plates were secured as described in Table 1.

TABLE 1

Plates of *Weisse I*, 5^h 592

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
102 . . .	1903 Sept. 20	$-0^h.7$	S	Fair	
127 . . .	Oct. 11	+0.1	S, Su, S	Good	Second exposure poor
131 . . .	Oct. 18	+1.3	S	Poor	Telescope east
133 . . .	Oct. 20	+0.1	S, Su, S	Fair	
204 . . .	1904 Jan. 31	+0.2	S, Su, S	Fair	
211 . . .	Feb. 4	+0.7	S	Fair	
223 . . .	Feb. 14	−0.4	S, Su, S	Good	
234 . . .	Mar. 3	+0.4	S, Su, S	Good	
465 . . .	Sept. 11	−1.9	S	Fair	
478 . . .	Sept. 25	−1.0	S, Su, S	Good	
527 . . .	Nov. 17	+0.3	S, Su, S	Fair	
589 . . .	1905 Jan. 29	−0.2	S, Su, S	Good	
594 . . .	Feb. 14	−1.0	S, Su, S	Fair	
602 . . .	Feb. 17	−0.9	S, S, Su	Poor	
623 . . .	Feb. 28	+0.6	S, Su, S	Good	
803 . . .	Oct. 3	0.0	Su, J, Su	Poor	
856 . . .	Nov. 21	−0.4	Su, J, Su	Fair	
857 . . .	Nov. 21	+0.1	Su, J, Su	Good	

COMPARISON STARS.

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
2.	1.06	−373	− 43	+.240	+.24
7.	0.84	−188	− 65	+.201	+.20
11.	0.79	− 33	−131	+.166	+.17
17.	1.00	+112	− 5	+.144	+.15
21.	1.20	+251	− 71	+.112	+.10
22.	0.95	+231	+315	+.139	+.14
Parallax star.	1.70	− 57	− 10		

The plates of this star were all measured by Miss Ware.

TABLE 2

Reductions for *Weisse I*, 5^h *592*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p} \cdot v$ in Arc
102........	0.148	0.3	+0.997	−422	+.026	+″04
127........	0.128	0.7	+0.887	−401	0	.00
131........	0.122	0.2	+0.823	−394	− 6	−.01
133........	0.145	0.7	+0.804	−392	+ 17	+.04
204........	0.106	0.7	−0.757	−289	+ 20	+.04
211........	0.087	0.3	−0.800	−285	+ 1	.00
223........	0.092	0.9	−0.890	−275	+ 5	+.01
234........	0.063	0.9	−0.983	−257	− 29	−.07
465........	0.359	0.3	+1.007	− 65	− 2	−.00
478........	0.359	0.9	+0.980	− 51	− 11	−.03
527........	0.332	0.7	+0.419	+ 2	− 33	−.07
589........	0.310	0.9	−0.743	+ 75	− 21	−.05
594........	0.353	0.7	−0.897	+ 91	+ 22	+.05
602........	0.350	0.3	−0.917	+ 94	+ 18	+.03
623........	0.348	0.9	−0.973	+105	+ 13	+.03
803........	0.626	0.4	+0.940	+322	+ 9	+.02
856........	0.606	0.7	+0.360	+371	− 3	−.01
857........	0.619	0.9	+0.360	+371	+ 10	+.03

The normal equations are:

$$\begin{aligned} +7.621\pi + 0.581\mu - 0.837c &= +0.301 \\ +80.367 - 6.148 &= +3.372 \\ +11.400 &= +3.338 \end{aligned}$$

These yield

$$c = +0.334$$
$$\mu = +0.0670 = +0\overset{\prime\prime}{.}178$$
$$\pi = +0.0710 = +0\overset{\prime\prime}{.}189 \pm 0\overset{\prime\prime}{.}010$$

Probable error corresponding to unit weight, $\pm 0.0102 = \pm 0\overset{\prime\prime}{.}027$.

The only other determination of this parallax that has been published is by Flint, who obtained $+0\overset{\prime\prime}{.}06 \pm 0\overset{\prime\prime}{.}036$ by the transit circle method.

S Monocerotis (6^h 35^m, $+9°$ $59'$)

This is a star of magnitude 4.7 with a helium-type spectrum, being classified as O*e*5. As the above designation implies, the star was at one time thought to be variable in its light, but this must now be considered as very doubtful. In 1904 Frost and Adams showed that this object is a spectroscopic binary with a small range in velocity. There is a 9th-magnitude companion at a distance of $3''$ and there seems to be present a little orbital motion. The rotating sector was employed to reduce the light of *S Monocerotis;* no trace of the companion appears upon the plates, since its apparent magnitude is reduced by the disk to about the 14th.

TABLE 1

PLATES OF *S Monocerotis*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
502 . . .	1904 Oct. 16	$+0^h3$	S, Su, S	Fair	
528 . . .	Nov. 17	0.0	S, Su, S	Good	
628 . . .	1905 Mar. 12	+0.2	Su, S	Poor	
636 . . .	Mar. 26	+1.0	S	Poor	
837 . . .	Oct. 15	−0.3	Su, Su, Su	Poor	
846 . . .	Nov. 7	−1.4	Su, J, Su	Poor	
858 . . .	Nov. 21	−0.6	Su, J, Su	Fair	
925 . . .	1906 Apr. 1	+2.0	Su, J, Su	Fair	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
1	1.18	−168	+ 25	+.311	+.30
2	1.02	−110	+ 62	+.213	+.20
3	0.77	− 82	+ 88	+.156	+.166
6	0.84	+136	+122	−.074	−.066
10	0.93	+224	−297	+.396	+.40
Parallax star.	0.86	− 9	− 92		

The writer had measured the first two of these plates, but these measures were not used, as both plates have positive parallax factors. If there is a slight systematic difference between the two observers, the use of these measures would affect the deduced parallax. The following results therefore rest upon Miss Ware's measures alone.

TABLE 2

REDUCTIONS FOR *S Monocerotis*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
502	0.066	0.7	+0.964	−230	+.012	+″03
528	0.049	0.9	+0.679	−198	− 5	−.01
628	0.057	0.3	−0.950	− 88	− 2	.00
636	0.048	0.2	−0.996	− 69	− 10	−.01
837	0.020	0.4	+0.970	+134	− 31	−.05
846	0.074	0.4	+0.800	+157	+ 22	+.03
858	0.050	0.7	+0.630	+171	− 2	.00
925	0.063	0.7	−0.999	+302	+ 7	+.02

The normal equations are:

$$
\begin{aligned}
+3.142\pi - 2.709\mu + 1.252c &= +0.059 \\
+17.693 \quad +0.681 &= +0.036 \\
+4.300 &= +0.234
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.055 \\
\mu &= -0.0007 = -0\rlap{.}''002 \\
\pi &= -0.0028 = -0\rlap{.}''007 \pm 0\rlap{.}''012
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.082 = \pm 0\rlap{.}''022$.

No other determination of this parallax has been published.

Lalande 13427 (6^h 54^m, $+48°$ $32'$)

This 8th-magnitude star has a proper motion of 0".7 per annum. Ten plates were secured as shown in Table 1.

TABLE 1

PLATES OF *Lalande 13427*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
130 . . .	1903 Oct. 18	$-0^h.8$	S, S	Fair	
220 . . .	1904 Feb. 9	−0.4	S, Su, S	Good	
224 . . .	Feb. 14	−1.3	S, Su, S	Good	
479 . . .	Sept. 25	−1.6	S, Su, S	Fair	
543 . . .	Nov. 20	−0.6	S, Su, S	Good	Second exposure only fair
596 . . .	1905 Feb. 14	−0.5	S, Su, S	Fair	
624 . . .	Feb. 25	−0.4	S, Su, S	Good	
859 . . .	Nov. 21	−0.5	Su, J, Su	Fair	
861 . . .	Dec. 5	−0.8	Su, J, Su	Good	
872 . . .	Dec. 12	−0.2	Su, J, Su	Good	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
3a	1.44	− 84	−262	+.229	+0.25÷1.05
5	0.73	− 84	+127	+.186	+0.20÷1.05
8	0.66	+168	+135	+.583	+0.60÷1.05
Parallax star . .	1.60	+ 62.7	+ 42.0		

There is another good comparison star at $X=150$, $Y=+131$, but the dependence comes out small, +0.033, and it was accordingly not used. Each of the first seven plates was measured by the writer as well as by Miss Ware, the remaining three by Miss Ware alone.

TABLE 2

REDUCTIONS FOR *Lalande 13427*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p} \cdot v$ in Arc
130	0.018	0.6	+0.962	−394	+.009	+″02
220	0.047	1.0	−0.635	−280	− 9	−.02
224	0.048	1.0	−0.700	−275	− 11	−.03
479	0.237	0.8	+0.995	− 51	− 6	−.01
543	0.288	0.9	+0.652	+ 5	+ 13	+.03
596	0.332	0.8	−0.710	+ 91	+ 23	+.05
624	0.318	1.0	−0.857	+105	+ 2	+.01
859	0.523	0.7	+0.642	+371	− 3	−.01
861	0.516	0.9	+0.438	+385	− 15	−.04
872	0.534	0.9	+0.326	+392	0	.00

The normal equations are:

$$
\begin{aligned}
+4.317\pi + 3.968\mu + 0.336c &= +0.438 \\
+63.425 \quad +3.091 \quad &= +5.215 \\
+8.600 \quad &= +2.450
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.260 \\
\mu &= +0.0684 = +0''.182 \\
\pi &= +0.0184 = +0''.049 \pm 0''.011
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0082 = \pm 0''.022$.

By means of heliometer measurements Chase has derived $+0''.01 \pm 0''.05$ for this parallax.

Lalande 15290 ($7^h\ 47^m$, $+30^\circ\ 55'$)

This 8th-magnitude star has a proper motion of 2″ per annum. Fourteen plates were secured as described in Table 1.

TABLE I

PLATES OF *Lalande 15290*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
134 ...	1903 Oct. 20	$-1^{h}.5$	S, Su, S	Fair	Star (28) lacking on first and second exposures
146 ...	Oct. 25	-1.1	S, Su, S	Poor	
155*A*..	Nov. 1	-0.3	S, Su, S	Poor	Parallax star triangular
160 ...	Nov. 8	-1.5	S, Su, S	Poor	Star (28) lacking on first exposure
176 ...	Nov. 26	-0.5	S, S	Fair	
236 ...	1904 Mar. 3	-0.8	S, Su, S	Fair	
260 ...	Mar. 27	0.0	S, Su, S	Fair	
529 ...	Nov. 17	-0.8	S, Su, S	Fair	
545 ...	Nov. 20	-0.3	S, Su, S	Fair	
597 ...	1905 Feb. 14	-0.6	S, Su, S	Fair	First exposure poor
603 ...	Feb. 17	0.0	S, Su, S	Poor	
626 ...	Feb. 28	-0.5	Su, S	Fair	
863 ...	Dec. 5	-0.5	Su, J, Su	Poor	
874 ...	Dec. 12	-0.1	Su, J, Su	Fair	

COMPARISON STARS

No.	Diameter	*X* (longitude)	*Y* (latitude)	Dependence	
				Computed	Adopted
8..........	0.57	+290	+156	+.152	+.15
10..........	0.84	+119	−269	+.221	+.22
12..........	0.49	+ 35	− 77	+.206	+.20
16..........	0.70	+ 33	+205	+.172	+.18
28..........	0.51	−477	− 15	+.249	+.25
Parallax star..	1.53	− 35.4	− 20.3		

All of these plates were measured by Miss Ware, and all but the last two by the writer as well.

TABLE 2

REDUCTIONS FOR *Lalande 15290*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p} \cdot v$ in Arc
134	0.100	0.6	+0.992	−292	+.068	+″14
146	0.023	0.5	+0.982	−287	− 13	−.02
155*A*	0.034	0.5	+0.955	−280	− 10	−.02
160	0.047	0.4	+0.914	−273	− 5	−.01
176	0.060	0.6	+0.746	−255	− 7	−.01
236	0.126	0.8	−0.759	−157	− 8	−.02
260	0.139	0.8	−0.958	−133	− 16	−.04
529	0.412	0.8	+0.835	+102	− 50	−.12
545	0.468	0.8	+0.805	+105	+ 3	+.01
597	0.526	0.7	−0.534	+191	+ 3	+.01
603	0.560	0.5	−0.576	+194	+ 36	+.07
626	0.543	0.6	−0.722	+205	+ 9	+.02
863	0.869	0.4	+0.627	+485	− 8	−.01
874	0.901	0.7	+0.529	+492	+ 19	+.04

The normal equations are:

$$\begin{aligned} +5.500\pi - 2.420\mu + 1.843c &= +0.484 \\ +58.448 \quad +1.048 &= +6.729 \\ +8.700 &= +3.013 \end{aligned}$$

These yield

$$c = +0.327$$
$$\mu = +0.1104 = +0\overset{''}{.}294$$
$$\pi = +0.0269 = +0\overset{''}{.}072 \pm 0\overset{''}{.}019$$

Probable error corresponding to unit weight, $\pm 0.0159 = \pm 0\overset{''}{.}042$.

Other determinations of this parallax are:

Peter (heliometer)	+0″.02	±0.014
Flint (transit circle)	−0.01	.030
Chase (heliometer)	+0.08	.046

Fedorenko 1457–8 ($9^h\ 8^m$, $+53°\ 7'$)

This is a well-known double star, otherwise designated as Σ 1321. The two components are nearly equal in brightness and are of the 8th magnitude. The system has a proper motion of 1″.7 per

annum; but this is not quite the same for both, doubtless because of orbital motion. The separation is now a little over 19″, whereas, according to Struve, it was over 20″ about seventy years ago. The position angle is increasing at the rate of 1° in four years.

This pair is so situated that the parallax may be derived from the displacements in declination with some weight. The plates were therefore measured first in right ascension and then in declination. There are thus four determinations of the parallax—two for each of the components. Twelve plates were secured as described in Table 1.

TABLE 1

PLATES OF *Fedorenko 1457–8*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
161 ...	1903 Nov. 8	-2^h1	S, Su	Fair	
184 ...	Dec. 6	−1.0	S, S, S	Good	
239 ...	1904 Mar. 3	−0.4	S, Su, S	Good	Second exposure only fair
262 ...	Mar. 27	−0.2	S, Su, S		Images triangular
570 ...	1905 Jan. 7	−0.7	S, Su, S	Fair	
599 ...	Feb. 14	−1.1	S, Su, S	Fair	
604 ...	Feb. 17	−0.8	S, Su, S	Fair	
630 ...	Mar. 12	−0.4	S, Su, S	Fair	
639 ...	Mar. 26	−0.4	S, Su, S	Fair	
865 ...	Dec. 5	−0.4	Su, J, Su	Poor	
870 ...	Dec. 10	−0.3	Su, J, Su	Fair	
876 ...	Dec. 12	−0.1	Su, J, Su	Poor	

COMPARISON STARS

No.	Diameter	X (right ascension)	Y (declination)	Dependence	
				Computed	Adopted
1..........	0.72	−446	− 60	+.058	+.06
2..........	0.50	−424	+199	+.252	+.25
14..........	0.52	+ 3	+200	+.314	+.31
24..........	0.54	+251	−170	+.078	+.08
26..........	1.04	+299	−109	+.130	+.13
27..........	0.86	+317	− 60	+.168	+.17
Parallax stars	1.60	− 23.1	+ 70.9		
	1.57	− 16.5	+ 73.8		

These dependences were computed with the means of the coordinates of the two parallax stars, and they were used in the reductions for both. As we have already stated, the same dependences apply to both the right ascension displacements and those in declination.

Each of the first four plates was measured by the writer as well as by Miss Ware, the others by Miss Ware alone.

TABLE 2

REDUCTIONS IN RIGHT ASCENSION FOR THE PRECEDING STAR OF *Fedorenko 1457–8*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p} \cdot v$ in Arc
161........	1.330	0.6	+0.947	−373	+.022	+″05
184........	1.254	1.0	+0.820	−345	− 3	−.01
239........	1.002	0.9	−0.456	−257	− 38	−.10
262........	0.944	0.3	−0.759	−233	− 39	−.06
570........	0.646	0.7	+0.428	+ 53	+ 24	+.05
599........	0.567	0.7	−0.187	+ 91	+ 44	+.10
604........	0.521	0.7	−0.236	+ 94	+ 5	+.01
630........	0.482	0.7	−0.579	+117	+ 23	+.05
639........	0.425	0.7	−0.746	+131	− 1	.00
865........	0.118	0.4	+0.822	+385	− 19	−.03
870........	0.085	0.7	+0.779	+390	− 42	−.09
876........	0.130	0.4	+0.778	+392	+ 6	+.01

The normal equations are:

$$
\begin{aligned}
+3.323\pi - 0.022\mu + 1.012c &= +0.900 \\
+54.101 \quad +0.540 &= -7.934 \\
+7.800 &= +5.245
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.675 \\
\mu &= -0.1534 = -0.''408 \\
\pi &= +0.0645 = +0.''172 \pm 0.''025
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0167 = \pm 0.''044$.

TABLE 3

REDUCTIONS IN RIGHT ASCENSION FOR THE FOLLOWING STAR OF *Fedorenko 1457–8*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
161........	1.262	0.6	+0.947	−373	+.027	+″06
184........	1.185	1.0	+0.820	−345	− 1	.00
239........	0.937	0.9	−0.456	−257	− 44	−.11
262........	0.888	0.3	−0.759	−233	− 40	−.06
570........	0.586	0.7	+0.428	+ 53	+ 23	+.05
599........	0.506	0.7	−0.187	+ 91	+ 35	+.08
604........	0.464	0.7	−0.236	+ 94	0	.00
630........	0.461	0.7	−0.579	+117	+ 52	+.12
639........	0.372	0.7	−0.746	+131	− 7	−.02
865........	0.080	0.4	+0.822	+385	− 4	−.01
870........	0.034	0.7	+0.779	+390	− 40	−.09
876........	0.054	0.4	+0.778	+392	− 17	−.03

The normal equations are:

$$+3.323\pi - 0.022\mu + 1.012c = +0.816$$
$$+54.101 \quad +0.540 = -7.836$$
$$+7.800 = +4.801$$

These yield

$$c = +0.619$$
$$\mu = -0.1510 = -0''.402$$
$$\pi = +0.0562 = +0''.149 \pm 0''.028$$

Probable error corresponding to unit weight, $\pm 0.0187 = \pm 0''.050$.

TABLE 4

REDUCTIONS IN DECLINATION FOR THE PRECEDING STAR OF *Fedorenko 1457–8*

Plate	Solution (l)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
161........	0.′518	0.6	−0.217	−373	+.011	+″02
184........	0.499	0.9	+0.091	−345	− 9	−.02
239........	0.497	0.8	+0.629	−257	+ 10	+.02
262........	0.414	0.3	+0.547	−233	− 55	−.08
570........	0.319	0.7	+0.418	+ 53	+ 21	+.05
599........	0.270	0.7	+0.624	+ 91	− 17	−.04
604........	0.291	0.7	+0.628	+ 94	+ 6	+.01
630........	0.292	0.7	+0.611	+117	+ 17	+.04
639........	0.251	0.7	+0.554	+131	− 9	−.02
865........	0.089	0.4	+0.086	+385	0	.00
870........	0.074	0.7	+0.141	+390	− 15	−.03
876........	0.093	0.4	+0.162	+392	+ 4	+.01

The normal equations are:

$$\begin{aligned} +1.617\pi+\ 1.275\mu+2.803c&=+0.869\\ +52.249\ +1.142\ &=-2.582\\ +7.600\ &=+2.403 \end{aligned}$$

These yield

$$c=+0.305$$
$$\mu=-0.0574=-0.''153$$
$$\pi=+0.0544=+0.''145\pm0.''036$$

Probable error corresponding to unit weight, $\pm 0.0103=\pm 0.''027$.

TABLE 5

REDUCTIONS IN DECLINATION FOR THE FOLLOWING STAR OF *Fedorenko 1457–8*

Plate	Solution (l)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
161........	0.579	0.6	−0.217	−373	+.013	+.″03
184........	0.553	0.9	+0.091	−345	− 8	−.02
239........	0.528	0.8	+0.629	−257	+ 2	.00
262........	0.462	0.3	+0.547	−233	− 45	−.07
570........	0.337	0.7	+0.418	+ 53	+ 22	+.05
599........	0.295	0.7	+0.624	+ 91	− 4	−.01
604........	0.288	0.7	+0.628	+ 94	− 9	−.02
630........	0.304	0.7	+0.611	+117	+ 22	+.05
639........	0.265	0.7	+0.554	+131	− 5	−.01
865........	0.077	0.4	+0.086	+385	− 8	−.01
870........	0.083	0.7	+0.141	+390	− 1	.00
876........	0.062	0.4	+0.162	+392	− 21	−.04

The normal equations are:

$$\begin{aligned} +1.617\pi+\ 1.275\mu+2.803c&=+0.915\\ +52.249\ +1.142\ &=-2.982\\ +7.600\ &=+2.564 \end{aligned}$$

These yield

$$c=+0.332$$
$$\mu=-0.0652=-0.''173$$
$$\pi=+0.0421=+0.''112\pm0.''032$$

Probable error corresponding to unit weight, $\pm 0.0093=\pm 0.''025$.

Collecting these results for the parallax, we have

From Right Ascensions of the Preceding Star.....	+0.″172	±0.″025
From Right Ascensions of the Following Star.....	+ .149	.028
From Declinations of the Preceding Star.........	+ .145	.036
From Declinations of the Following Star.........	+ .112	.032

Adopting the mean by weight as the definitive parallax we obtain

$$+0\overset{''}{.}148 \quad \pm 0\overset{''}{.}015$$

If we were to compute the probable error from the accordance among the four values we should get $+0\overset{''}{.}148 \pm 0\overset{''}{.}007$; or if we assume that the four determinations are of equal weight and compute the probable error from the four differences from the simple mean, we obtain $+0\overset{''}{.}144 \pm 0\overset{''}{.}008$. Otherwise expressed, the four separate values agree better than we should expect from a consideration of their probable errors.

Other determinations of the parallax of *Fedorenko 1457–8* are as follows:

Kapteyn (transit circle)	$+0\overset{''}{.}074$	$\pm 0\overset{''}{.}027$
Flint (transit circle)	$+\ .10$	$.032$
Peter (heliometer)	$+\ .18$	$.007$

A preliminary parallax from four plates was published by the writer in the *Astrophysical Journal*, **20**, 129, 1904; this is entirely superseded by the present work, where the same plates are definitively discussed in connection with eight later ones.

If we compare the residuals in right ascension in Table 2 with those in Table 3, we see that they are almost always of the same sign for the same plate. A similar agreement exists between each pair of residuals in declination (Tables 4 and 5). There is therefore present some source of error that apparently shifts the two parallax stars in the same direction. Without making any assumption as to the cause of this systematic tendency, let us represent its probable value by ϵ (in a sense analogous to probable error); and let ϵ_1 be the probable error of bisection. Then by taking the weighted sum of the squares of the residuals in Tables 2 and 3, we have for the right ascensions,

$$\epsilon^2+\epsilon_1^2=(0.6745)^2\frac{[p.\ v^2]}{12-3}=\begin{cases}0.000283 \text{ (Table 2)}\\ 0.000357 \text{ (Table 3)}\end{cases}$$

If now we substract each residual in Table 3 from the corresponding one in Table 2, these differences are, by definition,

free from the systematic tendency. Denoting these differences by Δv we therefore have

$$2\epsilon_1^2 = (0.6745)^2 \frac{[p. (\Delta v)^2]}{12-3} = 0.000052$$

Consequently

$$\epsilon_1 = \pm 0.0051 = \pm 0.''014$$
$$\epsilon = \pm 0.0160 = \pm 0.''043 \text{ (from Table 2)}$$
$$\epsilon = \pm 0.0182 = \pm 0.''048 \text{ (from Table 3)}$$

These results apply only to the right ascensions; if we treat the declinations in the same way we get,

$$\epsilon_1 = \pm 0.0045 = \pm 0.''012$$
$$\epsilon = \pm 0.0097 = \pm 0.''026 \text{ (from Table 4)}$$
$$\epsilon = \pm 0.0079 = \pm 0.''021 \text{ (from Table 5)}$$

Thus we see that the error of bisection comes out practically the same for the two co-ordinates; on the other hand, we notice (and this is the significant point) that the systematic part of the error is twice as great in right ascension as in declination. This circumstance excludes as an explanation any purely accidental cause, distortions of the film, for example, since there is no reason why an error of this kind should not affect the two co-ordinates to about the same extent.

We are thus led to seek a disturbing effect that is greater in right ascension than in declination. Two such (and only two) present themselves: *guiding error* and *hour-angle error*. The first of these, at least, would seem to be a plausible explanation; for, in the first place, the parallax stars are about one and a half magnitudes brighter than the mean of the comparison stars, and we should therefore expect the guiding error to be large. The diameters of the parallax stars exceed 4″, so that the observed effect is only about 1 per cent of these diameters. Again, we should expect the guiding error to be less in declination than in right ascension, since in the latter direction the observer must contend with irregularities in the driving, in addition to fluctuations due to atmospheric causes.

It remains to inquire whether the hour-angle error is not also competent to account for this systematic tendency. Earlier in

these papers I said that atmospheric dispersion (which may be assumed to form a large part of the hour-angle error) is less to be feared with this instrument than with a photographic refractor or with a reflector, since only a narrow region of the spectrum is concerned in making the image, no matter what the color of the star may be. Furthermore, dispersion by our atmosphere is *prismatic* and is therefore far from being normal. The yellow portion of the spectrum is crowded together more than the blue, and for a given difference in effective wave-length the shift due to dispersion would consequently be smaller in our case than in others.[1] Finally an inspection of the hour angles in Table 1 shows that they are nearly the same for all the plates, only the first (which has rather low weight) differing from the mean of the others by more than half an hour; and the mean hour angle for the morning plates differs from that for the evening plates by only one quarter of an hour. It is at once obvious that atmospheric dispersion can play at most only a small part in explaining the systematic tendency that we are discussing. It will nevertheless be of general interest to examine the residuals from this point of view.

The spectra of both parallax stars are classified as Ma by the Harvard observers. We have no determinations of the corresponding datum in the case of the comparison stars, but it is safe to assume that they are on the whole much whiter than the parallax stars. As a result the latter are photographed relatively farther from the zenith than if they were of the same color as the comparison stars. Consequently the farther east the telescope is pointed during the exposure, the more should the parallax stars be apparently shifted toward the west. An examination of Tables 1, 2, and 3 shows that if anything the opposite is the case, and we are forced to look elsewhere for an explanation.

Bergstrand has recently shown, in connection with his work on *61 Cygni*,[2] that even when differences of spectra are not involved atmospheric dispersion may still cause apparent shifts. Only the

[1] This may also be at least part of the reason why visual observations suffer less from atmospheric dispersion than ordinary photographic measures.

[2] *Astronomische Nachrichten*, **167**, 241, 1905. Nijland appears also to have called attention to the same effect in 1897, but in a work that was printed in Dutch. See *Astronomische Nachrichten*, **168**, 333, 1905.

brightest portion of the spectrum formed by the atmosphere produces the photographic image of a faint star, while for brighter stars the outlying portions are also effective. The curve of color-sensitiveness of the plates here used (Cramer Instantaneous Isochromatic) is steeper on the side toward longer wave-lengths than upon the other, and the images of bright stars will consequently appear nearer the zenith than they otherwise would.[1] In the case of *Fedorenko 1457–8*, the parallax stars are one and a half magnitudes brighter than the mean of the comparison stars, and the effect is therefore contrary in sign to that which arises from the difference in color. Following the usual notation, let θ be the zenith distance, and q the parallactic angle. The effect of atmospheric dispersion may then be represented by

$$b \cdot \tan \theta \cdot \sin q, \text{ in right ascension,}$$
$$b \cdot \tan \theta \cdot \cos q, \text{ in declination,}$$

where b represents the difference between the effects due respectively to color and to magnitude.

The coefficients of b in right ascension[2] were computed for the twelve plates and new sets of normal equations were formed in which b was introduced as a fourth unknown. As might have been anticipated, the weight of b comes out very small (only one-eightieth of that of π in the original solution), on account of the small range in hour angles at which the plates were exposed.

These plates are therefore not suited to determine the numerical value of the effect of atmospheric dispersion. It appears, however, from the observations of other astronomers, that that part of the dispersion which depends upon the brightness is of the order of $0''.10$ for a difference of one magnitude. If we adopt this value in the present case we shall probably be exaggerating its effect; for although the difference in brightness between our comparison stars and the parallax stars is greater than one magnitude, this will be more than compensated for by the effect due to difference in color, and because of the greater crowding of the yellow

[1] This is also true in the case of a photographic refractor or a reflector, and ordinary plates.

[2] It would be futile to discuss the declinations from this point of view, since in this co-ordinate the factors of b are practically the same for all the plates.

rays as compared with the blue, the value of 0."10 having been derived from observations with ordinary photographic telescopes. Let us then adopt this value tentatively and see what it will amount to for the various plates. As atmospheric dispersion would be of no consequence if it were the same for all the plates, I have subtracted its weighted mean from the individual values and have put the results into the accompanying table.[1] The residuals from Tables 2 and 3 are also given, reduced to seconds of arc.

TABLE 6

EFFECT OF ATMOSPHERIC DISPERSION IN RIGHT ASCENSION

Plate	0."10 tan $\theta \cdot \sin q$ *minus* 0."014	Residuals	
		From Table 2	From Table 3
161	+0."027	+0."059	+0."072
184	+ .007	− .008	− .003
239	− .007	− .101	− .117
262	− .010	− .104	− .106
570	.000	+ .064	+ .061
599	+ .008	+ .117	+ .093
604	+ .002	+ .013	.000
630	− .007	+ .061	+ .138
639	− .006	− .003	− .019
865	− .005	− .051	− .011
870	− .007	− .112	− .106
876	− .011	+ .016	− .045

An examination of this table reveals little evidence of the presence of atmospheric dispersion. It is true that the numbers in the second column have, more often than not, the same signs as those in the third and the fourth; but this is only because we have assumed that the part of atmospheric dispersion depending upon the difference in magnitude numerically exceeds the part that depends upon the difference in spectra. At all events, the numbers in the first column are small and inadequate to account for the strong tendency toward residuals of the same sign for the two parallax stars. The only plausible explanation that remains is guiding error, a conclusion that is borne out by the results for other stars in this list.

[1] A similar computation for the declinations yields 0."006 for Plate 161, and 0."002 or under for each of the other plates.

If we retain b as a fourth unknown in the least-squares solutions in right ascension, and differentiate the reduced normal obtained by eliminating c and μ, we get

$$\Delta\pi = 0.068b.$$

That is, if b were 0.″10, the parallaxes derived above from the right ascensions would be in error by 0.″007. Those from the declinations are practically independent of b, so that the error arising from this source in the definitive mean cannot be more than a few thousandths of a second. The fair agreement between the results from right ascensions and from declinations is additional evidence against the presence of sensible atmospheric dispersion.

Lalande 23917 ($12^{h}\ 45^{m}$, $+1^{\circ}\ 45'$)

This is an 8th-magnitude star with a proper motion of about 0.″7 per annum. The thirteen plates secured were measured in both right ascension and declination. The parallax was derived from the displacements in both directions, but the weight from the declinations comes out very small.

TABLE 1

PLATES OF *Lalande 23917*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
308 . . .	1904 May 1	$+0^{h}.4$	S, Su, S	Good	Star (20) too faint on first and second exposures
318 . . .	May 15	+0.4	S, Su, S	Fair	Third exposure good
332 . . .	May 20	+1.1	S, Su, S	Fair	
340 . . .	May 26	+0.9	S, Su, S	Fair	
569 . . .	1905 Jan. 3	+0.1	S, Su, S	Fair	
590 . . .	Jan. 29	−0.3	S, Su, S	Fair	
606 . . .	Feb. 17	−0.9	S, Su, S	Poor	
615 . . .	Feb. 25	−1.1	S, Su, S	Fair	
650 . . .	Apr. 16	0.0	F, Su, F	Poor	
652 . . .	Apr. 22	+0.7	F, Su, F	Poor	
681 . . .	May 22	+0.8	F, Ware, F	Fair	Second exposure poor
884 . . .	Dec. 26	−0.1	Su, J, Su	Fair	
897 . . .	1906 Jan. 9	−1.2	Su, J, Su	Poor	

COMPARISON STARS

No.	Diameter	X (right ascension)	Y (declination)	Dependence	
				Computed	Adopted
4	0.68	−294	−248	+.334	+.33
12	1.28	− 17	+220	−.303	−.30
14	0.69	+ 15	−103	+.521	+.52
20	0.47	+296	+131	+.447	+.45
Parallax star	1.19	+ 47.8	−144		

It will be seen that the distribution of available comparison stars in this field is poor. The sum of the squares of the dependences is 0.67. Had the mean position of the comparison stars been close to that of the parallax star, this sum would have been only 0.25; that is, the reciprocal of the number of comparison stars. The ratio of these two sums indicates the greater effect that errors in the measurement of the comparison stars have upon the accuracy of m. (See Equation 9.)

The plates were all measured by Miss Ware alone.

TABLE 2

Reductions in Right Ascension for *Lalande 23917*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
308	0.102	0.6	−0.453	−289	+.006	+″01
318	0.116	0.8	−0.631	−284	+ 24	+.06
332	0.086	0.5	−0.688	−279	− 5	−.01
340	0.086	0.7	−0.748	−273	− 3	−.01
569	0.054	0.7	+0.905	− 51	− 35	−.08
590	0.102	0.7	+0.803	− 25	+ 18	+.04
606	0.068	0.4	+0.622	− 6	− 12	−.02
615	0.076	0.7	+0.524	+ 2	− 3	−.01
650	0.048	0.4	−0.230	+ 52	− 16	−.03
652	0.046	0.4	−0.321	+ 58	− 16	−.03
681	0.041	0.6	−0.706	+ 88	− 13	−.03
884	0.083	0.7	+0.898	+306	+ 29	+.06
897	0.051	0.4	+0.897	+320	− 1	.00

The normal equations are:

$$
\begin{aligned}
+3.691\pi + 6.744\mu + 0.511c &= +0.017\\
+32.474 \quad -3.518 &= -0.488\\
+7.600 &= +0.587
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.072\\
\mu &= -0.0099 = -0\overset{''}{.}026\\
\pi &= +0.0130 = +\mathbf{0\overset{''}{.}035 \pm 0\overset{''}{.}020}
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0107 = \pm 0\overset{''}{.}028$.

TABLE 3

REDUCTIONS IN DECLINATION FOR *Lalande 23917*

Plate	Solution (l)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
308	0.350	0.6	+0.291	−298	−.013	− ″03
318	0.360	0.8	+0.351	−284	+ 5	+ .01
332	0.370	0.5	+0.367	−279	+ 18	+ .03
340	0.366	0.7	+0.384	−273	+ 18	+ .04
569	0.209	0.7	−0.378	− 51	− 18	− .04
500	0.208	0.7	−0.286	− 25	− 3	− .01
606	0.201	0.4	−0.180	− 6	+ 3	+ .01
615	0.186	0.7	−0.129	+ 2	− 7	− .02
650	0.175	0.4	+0.208	+ 52	+ 14	+ .02
652	0.160	0.4	+0.242	+ 58	+ 3	+ .01
681	0.097	0.6	+0.373	+ 88	− 42	− .09
884	0.020	0.7	−0.392	+306	+ 1	.00
897	0.054	0.4	−0.365	+320	+ 44	+0.07

The normal equations are:

$$
\begin{aligned}
+0.787\pi - 3.389\mu + 0.266c &= +0.240\\
+32.474 \quad -3.518 &= -2.534\\
+7.600 &= +1.669
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.193\\
\mu &= -0.0582 = -0\overset{''}{.}155\\
\pi &= -0.0093 = -\mathbf{0\overset{''}{.}025 \pm 0\overset{''}{.}044}
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0110 = \pm 0\overset{''}{.}029$.

The two determinations differ by as much as $0\overset{''}{.}06$, but this is a little less than the sum of their probable errors. Combining them

in accordance with these errors, we have, as the definitive parallax of *Lalande 23917*,

$$+0''.025 \quad \pm 0''.018$$

With the Yale heliometer, Chase has recently obtained $+0''.01 \pm 0''.05$ for this parallax.

Berlin A. 4999 ($13^h\ 40^m$, $+18°\ 24'$)

This is a faint star, below the 9th magnitude, with a proper motion of 1''.9 per annum. Eleven plates were secured as described in Table 1.

TABLE 1

PLATES OF *Berlin A. 4999*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
319 ...	1904 May 15	$+0^h.1$	S, Su, S	Good	
333 ...	May 20	+0.9	S, Su, S	Poor	
580 ...	1905 Jan. 15	−1.1	S, Su, S	Good	Second exposure only fair
608 ...	Feb. 17	−0.4	S, Su, S	Fair	
617 ...	Feb. 25	−0.6	S, Su, S	Good	
642 ...	Apr. 8	+0.5	F, Su		First exposure poor, second good
651 ...	Apr. 16	−0.2	F, Su, F	Poor	
682 ...	May 22	+0.5	F, Su, F	Fair	
909 ...	1906 Feb. 6	−0.4	Su, J, Su	Poor	
915 ...	Feb. 18	−0.1	Su, J, Su	Good	
918 ...	Feb. 20	−0.1	Su, J, Su	Fair	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
1..........	0.57	+214	−244	+.264	+.27
4..........	0.90	+125	+ 80	+.221	+.22
16..........	0.44	−339	+164	+.514	+.51
Parallax star..	0.77	− 89.3	+ 37.3		

Plates 319 and 580 were measured by both Miss Ware and the writer, the others by Miss Ware alone.

TABLE 2

REDUCTIONS FOR *Berlin A. 4999*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
319........	0.048	1.0	−0.636	−284	+.012	+″03
333........	0.003	0.4	−0.701	−279	− 38	−.06
580........	0.380	0.9	+0.969	− 39	+ 8	+.02
608........	0.390	0.7	+0.720	− 6	− 14	−.03
617........	0.413	0.9	+0.620	+ 2	+ 3	+.01
642........	0.442	0.5	−0.050	+ 44	+ 1	.00
651........	0.456	0.4	−0.186	+ 52	+ 10	+.02
682........	0.473	0.7	−0.722	+ 88	0	.00
909........	0.815	0.4	+0.838	+348	− 10	−.02
915........	0.836	0.9	+0.712	+360	0	.00
918........	0.839	0.7	+0.688	+362	+ 2	.00

The normal equations are:

$$
\begin{aligned}
+3.603\pi + 6.954\mu + 1.872c &= +1.655 \\
+37.745 + 3.879 &= +6.168 \\
+7.500 &= +3.435
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.390 \\
\mu &= +0.1180 = +0\overset{''}{.}314 \\
\pi &= +0.0291 = +0\overset{''}{.}077 \pm 0\overset{''}{.}014
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.075 = \pm 0\overset{''}{.}020$.

Russell has derived $+0\overset{''}{.}105 \pm 0\overset{''}{.}020$ for this parallax from plates taken with the coudé telescope at Cambridge, England.

ALLEGHENY OBSERVATORY
March 1911

[*To be continued*]

Minor Contributions and Notes

CORRESPONDENCE CONCERNING THE CLASSIFICATION OF STELLAR SPECTRA[1]

At the meeting of the International Solar Union held at Mount Wilson in September 1910, part of the last session was devoted to the question whether the Union should not extend its activities to include general astrophysics, instead of limiting itself to solar physics. An interesting general discussion took place and resulted in a unanimous decision so to extend the scope of the Union. This important action was immediately followed by the appointment of a Committee on the Classification of Stellar Spectra.

At the present time there are in extensive use three distinct systems for classifying stellar spectra: (1) the Draper Classification, developed by the Harvard observers, in their systematic surveys of the heavens; (2) Miss Maury's Classification, also having its origin at this observatory in the course of similar work; and (3) Vogel's Second Classification, devised at Potsdam and extensively used in spectrographic observations at that observatory and many others. In addition, the classic types of Secchi, upon which Vogel's are based, are still in common use; and Lockyer's designations, as well as those of McClean, have been used in connection with important researches.

Astronomers and physicists have long recognized the desirability of doing away with the confusion that the use of so many distinct systems has entailed. The general feeling on this subject at the beginning of this century was well summarized in the paper[2] read by Professor Frost before the Section of Astrophysics of the International Congress of Arts and Sciences, held at St. Louis in September 1904. At that time, however, there was no international organization whose scope would properly have included questions of this character.

With the object of bringing out the opinions of astrophysicists on

[1] Edited by FRANK SCHLESINGER, Secretary of the Committee on the Classification of Stellar Spectra of the International Union for Co-operation in Solar Research.

[2] "A Desideratum in Spectrology," *Astrophysical Journal*, **20**, 342, 1904. This paper also contains a brief bibliography of the subject of stellar classification according to their spectra.

this subject, the following self-explanatory letter was widely distributed shortly after the meeting at Mount Wilson:

Nov. 7, 1910

Dear Sir: At the Fourth Conference of the International Union for Co-operation in Solar Research, the following gentlemen were appointed to serve as a "Committee on the Classification of Stellar Spectra": Messrs. Adams, Campbell, Frost, Hale, Hamy, Hartmann, Kapteyn, Newall, Pickering (*chairman*), Plaskett, Russell, Schlesinger (*secretary*), and Schwarzschild.

This committee met at Mount Wilson on September 2, immediately after the adjournment of the Conference itself. In accordance with power to add to their number, it was unanimously decided to ask Mr. Küstner to serve, and he was present at this meeting. Messrs. Hale and Campbell, who had already left the mountain, were the only absentees.

The chairman called upon each member in turn to express his views concerning the classification of stellar spectra and his opinion as to what the scope of the committee should be. A brief summary of this discussion follows:

Mr. Adams preferred the Draper Classification,[1] and thought that if the members of the committee themselves would use this classification exclusively, until say the next meeting of the Solar Union, it would go far toward establishing uniformity. Mr. Küstner preferred the Draper Classification and was using it exclusively. Mr. Hartmann thought that the Draper Classification was the best that had been proposed, but hoped that an effort would be made to retain the Roman numerals of Secchi, that have now become classic, and that the subdivisions be made as in the Draper system by the addition of Arabic numerals; thus, II, 3. Mr. Schwarzschild suggested that instead of the letters B, A, F, G, K, M, the numerals 0, 1, 2, 3, 4, and 5 might respectively be employed; thus A2 would become 1.2, etc. In this system the number denoting the type would be proportionate to the *color-index*. Mr. Russell suggested the advisability of substituting some method for *measuring* the type of spectrum for the *estimations* that are now employed, and asked whether this could not be applied to the Draper Classification. Mr. Plaskett preferred the Draper Classification, but said that as he believed uniformity to be the prime consideration he would gladly adopt whatever system could be agreed upon. Mr. Frost thought that the committee should make no recommendation at the present time but should first canvass the whole subject thoroughly; it appeared to him desirable to investigate the visual end of the spectrum in connection with the photographic before arriving at a definite conclusion. Mr. Schlesinger preferred the Draper Classification and had decided to use it

[1] This classification is described in the *Annals of Harvard College Observatory*, **56**, 66. The letters O, B, A, F, G, K, M, and N are used to designate the sequence of the spectra. Numerals from 1 to 9 after the letter denote intermediate spectra; thus, B3 would be assigned to a spectrum between B and A, but more nearly resembling the former.

exclusively; he called attention to the desirability of making further distinction among the numerous spectra that are now classified as A without any modifying number; he thought that any attempt to establish a temporary uniformity now might prove an obstacle to the universal adoption of some more definite system later. Mr. Newall asked whether a spectrum might be intermediate between two letters in the Draper Classification that are not consecutive, as A5G. Mr. Pickering said in reply that such cases have not arisen in practice. Mr. Newall raised the question whether the committee should not consider the matter of stellar evolution. The members present seemed to be of the opinion that this was legitimately within the scope of the committee but that its immediate business should be the establishment of a uniform system for classification. Messrs. Russell, Hartmann, Kapteyn, and Schlesinger urged that no evolutionary basis for a classification be adopted at the present time; astrophysicists are not agreed as to the proper sequence from this point of view; if our ideas upon this matter should be modified in the near future (as seems very possible), then it would be necessary to modify or to abandon altogether any system of classification based upon these ideas. For similar reasons Mr. Russell asked that the use of such terms as "early" and "late," now so frequently used in describing spectra, be discontinued in favor of "white" and "red."

The secretary was directed to secure by correspondence as full an expression of opinion as possible, from the members of the committee and others, on the matters that had been discussed. The meeting then adjourned.

In accordance with this request, the following questions have been framed,[1] and you are asked to reply to them at length. In addition it is hoped that you will give your views in full upon any other points that may occur to you as being important in this connection.

(1) It will be noticed that, at the meeting reported above, there seemed to be a practically unanimous opinion that the Draper Classification is the most useful that has thus far been proposed. Do you concur in this opinion? If not, what system do you prefer?

(2) In any case, what objections to the Draper Classification have come to your notice and what modifications do you suggest?

(3) Do you think it would be wise for this committee to recommend at this time or in the near future any system of classification for universal adoption? If not, what additional observations or other work do you deem necessary before such recommendation should be made? Would you be willing to take part in this work?

[1] The general form that these questions should take was discussed at several informal meetings on the train coming east from the meeting at Pasadena. There were present at these meetings the chairman and the secretary of the committee, Mr. Russell, and (by invitation) Father Cortie.

(4) Do you think it desirable to include in the classification some symbol that would indicate the width of the lines, as was done by Miss Maury in *Annals of the Harvard College Observatory*, Vol. 28?

(5) What other criteria for classification would you suggest?

Although it is not the intention of the committee to frame a formal report at once, it is desirable that your answers to some of these questions should be forthcoming very soon; this is particularly the case with the third question. May I therefore request that your reply be sent, if possible, so as to reach me not later than the end of this calendar year? If you can secure an expression of opinion from any other qualified astronomer, it will be very welcome.

Very respectfully yours,

FRANK SCHLESINGER

Secretary of the Committee

Replies to these questions were received from nearly all to whom they were addressed. With the approval of the chairman of this committee, and with that of the chairman of the Executive Committee of the Solar Union, these replies are reproduced herewith:

WALTER S. ADAMS

(1) I prefer the Draper Classification to any other which has been proposed as yet.

(2) It seems to me that in the Draper system of classification some rather valuable criteria are neglected. For example, in considering the gradual increase of intensity of the "metallic lines" in passing from type B through A to types F and G the metallic or solar lines are treated too much as though they were all affected alike. This, of course, is far from being the case. Lines enhanced in the spark, for instance, are among the first which appear in the A type and attain very great strength in such stars as α *Cygni*. They fall off steadily in intensity, however, in stars of the F, G, and K types, while, at the same time, many other groups of solar lines increase in intensity. I shall refer to this further in connection with question (5). I agree with Mr. Schlesinger that further distinction is needed among the spectra now classified as type A, and I am somewhat in doubt whether the range A to F is sufficient to include satisfactorily the vast number of stars which have this general type of spectra.

It appears to me doubtful whether it is advisable to increase the number of classes in the Draper Classification by "class R" referred to in Circular No. 145 of the Harvard College Observatory. Those of these stars observed at Mount Wilson, while showing marked differences from

the typical stars of classes N and O, particularly as regards the strength of the ultra-violet spectrum, could, I think, be assigned without serious difficulty to subclasses under these two letters.

(3) It does not seem to me desirable to recommend at present any system of classification for universal adoption. If observational material accumulates rapidly in the course of the next three years, the committee might possibly be ready to make some definitive proposals at the time of the next conference of the Solar Union. At present there is not sufficient material to warrant such a recommendation. The principal deficiency is the lack of spectra of stars taken with high dispersion and especially the dearth of material obtained in the less refrangible portions of the spectrum. It is unnecessary to say that the Mount Wilson Observatory will be most glad and most desirous to aid in every way possible in the collection of additional data.

(4) It seems to me most desirable to adopt in any definitive classification some symbol indicating the width of the spectral lines. At the same time, however, to add such a symbol to the Draper Classification at present would merely introduce confusion. Accordingly, I should favor the employment of the symbol only in connection with the material which is being collected with a view to use in a future classification.

(5) I have already referred to one point in this connection in question 2. Although astronomers and physicists are by no means agreed as to the interpretation of intensity changes among various lines as bearing on questions of temperature, I think there is little doubt that some special significance must be attached to the behavior of certain classes of lines, such as the enhanced lines, for example, in stellar spectra. Whether abnormal strength on the part of the enhanced lines indicates high temperature, or some particular condition in the radiating or absorbing gases analogous to that in the electric spark, we do not as yet know. We may hope, however, that this question will be definitively settled by physicists before very long, and, in the meantime, if we can accumulate data bearing on the behavior of this class of lines, we shall be in a position, as soon as the physical evidence is available, to make a considerable step forward, perhaps on the side of stellar evolution, but certainly from the point of view of classification.

The same reasoning holds true of numerous other classes of lines; for example, the lines of titanium, vanadium, scandium, and calcium, which are peculiarly strengthened in the spectrum of sun-spots and show a very similar behavior in stars of class K and M. Closely allied to these lines are the band lines of titanium oxide, magnesium hydride, and

calcium hydride, which are also present in the spectrum of sun-spots, and of which titanium oxide at least forms a prominent feature of stars of class M.

It would seem to me that in the estimation of the relative intensities of lines or groups of lines some reasonably constant standards should be employed. The hydrogen lines vary enormously in intensity between class A and class M, and the same is true of the line λ 4227 of calcium. I would suggest that possibly the strong arc lines of iron, which occur between λ 4045 and λ 4415, might be used for this purpose. These lines appear with considerable intensity in α *Canis Majoris* and change relatively less, I believe, in intensity than any other lines as we pass toward groups K and M.

A rough determination of the blueness or redness of a star may be most valuable in the construction of a system of classification. Could not such a determination of sufficient accuracy be obtained, when photographic plates of the same kind are employed, by measuring with the Hartmann microphotometer, or some similar instrument, the intensity of the continuous spectrum at a few selected points? Such measures could be made very rapidly, and, while of course they would not serve as a basis of temperature determinations for the stars, they would be most important in anything like an empirical classification. I need only refer in this connection again to certain stars of Harvard class R. These appear to have a spectrum in all respects identical with the stars of class N, except that they are relatively much more intense in the blue and violet regions of the spectrum. Actual measures of intensity would be very valuable in the classification of these objects.

Mount Wilson
December 7, 1910

Sebastian Albrecht

When the circular letter of November 7 in regard to the work of the Committee on the Classification of the Stellar Spectra reached me I was working on a paper entitled "A Quantitative Method for Determining the Stellar Spectral Types of the Brighter Stars."[1] The paper is based on the variation of the wave-lengths of some of the spectrum lines of stars, which variation is progressive with the spectral type in the Draper Classification. It is the inverse of the problem for which I published a preliminary paper in the *Astrophysical Journal* (24, 333, 1906) and in the Lick Bulletins.

[1] *Astrophysical Journal*, 33, 130, 1911.

I was therefore especially interested in the circular letter; I think the idea of a committee for this problem a good one. The Draper Classification is, in my opinion, the best one available at present, and I agree with Mr. Adams' suggestion that if this classification were generally used by astronomers it would tend toward establishing uniformity. The curves accompanying my paper show that the Draper Classification represents fairly approximately the actual consecutive steps between types. In my opinion, the greatest need for improvement in the classification is for the O and B types, though here my method does not promise to yield as satisfactory results as in the types A to Mb. My present paper is for types F to Mb, though I am extending it to the A types. I have also done some work along this line for the B types of spectra. For the present, at least, this method has the limitation of not being applicable to faint stars. For the bright stars I believe it will be possible to obtain a considerable accuracy.

It is my plan to make a thorough study of this problem, and accordingly a request is being sent to several observatories for the loan of a suitable series of spectrograms.

In reply to question (3) I should think it best to wait before any system of classification be recommended for universal adoption.

Córdoba, Argentina
December 26, 1910

W. W. Campbell

(1) The Harvard College Observatory classification of stellar spectra, which makes use of the distinguishing letters O, B, A, F, G, K, M, and N, is, in my opinion, the most useful system that has thus far been proposed. We have been using it almost exclusively for many years, both on Mount Hamilton and in the work of the D. O. Mills Expedition to the southern hemisphere.

(2) Any slight objections on my part to the Harvard classification are due to the errors which exist in assignments of types to individual stars, and do not pertain to the system itself. The numerous errors in assignments contained in earlier Harvard volumes appear to have been eliminated from the data as published in *Annals H.C.O.*, Vol. 50. The few existing errors are probably due in most cases to poor atmospheric conditions when the objective spectrograms were obtained, and in other cases to over-exposure or under-exposure of the individual spectra.

(3) I think we should not recommend at this time, and perhaps at no future time, any system of classification for universal adoption; but we should aim for a convenient system in harmony with all existing knowledge of spectra. We can scarcely hope to obtain a system so perfect at the epoch of publication that the years immediately following would not suggest modifications, at least in detail. It would seem that the question of "universal adoption" would almost take care of itself, or at least depend upon the success of our efforts.

I should like "to take part in this work," but just what I shall be able to do, either personally or through the assignment of duties to members of the Lick Observatory staff, can hardly be made clear until the ideas of the committee have taken more definite form. Stellar spectrograms in the possession of the Lick Observatory may be considered as available for the purposes of the committee in so far as they seem to apply to the problem.

(4) I think it would be desirable to arrange that the system of classification could be utilized to indicate the widths of the lines in individual stellar spectra.

(5) Before improvements on the existing Harvard classification are attempted, the committee should, in my opinion, arrange to obtain spectrograms of representative stars in the green, yellow, orange, and red. It is now easy to render commercial dry plates sensitive to exposures in these regions. It has been found from researches on the sun, on stars containing bright lines, and on stars containing both bright and dark lines, that the $H\alpha$ hydrogen line is frequently differentiated in characteristics from the hydrogen lines in the blue and violet regions. Again, the sodium D_1 and D_2 lines and the helium D_3 line can hardly fail to show significant variations from one spectral type to another. There is no doubt that the H and K calcium lines are important guides to classification. It is not impossible that $H\alpha$, D_1, D_2, and D_3, and many other lines, one or all, may be equally important.

The Lick Observatory has relatively very few photographs covering the visual regions of stellar spectra, and this is probably true of other observatories. I should advocate that the first work of the committee consist in making arrangements to secure a large number of representative stellar spectrograms covering the green, yellow, orange, and red regions.

Lick Observatory
January 31, 1911

Annie J. Cannon

(1) The Draper Classification, as described in Harvard *Annals*, 28, Part II, and abridged in Vol. 56, No. 4, seems to me the most useful thus far proposed. I have studied in detail the photographic spectra of about 4300 stars, situated from the North to the South Pole, and I find that the sequence in the order Oe, B, A, F, G, K, M, with intermediate subdivisions, is satisfactory for classifying all the observed spectra except peculiar objects.

(2) As suggested by Dr. Hertzsprung, the principal divisions heretofore designated by B, A, etc., should be called Bo, Ao, etc. This has been done in a new catalogue of the spectra of about 1660 southern stars I have recently completed, which is now nearly all in print.

Instead of Ma, Mb, and Mc, numerals should have been continued for the M stars. Ma should be called Mo; Mb, M5; and Mc might be designated by another letter, as there is as much difference between spectra of classes Ma and Mc as between Ko and Ma.

The subdivisions described in Harvard *Annals*, 28 and 56, represent, as nearly as I could make them at the time of writing, the progressive changes in the spectra. But in the light of today's knowledge, some of the intervals do not seem to be exactly true. For instance, A5 should, I think, be called A8, as it is nearer to Fo than to Ao. A3 should be called A5. However, I have always tried to conform to the adopted intervals, so that the same letters should mean as nearly as possible the same in the different catalogues containing spectra of my classification.

(3) Not at the present time. The Harvard classification is based primarily upon the portion of the spectrum from λ 3889 to λ 4922. Other portions should be studied. I have examined the portion from $H\beta$ to D_3 for six stars of Class B and found very little that helped in the classification except the absence or presence of $H\beta'$, λ 5413. For stars of Classes G and K, it seems to me this portion should be studied to determine what relation the varying intensities of the numerous metallic lines have to the progressive changes in the spectra.

Spectra taken with the slit spectroscope should be studied with reference to their adaptation to the Harvard classification. I have made a preliminary study of the spectra of 20 stars, photographed with the slit spectroscope at the Yerkes Observatory, and of 49 at the Lick Observatory. After a little study of these spectrograms, I tried the experiment of classifying, according to the Draper notation, each of them having the dispersion of one prism. The name of the star was

concealed, and no comparison was made with objective-prism plates. It was found that the class assigned to these spectra was the same as that already adopted here, or differed by only one subdivision. Some changes appeared in the relative intensities of a few lines, which must be carefully studied. I think it is very important that all these differences should be investigated, so that we may be sure we are dealing with realities, before any definite, universal system is adopted.

(4) Yes. We must be very careful, however, to distinguish between real and apparent width. This can perhaps be done to greatest advantage with slit spectroscope photographs. The spectra classed as divisions *c* and *ac* by Miss Maury are rare and peculiar objects; for of 4800 stars whose spectra have been studied here in detail, only about 90 have been placed in those divisions. It appears from a comparison of some of the spectra of division *c* and other spectra, taken with the slit spectroscope, that the difference is more in the peculiar intensity of numerous strongly marked lines than in the real width of the lines.

(5) Peculiar spectra should be investigated. I find groups of stars in classes A0 to G0 in which lines due to certain elements are strong, as, for instance, the silicon lines λ 4128.5 and λ 4131.4, or the strontium lines λ 4077.9 and λ 4215.7.

The position in the sequence of stars from B0 to B8, having also bright hydrogen lines, should be studied.

Harvard College Observatory
February 6, 1911

A. L. Cortie

(1) There is no doubt, I think, that the Draper Classification is most useful, but at the same time I think that Secchi's should be retained as expressing the broad classes of the stellar spectra, i.e., stars with few lines, stars with many lines, and stars with banded spectra. It has also the advantage that one can visualize at once mentally a star of type I, II, III or IV.

(2) Not so in the Draper system of classification, especially when the numbers are appended to the letters. One does not form easily a mental picture of the difference, say, between a B3 and a B4 star. In Sir Norman Lockyer's system of classification generic names are given to classes of spectra founded upon some typical star in the class. It seems possible that a system could be devised that would retain the advantages of Secchi's, the Draper, and Lockyer's nomenclatures. In the discussion at Mount Wilson, Professor Hartmann suggested that

both Secchi's numerals and Pickering's letters should be employed; thus, IIG5. If now a typical star of this particular variety were named after this combination a visual picture of the spectrum would force itself on the observer's mind. This system, too, would be truly philosophical. We should have class II, Genus G, Species 5, and the name in brackets of an individual star.

(3) I think it would be well to recommend some scheme for universal adoption in the near future, which would only require secondary modifications with further progress of observation. Secchi's numerals will always remain, and from the general consensus of opinion at Mount Wilson, so too will the Draper letters. But the numbers following the letters may have to be altered with the progress of observation. It is well to devise a system in which the skeleton will be permanent. I should personally be willing to take part with Father Sidgreaves in any observations required for elucidating any system of classification proposed by the committee.

(4) I do not think it would be advisable to include in the classification any symbol to indicate the width of the lines; certainly not yet. It would make a system of classification too complicated.

(5) No other criteria but those already mentioned.

Stonyhurst, England
November 27, 1910

Heber D. Curtis

I am firmly of the opinion that no benefit could be derived by introducing any new system of classification of stellar spectra; we have too many systems at present. Any attempt to classify spectra on an evolutionary basis would, in my opinion, be most unwise at present, and would lead to further confusion in the future. I consider the Draper Classification to possess ample elasticity to allow for any expansion which may become necessary in the future. I would therefore answer the questions propounded as follows:

(1) I consider the Draper Classification the most useful that has thus far been proposed.

(2) As far as its present use by astrophysicists is concerned, I would suggest no modifications. It is quite possible that the need may arise, with the progress of spectrographic surveys with higher dispersions, to increase the number of subdivisions in the various classes of the Draper System. Such new subclasses, if needed, should be added with considerable conservatism, and their adoption should be in the hands of some central body, such as the present committee.

(3) I believe the Draper System possesses sufficient elasticity to allow for any probable future development, and thoroughly approve its immediate adoption as a standard system.

(4) I do not consider advisable the present adoption of any added symbol to indicate the width of the lines. The number of binaries showing the spectra of both components is increasing rapidly, and for such stars "wide" and "narrow" lines will recur at regular epochs. Moreover, lines classified as "narrow" in low-dispersion surveys might frequently be "wide" and unusable with higher dispersion.

LICK OBSERVATORY
November 17, 1910

R. H. CURTISS

(1) I concur in the opinion of the committee.

(2) One objection to the Draper system, which I am not inclined to urge, is that the notation is arbitrary. The original order has been disturbed and now the classification does not rest upon any suggestive scheme. Again I am not sure that the spectra now known as peculiar or composite or both will fall in with the Draper system, though they will probably yield to classification.

(3) I believe that a uniform system even if temporary should be recommended by the committee for universal adoption. Work could then be prosecuted with the object of proving such a system. Especial attention should be given to missing links, to abrupt changes between successive divisions, and to spectra vaguely known as peculiar.

(4) I think a symbol indicating the character of the lines should be used.

(5) If the wave-lengths of the lines of a spectrum are variable I think that fact should be denoted by a symbol attached to the classification letter.

ANN ARBOR
January 3, 1911

H. LUDENDORFF AND G. EBERHARD

Auf die in dem Brief vom 7ten November enthaltenen Fragen geben wir gemeinschaftlich die folgenden Antworten:

(1) Wir sind der Ansicht, dass die Draper-Klassifikation die zweckmässigste unter den bisher aufgestellten Klassifikationen ist.

(2) Gegen sie haben wir im Wesentlichen nur den Einwand, dass die Reihenfolge der Buchstaben O, B, A, F, ... wenig schön ist. Trotzdem würden wir dafür sein, diese Buchstaben beizubehalten, da

die Einführung neuer Buchstaben und Zahlen nur die Verwirrung vergrössern würde.

(3) Wir würden es für zweckmässig halten, wenn die Draper-Klassifikation *vorläufig* zur allgemeinen Anwendung empfohlen würde. Im übrigen schliessen wir uns aber der Meinung von Professor Frost an, dass namentlich erst noch das visuelle Ende der Spektra näher untersucht werden muss, bevor man entscheiden kann, ob die Draper-Klassifikation als definitiv angesehen werden kann.

(4) Es wäre sehr wünschenswert, wenn der Charakter der Linien durch irgendwelche Symbole gekennzeichnet würde. Es wird aber nicht leicht sein, zu einer befriedigenden Lösung dieser Frage zu gelangen, da in denselben Spektren Linien verschiedenen Charakters auftreten können, und da bei Anwendung verschiedener Dispersionen die Linien desselben Spektrums sehr verschieden beurteilt werden können. Ein Beispiel für die letztere Tatsache bietet der Stern η *Aquilae*, den Miss Maury zu der "Division *ac*" (Spektra mit ziemlich scharfen Linien) rechnet, während Wright von den Spektrallinien dieses Sternes sagt: "They have the general characteristics of breadth and haziness, which tend to make them objectionable for purposes of accurate measurement."

Um die allgemeine Einführung und richtige Anwendung der Draper-Klassifikation zu erleichtern, wäre es ratsam, wenn das Komité eine Liste von Normalsternen für die einzelnen Klassen veröffentlichte, und damit zugleich eine Tabelle, welche die Umwandlung der Bezeichnungen von Miss Maury, Vogel, u.s.w., in die der Draper-Klassifikation ermöglicht.

POTSDAM
Dezember 14, 1910

WILLIAMINA P. FLEMING

Having discussed the questions of classification and notation fully with Professor E. C. Pickering before his answers were sent, and having acquiesced in his statements therein, there seems very little left for me to add in detail, except what has come to me through direct, and partly recent, observations.

(1) The Draper Classification seems that best adapted for use in the study of stellar spectra, at the present time.

(2) For spectra of classes M to R, the present nomenclature seems altogether inadequate to provide for the numerous classes into which these may be subdivided, especially those of classes M, N, and R.

(3) Not at this time, nor in the near future, so far as I can judge from my own experience in classifying spectra of faint stars, obtained with

photographic doublets and using small prisms. The objective-prism plates received here recently from Mr. J. A. Parkhurst, for classification of spectra, show the faint stars with such wonderfully good definition that, from them, many differences may be obtained with certainty, which might be seen only imperfectly on our photographs here. Such, also, was the case with a photograph from which I classified the spectra of a number of stars for Professor Schwarzschild, although on his plate the definition was not good in the region between $H\beta$ and $H\gamma$.

(4) Any peculiarity in the appearance of the lines, for which no provision has been made in the Draper Classification now in use, should be indicated in some way, in order to avoid the necessity for writing notes or remarks on them.

(5) My work at the present time on "Peculiar Spectra," especially those of classes M, N, and R, seems to indicate that, in assigning any new notation, if such should be done before this work is completed in detail, scope for interpolation between spectra so designated should be allowed. Class M has already been divided into four subclasses, Ma, Mb, Mc, and Md, and it has been found necessary to further subdivide the last of these, class Md, into eleven, Md to Md 10, with one (Md 1.5) inserted even after this work was supposed to be covered. Further, the material is now available here for connecting classes N and R, and even class R might be subdivided if better photographs with larger dispersion were available.

Harvard College Observatory
January 3, 1911

Edwin B. Frost

The opportunity seems particularly favorable for an examination of the whole question, *ab initio*, as if no system of classification had ever been proposed; for the organization of the Solar Union is representative of so many and widespread scientific societies that if a formal "international classification" should be proposed by the committee and then be thoroughly discussed and amended, and finally adopted by the Union at some meeting, six, nine, or more years hence, it would seem that its universal adoption ought to be assured.

In urging this thorough sort of consideration of the matter, I am not depreciating the insight and labor of those who have proposed the earlier or the later classifications: on the contrary, the acumen shown in their discriminations seems to me to be remarkable, and the later additions to and modifications of the earlier work have represented actual and

important differences in spectra. But it seems to me clear that none has adequate elasticity in providing subdivisions, or groups, for future studies made with improved apparatus and higher dispersion, and covering a greater extent of spectrum.

An examination and discussion of the possible bases of a classification would appear to be a natural preliminary to the investigations of the committee. Among these would be considered the physical differences in stars as indicated by the continuous spectrum—the wave-length of its maximum of intensity, and its extension to the ultra-violet, etc.—the condition of the star's atmosphere, as indicated by the character of its lines, whether of emission or absorption, broad or narrow, diffusely or sharply bounded. A knowledge of how their character corresponds to different kinds of electrical excitation, or how their wave-lengths vary in different spectral types, may lead to a diagnosis of the conditions of pressure in the star. Information as to the level in which the lines originate will be very useful here.

At the start the question arises: How far should a classification be made a chemical one? Obviously to a considerable extent, as this affords a very natural basis of discrimination. The presence of the lines of helium in a stellar spectrum, for instance, is most distinctive. But we shall not construe the lack of the lines of a substance necessarily to imply the absence of that element. Much valuable experimentation could be made in physical and chemical laboratories on the mutual compatibility of the different elements and compounds. Very little work has yet been undertaken in this direction.

Doubtless all agree in understanding by a chemical discrimination that the particular star is in that phase of development where the conditions favor the spectral predominance of certain elements, which may have been obscure in previous phases, and may vanish in later phases. May not this ultimately be reduced largely to a matter of temperature? It is certainly significant that the recent visual determinations of stellar temperatures by the spectrophotometer methods, by Wilsing and Scheiner, should so closely follow the commonly accepted sequence of spectral types; whether or not the absolute temperatures are correct is not material here, so long as the relative effective temperatures are reliable.

It is desirable that both branches of the temperature-curve should be recognized, in so far as their validity can be established, in any new classification, particularly for the red stars. One cannot help feeling that Sir Norman Lockyer must be right, to some considerable extent,

in his contentions on this point, even if the evidence he has thus far presented may not be entirely convincing.

The suggestions on this topic by Professor H. N. Russell in his paper at the meeting of the Astronomical Society at Harvard last summer may require careful consideration in this connection.[1] If it can be proven that of two red stars having very similar spectra one is on the rising branch and the other on the descending branch of a temperature-curve, then the classification ought to show it. If other collateral facts enable us to discriminate *where the spectra themselves do not*, it ought to be indicated; and it should not be difficult to devise symbols for the purpose. It would be safe to expect, if the conditions were real, that in the future differences in the spectra would be recognized which we do not at present realize.

The symbols adopted ought to give all the information which could be conveyed without becoming too complicated: if possible the predominant chemical elements; the nature of the lines, distinguishing between sharp and diffuse lines,[2] and indicating the presence of bright lines, if any.

Serious consideration ought to be given to whether any analogies could be adopted from the classification of natural history, as subkingdom, class, order, family, genus. For many astrometrical studies only the broadest distinctions are needed, as of the four types of Secchi. These might, for instance, correspond to the "class" in the sense used in natural history. It certainly would be advantageous to consider families and genera of spectra, as a minute study will in my opinion reveal numerous cases where the sequence of spectra (not thereby implying any theory of stellar evolution) has a choice of collateral branches to follow before the next group is reached. To adapt the terms, in so far as they were not letters or numerals, to universal use, it might be advantageous to employ the Latin.

In reply to the specific inquiries of the secretary of the committee, I will say:

(1) Under present conditions the Draper Classification seems to be the best one for provisional use until some action shall be taken by the Solar Union regarding an "international classification."

(2) (*a*) Some of the objections originally urged against the early form of the Draper Classification have been met by dropping the use of

[1] *Science*, 32, 883, 1910.

[2] As was done by Miss Maury in her classification, which has many points of remarkable excellence.

certain letters, such as C, D, E, and H, which depended in part upon instrumental adjustments, and did not represent real differences in the stellar spectra. But this rectification has brought with it the serious disadvantage of discontinuity, and lack of logical sequence. This is of small consequence to us who have in a sense grown up with the photographic classifications. But we must have in mind our colleagues in other branches of science, the physicists, chemists, mathematicians, and geologists. To them a system having the order P, O, B, A, F, G, K, M, N, R surely could not carry any definite impression of a logical sequence of spectra. We must also remember the difficulties of our friends who are each year trying to teach to hundreds of students the main points of difference in stars as indicated by their spectra.

(*b*) The Draper Classification is essentially based upon plates taken with low dispersion. No one can have more admiration than I for the skill and discrimination of Professor E. C. Pickering, Mrs. Fleming, Miss Maury, and Miss Cannon in their work on the classifications successively evolved at Harvard; but there are in many respects very marked differences between small-scale objective-prism plates and large-scale spectrograms obtained with slit-spectrographs. Some of the distinctive features on the small scale are lost on the large scale. If only one scale were available, probably the low dispersion would be the one to choose for purposes of broad classification; but any system is incomplete which does not depend upon plates taken with both the low and high dispersion.

(*c*) The Draper Classification lacks subgroups at certain points. This applies particularly to type A. Under both A and A2 are necessarily included many spectra which with high dispersion are quite widely different. Further, the addition of the letter "p," for peculiar, as Ap, Bp, while doubtless necessary in the Draper Classification, cannot be permanently justified. It stands for the avoidance of an assignment. In an international study and investigation, there ought to be subdivisions enough to provide for all peculiar groups, families, or genera. One hundred groups might possibly satisfy present requirements, but the system ought to be elastic enough to provide for an indefinite number, 1000 or more.

(*d*) It seems inappropriate to discuss at this time any modifications of the Draper Classification. With his habitual courteous consideration for the views of others, Director Pickering might adopt those of the suggestions made with some unanimity, and then we should have still another classification to add to the present confusion. If the result

of the present preliminary inquiry should be favorable to the general use of the Draper system until international action is taken, it would surely be desirable to use that system as it is incorporated in the *Harvard Revision* (*Annals*, **50**, 1908), in Miss Cannon's valuable study of 1477 stars (*Annals*, **56**, 1910), and in Mrs. Fleming's forthcoming memoir on stars with peculiar spectra.

(3) I think it would be exceedingly unwise "for this committee to recommend at this time or in the near future any system of classification for universal adoption." The duty of this committee, as it seems to me, is to report to the Solar Union at its meeting in 1913, making such recommendations as it may agree upon prior to that meeting. Perhaps by 1916 the committee might have time to carry out the necessary investigations so that proposals could be made to the Union regarding an international classification.

Special and minute studies should be made of the different varieties of spectra, with the use of plates covering all the spectrum photographable, with both low and high dispersion. Perhaps the varieties of the different types would be assigned to different observers for their special investigation. Relations to stellar temperatures as derived from spectral photometric measurements and from those made with the heterochrome photometer should be examined. Work should be done in physical and chemical laboratories on the spectra of mixed gases and the suppression of spectra in such mixtures; on the relative shifts of different kinds of lines under different conditions. The physicists or chemists ought to submit hydrogen to every possible variation of laboratory conditions with a view to producing the lines discovered by Professor Pickering in ζ *Puppis* and in certain other stars. A general attack on the problem of the spectral lines not yet found on the earth might lead to new information regarding nebulum.

I should be willing to take part in the work, particularly in utilizing our collection of some four thousand spectrograms obtained with the Bruce spectrograph.

(4) As already stated, I should certainly regard it as desirable that the nature (width, sharpness, etc.) of the lines should be indicated in the classification.

(5) I have suggested above several criteria, and it would unduly protract this already over-long communication to propose others at this time.

THE YERKES OBSERVATORY
February 16, 1911

M. Hamy

Comme les autres membres du comité chargé de s'occuper de la question, je pense que la Classification de Draper répond assez bien aux besoins de l'astronomie. Je lui reproche cependant de ne pas parler assez à l'esprit. Les notations employées pour désigner les différents genres de spectres ont la mérite d'être brèves; mais je préférerais des dénominations rapellant un peu ce qui caractérise chacun des spectres. Je pense, par ailleurs, qu'il serait très utile, dès maintenant, de s'entendre au sujet de l'adoption universelle d'un système de classification, quel qu'il soit; et pour ma part, je le répéte, j'adopterais volontiers celui de Draper. Toutefois, dans mon esprit, cette adoption ne saurait que provisoire. Le jour, en effet, ou l'on sera en possession de données certaines sur la température des étoiles, j'estime qu'il faudra laisser de côté l'empirisme et classer les spectres stellaires en se fondant sur les températures de ces astres, ainsi que Lockyer a voulu le faire en partant de considérations theoriques.

Au cas où le système de Draper serait adopté, je suis d'avis de rejeter l'emploi de symboles complémentaires destinés à fournir des indications sur l'aspect des raies, non que cette indication soit inutile, mais parce que je suis concerné de la complication. Un instrument perfectionné mais de maniement complexe rend souvent moins de service qu'un autre plus rudimentaire qui a pour lui l'avantage de la simplicité.

Paris
Novembre 19, 1910

J. Hartmann

(1) Als *vorläufiges* System ist das Draper System das beste, wenn es auf genauere Bezeichnung des Spektraltypus ankommt. In *sehr* vielen Fällen ist es jedoch gar nicht nötig und in noch viel mehr Fällen überhaupt gar nicht *möglich* einen Stern genau in eine der Draper-Abteilungen einzuordnen. Ich halte es daher für das *erste* Erfordernis einer allgemein brauchbaren Einteilung, dass sie zunächst alle Sterntypen in ganz wenige *grosse* Gruppen einteilt, die sich stets, auch bei schwachen Sternen leicht unterscheiden lassen. Jede Klassifikation, die sofort alle Sterne in zu viele verschiedene Gruppen einteilt, ist nicht nur zu schwerfällig, sondern überhaupt unbrauchbar. Historisch zeigt sich dies auch darin, dass alle Benutzer anderer Klassifikationen nebenher immer noch die Bezeichnungen Secchis I, II Typus benutzen.

Ich meine also dass man als *oberste* Einteilung die Secchi'schen Typen beibehalten soll—jeder kennt sie und jeder benutzt sie heute.

(2) Als zweite Grundregel schlage ich vor: Es soll nicht eine Klassification der Sterne oder ihres physikalischen Zustandes, sondern eine rein äusserliche Klassifikation der Spektra sein. Es kommt nur darauf an, dass jedes Spektrum durch ein Symbol möglichst kurz und genau charakterisiert wird, ohne dass durch das Symbol irgend etwas über den physikalischen Zustand des Sterns gesagt werden soll. Diese Verknüpfung des Symbols mit bestimmten physikalischen Begriffen ist erst Gegenstand besonderer Untersuchungen, die im Laufe der Zeit zu ganz verschiedenen Resultaten führen können; solche verschiedene Folgerungen sollen aber keinen Einfluss auf das Symbol und die Klassifikation haben.

(3) Es empfiehlt sich nicht, zu schnell ein neues Bezeichnungssystem aufzustellen. Am besten wird es sein, vorläufig die kombinierte Bezeichnung Secchi+Draper zu benutzen, also etwa IB5.

(4) Für die wichtigste Aufgabe halte ich zuerst die Aufstellung von "Repräsentanten." Man soll alle vollkommen identischen Spektra verschiedener Sterne zusammenordnen und aus jeder solchen Gruppe den hellsten Stern, dessen Spektrum am leichtesten zu erhalten und genauer zu untersuchen ist, als Repräsentanten der Gruppe betrachten. Ich schlage vor, diesen Teil der Arbeit zunächst in Angriff zu nehmen und die Debatte über die Repräsentanten zu eröffnen. Erst wenn die Repräsentanten festgestellt sind, wird man nur diese zu klassifizieren haben. Bis zur Annahme definitiver Symbole wird es sich empfehlen, zur Charakterisierung eines Spektrums den Repräsentanten des betreffenden Spektraltypus zu nennen.

(5) Ueber die Kriterien, nach denen die Klassifikation erfolgen soll, wird man besser erst beraten, wenn die Frage der Repräsentanten vollkommen erledigt ist. Es handelt sich dann nur um die Ersetzung des Sternnamens des Repräsentanten durch das Symbol.

So viel es meine Zeit und meine Hülfsmittel hier zulassen, will ich gern meine Mitarbeit zur Verfügung stellen.

Göttingen
Februar 7, 1911

Ejnar Hertzsprung

The spectral classification of stars should be made so that the designation of spectrum is connected in a simple way with other physical properties. If only one sequence of designations is used, as in the *D.C.*, the spectrum should be connected linearly with the color-index, m (*photogr.*) —m(*vis.*). Now the *D.C.* Classification nearly fulfils this condition, as the increase in color-index is about $0^{m}.4$ for each spectral class. I should

recommend that the *D.C.* Classification be modified so that the relation between the spectrum and mean color-index will be exactly linear. The modification needed will not be greater than the systematic differences now existing between the different Harvard Catalogues (the spectrum F in *D.C.* is generally called F5 or F8 by Miss Cannon) joined together in the *H.R.*

To distinguish the rough classification of the *D.C.* from the later more precise ones in the same system, I use A0 (A zero) for simple A, etc.

Great care should be taken to get the spectral classification independent of the brightness of the stars.

An imperfection in the present use of the *D.C.* scale is the inequality of the tenth-divisions. Certain tenths, as 5 and 2, are much more numerous than others, as 7, 4, and 6. It would perhaps be practicable to make a continuous scale of spectra exactly corresponding to the color-index scale and with the aid of an arrangement like that of the Hartmann microphotometer to determine the scale-reading corresponding to the spectrum examined.

A subdivision or second co-ordinate of spectral classification may in the future be connected with the absolute brightness of the stars. At the present time this can be done only in a very imperfect way, as we have still no spectral equivalent of the great difference in absolute brightness between such stars as α *Boötis* and *70 Ophiuchi* or α *Tauri* and *61 Cygni*, which are in pairs ranked in the same spectral class and subdivision (XVa, remark 184, and XVIa). For the "earlier" spectral types we have the valuable subdivisions *c* and *ac* of Miss Maury, which spectral distinctions are criteria of great absolute brightness. Using the *D.C.* classification the *c* and *ac* properties should at least be indicated by the letter *p* (peculiar), which has occasionally been omitted, for example from the very peculiar star α *Cygni*.

Therefore, with the above-mentioned modifications, I recommend at present the use of the *D.C.* classification. As the most important problem now I consider the exploration of the connection between absolute brightness and spectrum. The recent discovery of star-streams among the *Orion* stars by Kapteyn and Eddington holds out a prospect of getting the absolute brightness of such stars, the parallaxes of which are not directly measurable.

Accordingly I imagine for the future a spectral classification in two co-ordinates, the one giving the normal color-index and the other the normal absolute brightness corresponding to the spectrum observed.

POTSDAM
December 2, 1910

S. S. Hough

I concur in the general opinion put forward that the Draper Classification is the most useful that has thus far been used, and the most convenient one at the present time on which to base any system for general adoption.

The chief objection to the Draper system to my mind is to be found in its nomenclature rather than in its actual classification. While the essential feature of the system is the existence of a more or less regular sequence between the various types, the symbols by which the leading types are denoted follow one another in an apparently arbitrary manner, having no relationship to the order in which the types occur in this sequence. I would suggest that if any system is to be proposed for universal adoption, the Draper sequence should be utilized, but the various classes should be designated by a sequence of symbols following either a strictly numerical or a strictly alphabetical order. The former is doubtless open to objection on the grounds that it suggests quantitative relationships between some characteristic properties which differentiate the classes.

The chief objection to the latter is the departure from existing practice which has already in some measure become established. To avoid confusion with existing notation the principal types might be indicated by letters of the Greek alphabet. Arabic letters attached could then serve for the further subdivision of types and numerals for interpolation between types as in the Harvard notation.

Such a scheme would have the advantage that the committee would be in a position to decide the number of typical classes without reference to the existing classification, except so far as that the same sequence would be involved, and to give a precise definition of the characteristics peculiar to each class by reference to a particular typical star.

While the system should be made as elastic as possible to provide for typical deviations from the leading types selected, I think a too minute classification at present neither necessary nor desirable. What is required is a notation which should be suggestive in character and the significance of which should be absolutely free from ambiguity.

This I would consider the most far-reaching step which could with advantage be taken at the present time. It is not improbable that the early future may indicate some natural system of classification founded on a physical or chemical basis rather than a purely optical one. Any system previously introduced would almost certainly have to give way to a new one founded on such considerations. Until this stage is reached,

any modification of the Harvard system, which on the whole seems to meet the practical requirements of an empirical classification, appears to me undesirable except in the direction I have indicated.

Cape of Good Hope
December 20, 1910

F. Küstner

Bei unserer jetzigen Kentniss der Sternspectren erscheint es mir zur Zeit nicht möglich an die Stelle der Draper Classification eine besserer zu setzen.

Ich empfehle deshalb, diese Classification vorläufig und so lange, bis die Fortschritte der Beobachtung von selbst eine Aenderung verlangen werden, beizubehalten; und auch an ihrer Art der Bezeichnung nichts wesentliches zu ändern, da dies nur zu Verwirrung führen würde.

Für dringend notwendig halte ich es, das rothe Ende des Spectrums bei möglichst vielen Sternen photographisch genauer zu untersuchen.

Bonn
Dezember 19, 1910

H. C. Lord

There is one aspect of the classification of stellar spectra that I have had some experience with, and have found a great deficiency in the existing catalogues in this respect. My trouble can best be shown by an example. Suppose I wished to measure the velocity of a certain star, the first thing that I would want to know is what its spectrum looks like. I turn to the Draper catalogue and find it classed as B3; I then find a description of B3, but at best such a description falls far short of a good enlargement of a typical spectrum of class B3. It would seem to me, therefore, of the utmost importance that whatever classification is adopted, it should be accompanied with a photographic catalogue of typical stellar spectra, so that if one finds a star of type B3 he can turn to a picture and see what B3 looks like and not have to depend upon a description by someone else.

My second trouble is closely akin to the above. Suppose two stars, A and B, are photographed at Harvard with their greatest resolving power and A is found to have broad hydrogen lines and nothing more, while B is found to have the hydrogen lines narrower than A, though still broad, and in addition a number of very fine other lines. Now it can be easily shown from the principles of physical optics that if star B were to be photographed with a spectroscope of much smaller resolving

power, the resulting photograph would show very broad hydrogen lines and nothing else. It would seem to me, therefore, that the above-mentioned catalogue should contain photographic enlargements of spectra taken with different instruments covering as wide a range of resolving power as possible. Then any observer, knowing his own instrument, could tell at a glance about what to expect from spectrograms of a star classed as of a given type.

To answer your questions:

(1) I agree.

(2) Lack of photographic catalogue of typical spectra as explained above.

(3) (*a*) I think now is as good a time as any.

(*b*) Additional observations as outlined above should be made but they had better be made after the adoption of the classification than before.

(4) No. Pictures are better.

EMERSON MCMILLIN OBSERVATORY
January 9, 1911

J. LUNT

(1) It seems necessary to distinguish the several classifications emanating from the Harvard College Observatory at different times by quoting the year of publication. The name Harvard Classification seems to me preferable to Draper Classification and I would distinguish the following:

"Harvard Classification 1890."
"Harvard Classification 1897."
"Harvard Classification 1901."
"Harvard Classification 1909."

As further modifications are almost certain to be introduced in the future this would provide a short reference title for each classification. Although the 1909 Classification is based on and closely follows the 1901 Classification, there are differences of more or less importance; e.g., in the former classification we find the abbreviation of the symbols of the classes, the different choice of typical stars, the inclusion of Class N, the absence of reference to other classes (P and Q).

The 1897 Classification contains classes C and L, which are not so named in the later ones but still are used in recent work.

I concur in the opinion that the Harvard (or Draper) Classification 1909 should be the *basis* of a system for universal adoption, it being

understood that modifications in detail would be made as our knowledge of differences of stellar spectra became more complete.

In details, however, I think none of the published Harvard classifications is perfect and that even the latest should be modified in certain ways before being put forward for universal acceptance.

(2) The symbols for the subdivisions of the classes should be uniform throughout. Having started with Oa, Ob, Oc, Od, Oe, I think it would be an advantage to continue Ba, Bb, Bc, Bd, Be, uniformly with classes O and M, instead of using numerals which have a decimal meaning. This decimal division is unsuited for the subdivision of *all* the class intervals and there are great practical difficulties in deciding whether a given spectrum is 8 or 9 tenths of the interval between two types.

There should be only *one* typical star for each class and that one the brightest example of its class. The 1901 and 1909 classifications between them sometimes give three typical stars, which introduces confusion. Let each class symbol refer to one star only.

α *Boötis* as type star for class K is preferable to ϵ *Scorpii*.

α *Orionis* as type star for class Ma is preferable to γ *Hydri*.

Wherever possible I would refer to stellar types (except in tables) by naming the type star, e.g., *Sirius* type, *Canopus* type, *Procyon* type, *Aldebaran* type, using the noun as an adjective.

I agree with Mr. Schlesinger that a further subdivision of class A is necessary.

Spectra might with advantage be further differentiated in subgroups having additional suffixes to present symbols such as Ba, Baa, Baaa, or Ba, Ba^2, Ba^3. This would be a convenient way of including new typical stars which differ appreciably from the nearest type star. Where a bright-line star has the same composition as a dark-line star, I would class it with the dark-line star and add β to the suffix, e.g., $Ba^2\beta$. The suffix W might also be used to indicate spectra with wide diffuse lines.

In the 1901 Classification ϵ *Canis Majoris* and β *Crucis* are classed as B1A under type star β *Centauri*. I consider that the first-named star should be used as a type star as the best example of a star showing oxygen lines. In the 1897 Classification *R Leonis* is classed as XXa under type star o *Ceti*, although its spectrum is very different. The nature of composite spectra should be indicated by bracketed symbols.

(3) I think it would be wise for the committee to recommend, not at present but in the near future, a slightly modified and improved classification based on the Harvard 1909 Classification, on the understanding that it is always open to revision by a standing committee

deliberating yearly; and as a preliminary step I should be in favor of printing the classification—with whatever modifications are, in the immediate future, approved by the committee—as a separate circular containing a fuller description of each typical star, including wave-lengths and origins (where known) of its distinctive lines, and a photograph of its spectrum marked in such a way as to call attention to its distinguishing features. The photographs should be as far as possible on the same scale. This would give a working basis for the full discussion of the question. Very few observatories possess photographs of all the star spectra used as types in the Harvard Classification. In many cases objective-prism photographs are alone available, but wherever possible photographs taken with a slit spectroscope should be reproduced. This seems to me the first step toward a full discussion of the subject by all who are interested in it.

(4) I agree with Miss Maury in placing in separate subgroups spectra in which (*a*) all lines are exceedingly narrow and sharp and (*b*) all lines wide and hazy. The symbol for the latter I would suggest should contain *wd*—wide diffuse, and for the former *ns* or *nf*—narrow and strong or narrow and faint.

(5) The fundamental criterion for classification should be a chemical one. Wherever a new group of lines appears as we pass from simple hydrogen stars, a new symbol should be employed. Spectra in general need to be still further differentiated into subgroups and varieties by adding to existing symbols in the way I have suggested.

The Roman numerals of Secchi should in my opinion be discarded entirely as no longer adequate. Their retention would only add confusion and complicate the Harvard notation unnecessarily.

At the present time I think that an evolutionary basis of classification is premature, but the terms "early" and "late" are so convenient and so frequently used that I would continue to use them in preference to "white" and "red" (which have a more restricted meaning), even though they may ultimately have to be abandoned.

Cape of Good Hope
December 14, 1910

Antonia C. Maury

(1) The Draper Classification appears to me to express satisfactorily the most detailed information in regard to the largest number of stars. As a somewhat different classification was proposed by me and published in *Harvard Annals*, 28, Part I, I may say that the latter represented an

attempt to discover and exhibit the natural sequence of type as revealed in high-dispersion spectra of bright northern stars; and that, as this sequence showed continuous transformation by barely perceptible changes or gradations, the groups into which it was divided were necessarily somewhat arbitrary. The twenty groups named by successive Roman numerals, into which the series consisting of stars of Secchi's I, II, and III types and of those of *Orion* and helium type was divided, were based mainly on least readily definable differences. As these groups, with a few exceptions, have been represented in the Draper nomenclature, and as the Draper symbols, when placed in order O, B, A, F, G, K, M, with their appended arabic numerals, express the sequence presented in *Harvard Annals*, **28**, Part I, the change of symbol does not seem of moment.

(2) However, it does seem to me important that in a final classification this sequence, which shows in so very marked and wonderful a way the gradual transformation of spectral type, and must in some manner express the law of stellar evolution, should be represented either by numerals in natural sequence or by letters in alphabetical order. If this be not done, the attempt to grasp in thought the evolutionary changes and fix in memory the places of individual stars becomes bewildering.

As, however, main divisions of type clearly exist, it would seem well to represent these by letters, adding arabic numerals for intermediate stages, as has been done in the Draper system. But why could not the Draper letters be rearranged in alphabetical order for a final nomenclature?

Moreover, might not the classic numerals of Secchi be retained to represent, as they actually do with the exception of the helium stars, the great fundamental types? The alphabetic groups would then be subdivisions under them.

(3) It seems to me to be of supreme importance that the system to be finally and universally adopted should be *evolutionary*. The main difficulty at present in the way of this would seem to be our ignorance as to the place in the sequence of the groups of rare stars, as the bright-line stars of V type newly discovered at Harvard, as well as *Novae* and others.

We cannot but recognize, however, the gradual transformation of type from stars of Draper class B, through A, F, G, K, M, i.e., from the *Orion* or helium type through Secchi's types I, II, and III. This is shown in an unmistakable manner in the Harvard photographs of high disper-

sion, discussed in *Harvard Annals*, 28, Parts I and II. And the transition from Secchi's type III to type IV (Draper M to N) was proved by photographs taken at the Yerkes Observatory.

Since, then, the vast majority of stars evidently fall in this sequence, there would seem to be little doubt that it represents the main outlines of stellar evolution.

The transition from O to B, that is, from "V type" bright-line stars to the helium type, was also well shown by Miss Cannon in the case of stars discussed in *Harvard Annals*, 28, Part II. Yet the scarcity of stars of this kind raises a doubt whether they represent a universal phase of evolution or merely an occasional one. The same may be said of other rare types. Obviously, however, this small sprinkling of peculiar stars should hardly stand in the way of the evolutionary classification of the main body of stars.

As to the question whether the order O, B, A, F, G, K, M, N should be reversed, the evidence seems to me almost conclusive against this. For, if we reverse the order, we should have to assume either that cooling stars change from red through yellow and white to blue, instead of pursuing the opposite course, or else that such stars, as, for example, the sun, are growing hotter instead of cooler, which seems unlikely. The high light-intensity in the ultra-violet of B and A stars and the gradual falling off of light in this region and later in the blue, with advance toward Secchi's types III and IV, seem clear proofs of loss of energy by radiation.

Again, the vast atmospheres of hydrogen and helium of B and A stars, when contrasted with the heavier metallic atmospheres of solar and third-type stars, seem clear evidence of an early stage of condensation; while the banded spectra of stars of type IV, indicative of still heavier vapors, scarcely permit us to doubt that they are the most advanced in condensation of all. The fact also that nearly all *Algol*-variables are helium or B stars is necessarily accepted as evidence of light weight in the latter. So that, were we to reverse the commonly accepted order, we should have to assume that the solar and even the deep-red stars with fluted spectra are growing not only hotter but also rarer, which seems out of the question.

Finally the overwhelming predominance of helium or B stars in great nebulous regions as those of *Orion* and the *Pleiades* seems to prove irresistibly that the helium stars are in an early stage of formation from the nebulae.

These considerations strongly suggest the conclusion that the order

of the sequence from O to N is altogether unlikely to be reversed by further study and investigation.

(4) In regard to stars classed by me as series *b* and *c*, parallel to the normal series, I would say that the distinctions recorded were, in the majority of cases, unmistakable, and readily evident to anyone familiar with the photographs. The *b* variation consisted in unusual width and haziness of lines and did not affect their relative intensity, so that it would appear to call for some purely physical explanation, as, for example, rapid rotation.

The parallel series *c*, on the other hand, showed special lines of enhanced intensity as well as great sharpness in all the lines, and narrowness noticeable especially in lines of hydrogen and helium. In such stars as α *Cygni* and β *Orionis* these peculiarities were very striking. They must indicate important differences of constitution in these stars, which seem to pursue a line of evolution that, through a portion at least of its course, deviates from the normal. The study made by Dr. E. Hertzsprung (see *Astronomische Nachrichten*, **179**, 373, 1909; and articles by the same observer, "Zur Strahlung der Sterne" in *Zeitschrift für wissenschaftliche Photographie*, **3**, 429, 1905; **5**, 86, 1907) led him to the conclusion that these stars are bodies at great distance and of supernormal light-energy.

In *Harvard Annals*, **56**, No. IV, pp. 113, 114, Miss A. J. Cannon gives a list of stars showing enhanced silicon lines, and another list of those in which λ 4077.9, believed to be due to strontium, is unusually dominant. These classes of stars, like the *c*-stars, form collateral series, all such variations, however, ceasing as Secchi's type III is approached.

It would seem that these, and any other parallel series possible to trace, should in a final classification of spectral groups be clearly distinguished, since a study of their variant lines would seem likely to prove extremely suggestive in investigations as to the real nature of stellar evolution.

HASTINGS-ON-HUDSON, N.Y.
January 1, 1911

J. A. PARKHURST

(1) I have used the Draper Classification in classifying over a thousand spectra taken with the Zeiss objective-prism and doublet, the spectra having a dispersion of 2.34 mm between $H\beta$ and $H\epsilon$. The color-curve of the doublet is flat from $H\gamma$ well out into the ultra-violet, the focus for $H\beta$ being slightly longer; so that the region from $H\beta$ to

the ultra-violet was available for the classification. Though the scale of the spectra is very small, the resolution on good plates is somewhat better than the engraving in *Harvard Annals*, 64, for the types A to G, but rather poorer than the engraving for types K to M. For classification of spectra between B and M on this small scale the Draper system seems to me to be satisfactory. It is much more useful to me than Secchi's would be, and it would be practically impossible for me to use Vogel's system, since his criteria are mostly invisible on this scale.

The Draper system seems to be substantially complete within the limits mentioned, for I have found no spectra which fall without the sequence. Another reason for my liking this system is that the curve connecting the "color-index" and the spectral type is very nearly a straight line, when the distances between the letters A, F, G, K are platted equal.

(2) As a sort of a modification of the system I would suggest a little more definite statement of the criteria so that fractions of a division between types F, G, and K could be measured with a precision comparable with the fractions between A and F. It is possible to measure the width of the Fraunhofer K line, as compared with $H\delta$ and $H\epsilon$, and so determine the classification between A and F. It would seem possible to use the solar G group and the calcium 4227 line in a similar way, to get a numerical measure of the classification between F, G, and K. It would probably be necessary to measure also the opacity of the continuous spectrum, as a check on the measures of width of lines.

(3) It seems to me that the visual and the ultra-violet portions of the spectrum should be taken into account in fixing upon a plan to recommend for universal adoption. I expect to do some work on the visual region soon.

YERKES OBSERVATORY
January 5, 1911

EDWARD C. PICKERING

(1) The daily use of the Draper Classification of stellar spectra for more than twenty years has satisfied me that it is very convenient for a large portion of the stars. Also, that for the principal classes of spectra it is a natural system, and that while the names may be changed, the principal subdivisions must be retained, permanently. In the sequence B, A, F, G, K, M, the letters A and B might be interchanged. But A is a great group including half the stars in the sky, while B appears to represent a few very large, distant objects occurring in certain regions only.

(2) See also (4). The most important modifications to be recommended are to extend the classification. Thus, the classes Ma, Mb, Mc, and Md differ from each other more than classes G and K or K and M. These subdivisions had not been discovered when the letters were assigned. The divisions of Md also differ widely. In some, $H\gamma$ is ten times as bright as $H\delta$; in others, $H\delta$ is ten times as bright as $H\gamma$. If new letters should be assigned to the fundamental types (which is not advisable) separate letters should be assigned to Ma, Mb, and Mc, and Md should be further subdivided. The present notation does not lend itself well to peculiar spectra, like those of nebulae, novae, and stars of classes O and R.

(3) It would not be wise for this committee to recommend a system of classification until the laws of evolution of stars are fairly established. In order that the class of spectra should be measured instead of estimated, as proposed by Professor Russell, it would perhaps only be necessary to measure accurately the intensity of different portions of the spectrum. I hope to undertake this work, but the errors introduced by such measurements would probably be larger than those in classifying the spectra. The reverse would probably be the case if the intensity of the photographs was measured mechanically by means of a bolometer, radiometer, or other similar instrument. Approximate results could also be obtained by comparing the photographic and photometric magnitudes. This is the most practical method for very faint stars. The last of these methods is in use here, and the first method will probably be tried. I should be glad to do my share of any investigations that would aid the work of the committee.

Photographs of many stellar spectra have been taken here with isochromatic plates, but the yellow portions have not proved of much use in classifying the spectra.

By the courtesy of two members of the committee, fifty excellent photographs of stellar spectra taken with the slit spectrographs of the Yerkes and Lick Observatories have been sent here for examination. A careful study is now being made of them.

(4) Yes. With an objective-prism, increased width of lines is not easily distinguished from change in focus of the telescope. With a slit spectroscope, the comparison spectrum shows that a widening cannot be due to this cause. The width of two lines compared with their distance apart could in some cases be well estimated or measured.

Harvard College Observatory
November 21, 1910

J. S. Plaskett

(1) The Draper Classification is the most useful scheme hitherto proposed, but it is possible that it might be improved upon in some respects.

(2) The principal objection to the Draper Classification occurring to me is that the designations of the different types of spectra do not of themselves suggest anything in regard to the character of the spectra, and are in this respect arbitrary and unsatisfactory. It is true that familiarity with and use of the Draper system soon diminishes the weight of this objection, but for those using or referring to it occasionally, a system of nomenclature which would at once suggest the type of spectrum designated would be a decided advantage, and I would suggest that the committee consider the possibility of such a modification. Would it be possible to combine the simplicity and the universally known features of Secchi's nomenclature with the more complete, systematic, and consecutive division of the spectral types in the Draper Classification? There is, of course, the objection that one would have a tendency to associate the order of the numerals therein used with the order of stellar development, and this, considering the present state of our knowledge of stellar evolution, would be inadvisable. A similar objection may be urged to the designations of the Draper subdivisions in that they are always used in one order, thus always A4F never F6A, tacitly assuming that stars develop from the A to the F types, and not, as may be possible, from F to A.

(3) In my opinion, the question in all its bearings should be discussed as fully as possible by correspondence, so that at the next meeting of the Solar Union at Bonn in 1913, the committee may be prepared to recommend some scheme of spectral classification for universal adoption. It does not seem to me advisable to formulate any system before that date, as it can only be put into satisfactory shape after personal meetings and discussions among the members, and such meetings will not likely be possible until the next Solar Conference. On the other hand, the only thing that would justify a longer delay than that necessary for a full consideration of the question would be the chance of obtaining, in the near future, some more positive knowledge of the order and process of stellar evolution than we at present possess. The probability of a final solution of that problem is not, in my opinion, sufficiently great to justify a long postponement of the advantages that will undoubtedly accrue from the adoption of some uniform system of spectral classification.

It seems to me desirable before a definite classification is adopted that some work be done on the red end of stellar spectra up to and including $H\alpha$. It is possible that very valuable criteria for the division and distinction of the various types may be obtained from the behavior of some of the lines, such as the sodium D, the helium D_3, and the magnesium b, between $H\alpha$ and $H\beta$. It would be necessary to obtain, with a reasonably high dispersion, not less than 50 Å per mm, photographs of the red end of the spectrum of representative stars of the different spectral subdivisions, before it could be determined whether any modifications of existing divisions would be required. Such work and any further work that might develop I would be willing to take part in.

(4) It seems to me to be essential, or at least very desirable, in any complete system of classification, to introduce some method of representing the width of the lines. It is undoubtedly true that there is frequently much greater difference in the appearance of two stars of the same type, one with wide and one with narrow lines, than between two stars, each with narrow lines, of types one or more subdivisions apart. It may not be necessary to introduce a separate symbol to represent the character of the lines. If we consider all spectra with sharp or moderately sharp lines as normal, and represent them in the ordinary way, then spectra with diffuse lines might be differentiated from the normal by the use of the same distinguishing letters and figures but in different type, e.g., sloping or italic.

OTTAWA
January 26, 1911

HENRY NORRIS RUSSELL

(1) Yes.

(2) None, except some details of notation referred to later.

(3) It seems to be generally admitted that the classes O, B, A, F, G, K, M, N correspond in a general way to the principal types of stellar spectra. If the same unanimity of opinion on this point that was in evidence at Mount Wilson should appear on more extended correspondence, I think the committee might recommend at once that they be generally adopted in preference to other existing systems.

The detailed classification, however, demands a careful study of the minor differences among spectra and also of the most convenient notation for expressing them, and this will take some time, and require a good deal of discussion.

The most important observational work seems to me to be the study

of the yellow and green regions of the spectrum. This is obviously *necessary* before any definite classification can be attempted.

Next to this comes the study of plates taken with slit spectroscopes with reference to the width of the lines, etc. I believe this is already under way.

I would add the suggestion that a comparative study should be made of the spectra of stars of very different total luminosity but the same spectral class (Hertzsprung's "giant" and "dwarf" stars). If any definite and constant *spectroscopic* differences exist, they will be of value in classification.

(4) Hertzsprung's work (see *Astronomische Nachrichten*, 179, 373, 1909) shows conclusively that the fineness of the spectral lines is of great astrophysical importance. The object of the study suggested in the last paragraph would be largely to find the analogous difference among the redder stars. Such differences should be certainly taken into account in a satisfactory classification.

(5) In my opinion, the classification should be based exclusively on a study of the spectra, i.e., of the *line and band absorption*, without reference to color, intrinsic brightness, and the like, much less to theoretical considerations. External considerations should be admitted only (1) in the search for differences, *perceptible in the spectra themselves*, which might otherwise escape notice; (2) in determining which of numerous small differences are entitled to specific rank.

For example, I find while studying the relations of mass, brightness, etc., among the fainter stars, that those of type K seem less homogeneous than the others, and this suggests the desirability of closer definition of its limits, especially toward type M. Here the visual region will doubtless be of great importance.

The Draper Classification seems to me all the better because the letters are not in alphabetical order. This helps to keep the novice from thinking that it is based on some theory of evolution. The decimal notation for those finer differences which are of the nature of gradations between the types is admirable. It should be *reserved for this purpose.* If the types of spectra now denoted by Md1 to Md9 are not stages in a gradation from Md to something else, some other notation should be devised for them.

The use of small Roman letters a, b, c . . . . for other subdivisions of a type not in the nature of a transition toward the next seems also to be excellent. I would suggest for discussion the use of some definite letter (perhaps *l*) to denote the presence of bright lines. Other letters

might be used for other frequently occurring peculiarities, leaving *p* (which is now rather overworked) for the more exceptional cases, to be explained in footnotes; and, in rougher classification, as a general indication of abnormality. The width of the spectral lines might be denoted by Greek letters (e.g., α narrow, β medium, γ broad). The letters *a*, *b*, *c*, used by Miss Maury, are pre-empted for types O and M.

Absence of a given sign should in general denote *deficiency of information* about the characteristic in question; e.g., a spectrum between B9 and A1 should be called A0 (as suggested by Professor Pickering), leaving A (without affix) as a general rough designation for spectra not nearer B or F. Similarly, every case where the width of the lines is known (whether it is normal or not) should have a Greek letter.

The symbols denoting peculiarities of spectrum would of course be exceptions to this rule. It might perhaps be desirable to devise some notation for a moderately rough classification by half-classes, as say F+ for a spectrum nearer to F5 than to F or to G, reserving F5 for spectra definitely classified as not F4 or F6.

In defining the exact limits of the broader classes, e.g., whether B8 and B9 should be counted as B or A, extraneous data, such as the average proper motion, may in my opinion be used to advantage; and the same may be said of the assignment of those decimal subdivisions now practically disused, e.g., K3, K7, K8, K9. The subclasses of type K in particular may need revision with the aid of the visual region of the spectrum, and here observations of colors may be of use.

When agreement on the details of classification is reached, a series of type stars should be chosen, preferably several for each subdivision, and these should be taken as its permanent definition.

Princeton University Observatory
December 31, 1910

J. Scheiner

Zu dem Vorschlage, eine allgemein einzuführende Classificirung der Sternspectra aufzustellen, und der Anfrage ob hierzu die Draper Classification für geeignet gehalten wird, möchte ich folgendes bemerken.

Eine solche Classification muss eine so gute sein, dass sie nach unseren jetzigen Kenntnissen nicht verbesserungsdürftig ist. Sie muss mnemotechnisch einfach sein, und ganz bestimmte Grundgedanken müssen in ihr zum Ausdruck kommen.

(1) Die Draper Classification erfüllt beide Bedingungen nicht; sie ist nicht einfach, und die beiden Grundprincipien der Eintheilung,

Breite der Linien und das Auftreten bestimmter Elemente, wird nicht klar auseinander gehalten. Die Argumente, die Buchstaben A, B, etc., befinden sich nicht einmal in den richtigen Reihenfolge. Wenn sie sich den jetzigen Kenntnissen auch besser anschliesst als die 2te Vogel'sche Classificirung, so ist letztere dafür bequemer und einfacher; aber auch sie wäre nicht ohne wesentliche Aenderungen einzuführen. Ich kann mich daher dem Vorschlage, die Draper Classification allgemein einzuführen, nicht anschliessen.

(2) Einen bestimmten Vorschlag zu einer neuen Eintheilung oder gar den Entwurf zu einer solchen möchte ich nicht machen, da ich mit den neuesten Untersuchungen, welche die spectralen Erscheinungen mit anderen Factoren (Eigenbewegung, Parallax, etc.) in Verbindung bringen, nicht genügend vertraut bin. Es scheint mir aber so, als wenn auch diese Dinge in einer neu aufzustellenden Eintheilung eventuell zu berücksichtigen wären, wenn gleich vorher noch manches zu untersuchen sein dürfte.

(3) Ich halte den jetzigen Zeitpunkt, wo sich gerade neuere Untersuchungen eröffnen, *nicht* für die geplante allgemeine Einführung irgend eine Classification für geeignet. Es dürfte überhaupt verfehlt sein, hierbei eine Abstimmung vorzunehmen und eine Majoritätsbeschluss herbei zu führen. Vielmehr müsste eine aus zahlreichen Mitgliedern bestehende Specialkommission eingesetzt werden, die nach sorgfältigen Berathungen ein Votum nur dann abgeben dürfte, wenn sie sich über einen Vorschlag nahezu einstimmig einigt. Stellt sich eine solche Einigung als unmöglich heraus, so wäre das ein Zeichen dafür, dass überhaupt der Versuch der Einführung einer Classification der Sternspectra verfrüht ist und vorläufig der Zukunft überlassen bleiben muss.

Potsdam
Dezember 1910

Frank Schlesinger

(1) I concur in the general opinion as to the usefulness of the Draper Classification; it has been in exclusive use at the Allegheny Observatory for some time.

(2) This classification has responded well, and still does so, to the immediate needs of astronomers. Whatever objections I have to this system (except for one or two of minor importance) are concerned with the question as to its adoption as a permanent and universal classification. These objections are: First, that it deals with only the photographic region of stellar spectra. It appears probable, from the some-

what casual explorations that have been made in other regions, that the photographic portion of the spectrum is richer in practical criteria for classification than any other; but it can hardly be doubted that a thorough study of the visual portion would be well rewarded from this point of view, as well as from the more general and more important ones presented by the question of stellar evolution.

A second objection is that the observational material from which the Draper Classification was made is restricted in still another way; the plates were obtained by means of objective-prisms attached to telescopes that were not guided by hand. As a result, they do not show some important details that are well brought out in other photographs, especially those obtained by means of a slit spectrograph. This probably accounts, among other things, for the wide diversity of spectra that have been classified as A. Thus *Sirius*, α *Coronae*, and θ *Aquilae* are assigned to this one class, although they show considerable differences on plates taken with slit spectrographs.

A third objection to the universal adoption of this classification as a permanent one is that it neglects certain important criteria; to mention only the two of these that seem to me to be of greater importance than the others, we have the progressive changes in the form of the intensity-curve in the continuous spectrum as we go from one type to the next and the progressive changes with type in the wave-lengths of certain lines.

(3) I do not think that this committee can profitably recommend at this time any system of classification for permanent adoption. Nor do I believe that it will be possible to do this to advantage in the near future. Much observational work will be necessary in order to establish a system on so firm a basis as to render improbable a revision within a few years after it is set up. The general character of this observational work should be to *co-ordinate* the more important criteria, and the various regions of the spectrum, into a consistent whole. But I am opposed to considering in this connection (classification) any other facts than those revealed in the spectra themselves. If we do not draw the line here, we shall soon become involved in a piece of work that is almost coextensive with sidereal astronomy itself, and which is in fact inseparable from the question of stellar evolution.

I would suggest the following investigations as being the most profitable that this committee could undertake or encourage at the present time: first, the continuation and extension of the work of Parkhurst and Jordan, King and others, on the relation of star colors to spectral type; that is, the determination of the *color-index*, or difference

between photographic and visual magnitudes. These investigations have already shown that the color-index varies directly with the type in the Draper Classification, when the latter are arranged in the order O, B, A, F, G, K, M. It remains to invert this problem and to see whether the spectrum can be correctly classified from a knowledge of the color-index; or, in other words, whether any two stars that are of the same spectral type according to the Draper criteria always have precisely the same color-index. If not, the specific reasons for this should be ascertained. Such investigations are especially valuable, since they would seem to afford the possibility of determining the spectral type by quantitative methods. The same remark applies to Albrecht's recent work, in which the spectral type is co-ordinated with progressive changes in wave-length. This method of classifying spectra quantitatively is more laborious than by means of the color-index, and cannot be applied to as faint stars; but it should, in my opinion, be vigorously prosecuted.

Finally, as Professor Frost has urged, the visual portion of the spectrum should be carefully studied before any permanent system of classification can be adopted. For this purpose, it would greatly facilitate matters if the whole extent of the spectrum from λ 3600 to λ 6800 could be photographed on one plate. This has recently been made possible through the manufacture of plates that are almost equally sensitive to this whole range of spectrum. A reflecting telescope and a grating spectrograph would also be essential for this purpose. If circumstances warrant, the Allegheny Observatory is prepared to install such a spectrograph, to be used in connection with the Keeler Memorial Reflector.

(4) Symbols denoting the widths of lines would be very convenient. These symbols should be assigned from a study of slit spectrograms, and care should be taken that the spectra are of normal density. I regard this matter of specifying the width of lines as being of minor importance as compared with other questions that the committee is considering.

Allegheny Observatory
February 1911

K. Schwarzschild

(1) Die Draperklassifikation ist nach meiner Meinung die beste zur Zeit existierende.

(2) Der Hauptvorwurf gegen das Drapersystem ist die unglückliche Wahl der Buchstaben für die Spektraltypen. Wenn das System jetzt neu einzuführen wäre, würde ich den Vorschlag machen, die Buchstaben

überhaupt durch Zahlen zu ersetzen, und zwar 0 statt B, 1 statt A, 2 statt F, 3 statt G, 4 statt K, 5 statt M zu schreiben und die Zwischenstufen durch Dezimalen mit eventuell mehreren Stellen auszudrücken. Statt der Draperbezeichnung A2 würde ich also 1, 2 schreiben; die Zahl 1, 25 würde ein Spektrum in der Mitte zwischen den Drapertypen A2 und A3 bezeichnen. Bei dieser Bezeichnungsweise würde die den Spektraltypus angebende Zahl mit 0.4 multipliziert sehr nahe den Farbenindex ergeben.

(3) Ich halte es *nicht* für richtig, in nächster Zeit ein System zur allgemeinen Annahme zu empfehlen. Schon die bisherige Diskussion hat dem Drapersystem zu weiterer Verbreitung geholfen und es wird ganz von selbst in den nächsten Jahren die herrschende Stellung einnehmen. Doch ist das Studium der Sternspektren zu sehr im Fluss, als dass man jetzt abschliessende Bestimmungen treffen könnte.

Ueber die Zukunftsprojekte möchte ich folgendes sagen. Es ist das Verdienst vor allem der Harvard Sternwarte, nachgewiesen zu haben, dass die Sternspektren sich in eine Serie einordnen lassen, in der die Elemente sich nach einer festen Reihenfolge ablösen. An der Spitze steht das Helium, dann folgt der Wasserstoff, darauf Calcium, schliesslich Titan und Eisen, bis sehr zahlreiche Elemente und Banden auftreten. Es ist dies nebenbei bemerkt dieselbe Reihenfolge der Elemente, die man beim Eindringen in die Sonnenatmosphäre antrifft. Diese Gesetzmässigkeit steht fest für die kräftigen Linien, die in Spektren geringerer Dispersion sichtbar sind. Sehr wenig untersucht ist bisher das Verhalten der feineren Linien, die auf den Spektrogrammen grösserer Dispersion hervortreten. Es ist möglich, dass diese sich nicht mehr in eine Serie einordnen lassen—Frost und Adams finden z. B. für die Orion-Sterne aus den feinen Linien zwei Parallelserien. Vielleicht liegen die Verhältnisse in Wirklichkeit noch komplizierter. Die Draperklassifikation stellt die Spektra dar als Funktion einer Variabeln, die man als "Entwicklungsstadium" bezeichnen kann. Bei Miss Maury tritt bereits als zweite Variabel die Linienbreite hinzu, die nach den Untersuchungen von Hertzsprung ein physikalisch sehr wichtiges Kriterium ist. Es ist sehr wohl möglich, dass man noch mehr Variabler bedarf, um alle feinen Variationen der Spektra darzustellen. Ich möchte aber vermuten, dass die Anzahl der Variabeln begrenzt ist und vielleicht die Zahl 3 nicht überschreitet. Ich glaube nämlich nicht gern, dass die Verteilung der Elemente auf den Sternen zufällig ist, vielmehr möchte ich annehmen, dass die Mischung der Substanz der Sterne aus den chemischen Elementen überall die gleiche oder höchstens

eine vom Alter der Sterne gesetzmässig abhängige ist. Ist letztere Voraussetzung aber richtig, so kann das Spektrum eines Sterns schliesslich von nichts anderem mehr als seiner Masse, seinem Alter und seiner Temperatur (seinem Energiegehalt) abhängen.

Das Programm, das sich hieraus ergibt und das vor einem internationalen Beschluss über eine Klassifikation der Spektraltypen durchgeführt sein sollte, ist: die Klassifikation der Spektraltypen in der von der Harvard-Sternwarte inaugurierten Weise nochmals unter Berücksichtigung von Spektren grosser Dispersion vorzunehmen. Das Potsdamer Observatorium würde hierzu nicht viel beitragen können, da die aufgenommenen Spektra sich auf eine verhältnismässig kleine Zahl von Sternen beziehen. Die meisten Observatorien, die Sterne von veränderlicher Radialgeschwindigkeit verfolgen, werden in ähnlicher Lage sein. Vielleicht würde die Herstellung einer internationalen Sammlung möglichst guter Diapositive von möglichst vielen Sternspektren, wie sie für Radialgeschwindigkeiten gebraucht werden, das beste Mittel sein, um rasch zu dem nötigen Material zu gelangen.

Potsdam
Dezember 16, 1910

Walter Sidgreaves

(1) Yes, as a preliminary working step.

(2) Only that I personally prefer *stellar names* as Altairian, etc., to numbers and letters.

(3) I hesitate much as to this. We are still in want of some indication in the life of a star of declining age, and distinction between heat effect and electrical effect.

(4) Yes, especially of the hydrogen, helium, magnesium, and calcium lines.

My preference for names in place of numbers and letters will be best understood by the remark that a stellar name is a picture of the spectrum to any stellar spectroscopist. Subdivisions or intermediaries can be expressed by steps 1, 2, 3, etc., toward the next name of the list.

Stonyhurst College, England
December 1910

V. M. Slipher

(1) Yes.

(2) Would it not be advantageous, in the long run, to have the letters denoting the spectral sequence arranged alphabetically, by using A in place of O, and call in C or D to take the place now held by A?

(3) If there is a general agreement among spectroscopists on a uniform system of classification I see no reason why its recommendation for adoption should be postponed, except it be for acquiring corroborative knowledge of unexplored parts of the spectrum, such as the region from λ 4900 to λ 6900. Co-operative work should supply these needed facts at an early date. A few years ago I made spectrograms of this region of certain typical stars, and I could probably arrange to carry the work farther and with higher dispersion.

(4) A symbol to indicate the width of the lines should prove to be useful, as for certain studies such knowledge is almost indispensable, and moreover the cause of the abnormal width needs investigation.

(5) In the classification should not the relative intensities of different regions of the continuous spectrum from type to type be carefully considered?

LOWELL OBSERVATORY
January 5, 1911

J. WILSING

Zu Gunsten der Einführung der dem Draper Katalog zu Grunde liegenden Klassification der Sternspectra spricht die bereits bestehende Geneigtheit zahlreicher Fachgenossen für diese Einteilung. Die notwendige Aenderung der Buchstaben hat nur formale Bedeutung.

Da indessen der praktische Wert jeder einzuführenden Klassification von der Homogenität abhängen würde, so wären zuvor auf jeder beteiligten Sternwarte bestimmte characteristische Normalspectra aufzunehmen, wie eine Anzahl solcher Spectra bereits in *Harvard Annals*, **56**, bezeichnet worden sind. Auf Grund der Vergleichung der von den verschiedenen Beobachtern angefertigten Originalnegative würde die Stellung dieser Spectra in der Klassification festzusetzen sein und als Richtschnur sie die weiteren Bestimmungen dienen können. Dabei würde sich auch ergeben, ob die Einführung noch weiterer Kennzeichen notwendig und möglich ist, ohne die Homogenität zu gefährden.

POTSDAM
Dezember 13, 1910

Reviews

Transactions of the International Union for Co-operation in Solar Research. Edited by ARTHUR SCHUSTER, Chairman of the Executive Committee. Manchester: The University Press (Sherratt & Hughes). Vol. I, 1906; Vol. II, 1908. 7*s.* 6*d.* each.

These handsome volumes, of about 250 octavo pages each, record in full detail the useful work accomplished at the first three meetings of this truly international astronomical society.

The first conference, held at St. Louis in connection with the exposition of 1904, was of course occupied with organization, but arrangements had been made in advance for the presentation of papers upon co-operation in solar work and upon standards of wave-lengths.

Careful preparations were made by the Executive Committee for the second conference, which was held September 27–29, 1905, at Oxford upon invitation of the authorities of New College. These preparations are described in the first volume, together with the minutes of the conference, and many important reports. A valuable paper by Professor A. Fowler is appended, treating fully of the observations of sun-spot spectra.

Vol. II is occupied with the third conference, which was very successfully held at Meudon, May 20–23, 1907. The reports by committees and individuals printed in the volume are as follows:

No. 1. Détermination de la longueur d'onde de la raie rouge du cadmium, étalon fondamental des longueurs d'onde. Par MM. R. Benoît, Ch. Fabry et A. Perot.

No. 2. Mesures de longueurs d'onde pour l'établissement d'un système de repères spectroscopiques. Par MM. Ch. Fabry et H. Buisson.

No. 3. Zur Ermittelung neuer Wellenlängen-Normalen. Von H. Kayser.

No. 4. Bericht über einige Arbeiten in dem Gebiete der Sonnenstrahlung 1905–7. Von Knut Ångström.

No. 5. Report on the Work Carried out in the Smithsonian Astrophysical Observatory. By C. G. Abbot.

No. 6. Report of Committee on Work with the Spectroheliograph.

No. 7. The Measurement of Solar Photographs Made with the Spectroheliograph. By George E. Hale.

No. 8. Report of Committee on Sun-Spot Spectra. Drawn up by the Secretary.

No. 9. Projet d'entente au sujet des observations à faire et de la publication des résultats obtenus. Par A. de la Baume Pluvinel.

No. 10. Au sujet de l'observation des protubérances pendant les éclipses totales du soleil. Note de A. Riccò.

No. 11. Enregistrement de la surface et de l'atmosphère solaire à l'observatoire de Meudon. Par M. H. Deslandres.

This list shows what a significant movement in physics and astrophysics has been begun by the Union.

Vol. II is adorned with handsome reproductions of some of M. Deslandres' excellent spectroheliograms.

The preparation of the third volume, giving a record of the meeting at Pasadena, August 31 to September 2, 1910, at which important results already accomplished were reported by the committees, is now in hand. Science is greatly indebted to the chairman of the Executive Committee, Professor Arthur Schuster, for the publication of these volumes. Enough has been said in this brief notice to indicate the importance of securing the series for the libraries of observatories and laboratories and of individuals interested in astrophysical progress.

F.

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XXXIII MAY 1911 NUMBER 4

THE POSITION OF CERTAIN CORONAL STREAMS ON THE ASSUMPTION THAT THE CORONA IS A MECHANICAL PRODUCT

By JOHN A. MILLER

INTRODUCTION

The present investigation had its genesis in an attempt to apply Schaeberle's mechanical theory of the sun's corona[1] to the streams of the corona of 1905. I decided, after some investigation, to modify, in some respects radically, the assumptions made by Professor Schaeberle and to consider the streamers of the corona as the projection on a plane of streams of particles, the motion of which is produced by ejection, by the rotation of the sun, by the attraction of the sun, and by the radiant pressure of the sun.

The theoretical results made it desirable to examine a great number of large-scale photographs of the corona. Director Campbell generously placed at my disposal the most excellent series at the Lick Observatory. The present paper is a discussion of streamers found on these plates. Briefly stated, the results of the investigation indicate a mechanical origin of the corona, at least thus far:

If the streamers of the corona are formed of particles acted on by the forces named above and under the assumptions stated more explicitly later, streams of a certain shape must theoretically

[1] J. M. Schaeberle, *A Mechanical Theory of the Solar Corona*, p. 47; total solar eclipse of December 1889, etc., Lick Observatory.

result. Streams of this shape are found on the plates, and, under the same assumptions, it has been possible to find their heliocentric positions and the magnitude of the radiant pressure acting on particles in them.

I am under many obligations. Director Campbell not only put at my disposal all the coronal photographs of the Lick Observatory, but aided me in many ways when I was measuring them. Professor Marriott of Swarthmore College aided me, not only in making the computations, but discussed with me the entire theory and suggested improvements in the methods of solution. Mr. C. J. Olivier very effectively aided me in the computations, as did my own students, Mr. J. H. Pitman, Miss H. W. Sheppard, Mr. G. C. Carr, and Mr. J. A. White, Mr. Pitman sharing in the computation of six streamers.

THE THEORY

If it is assumed that the sun's corona is caused by light emitted and reflected by particles ejected from the sun by forces which in general act along lines normal to the sun's surface and that these particles follow each other closely enough to make a continuous stream, it follows from the laws of mechanics that of two particles of the same stream that one has the smaller angular velocity which is farther from the sun. Moreover, if any particle, after leaving the sun's surface, is acted upon by forces the resultant of which passes through the center of the sun, it will move in a plane passing through the sun's center, tangent to the small circle on the sun's surface described by the point of ejection. The plane of the subsequent orbit of the particle will therefore be perpendicular to a meridian of the sun passing through the point of ejection. In Fig. 1, P_1 is the pole of the sun; C its center; the xy-plane, the plane of the sun's equator; P_0 is the point of ejection; CX is the radius of the sun passing through its west side. The particle ejected from the point P_0 will move in a plane passing through P_0C and perpendicular to the arc P_1P_0.

These assumptions require that the sun be a sphere and that the resultant of all forces of radiant pressure acting on a particle is normal to the sun's surface.

If we assume, further, that the resultant of all forces that act

on a particle after it leaves the sun's surface varies inversely as the square of the distance of the particle from the center of the sun, it (the particle) will describe a conic section with the center of the sun as one focus. That is, under these assumptions a stream of the corona is made up of particles each moving in a conic section but no two of which are moving in the same plane. In fact, the

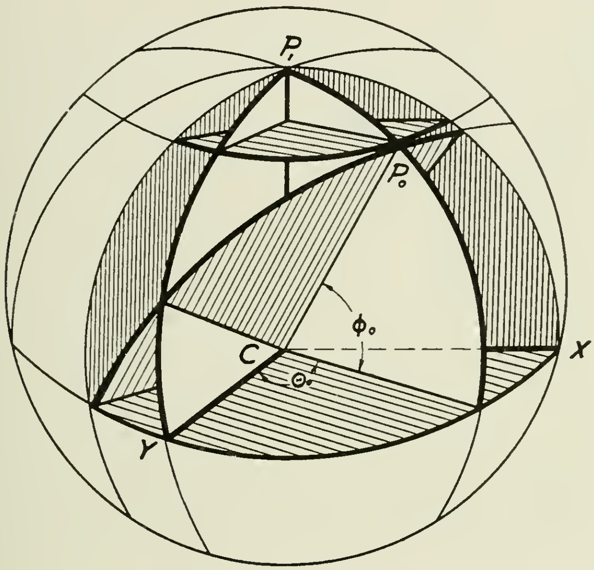


FIG. 1

planes of the orbits of the various particles will be tangent to a circular cone, the vertex of which is at the center of the sun and the base of which is a small circle described by the point of ejection. The stream itself is, of course, not a plane curve.

If what we see and photograph as a *streamer*[1] of the sun's corona is one of those *streams* of particles projected orthogonally on a plane perpendicular to the line joining the observer's eye to the center of the sun, it is possible to prove that some of these streamers

[1] Throughout this paper I shall mean by a *stream* the actual stream of the particles in the corona. By a *streamer* I shall mean the orthographic projection of a stream. By the *base* of a stream, I shall mean the intersection of the stream and the margin of the moon's shadow.

will curve throughout their entire length away from the projection of the pole of the sun, others will curve toward it, while a few others will curve first toward it and afterward away from it, or vice versa. That is, from the center of the sun, C, draw radii vectores to the successive points a, b, c in the streamer (Figs. 2, 3, 4). Choose as the initial line CW, W being the west side of the sun; then the vectorial angle WCa always decreases as the extremity of the radius vector describes the streamer, as in Fig. 2. Or it always increases as in Fig. 3; or it increases first and then decreases, or vice versa as in Fig. 4. It is hardly necessary to say that in these drawings the amount of curvature is exaggerated. I shall consider here the class of streamers last described — those shown in Fig. 4. For this class of streamers it is possible to find the heliocentric latitude and longitude of a particle at that point of the streamer at which the radius vector is tangent; as, for example, the point P in Fig. 4. We can find the distance of this particle from the

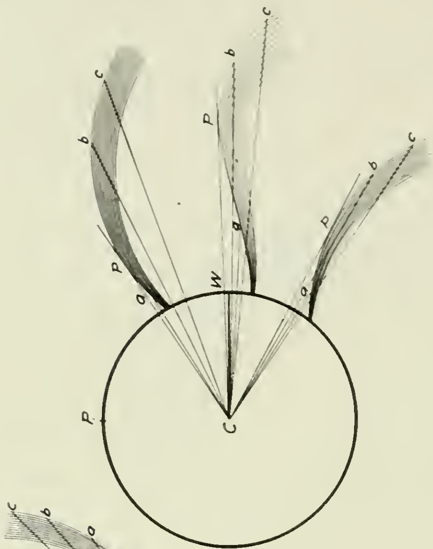


FIG. 4

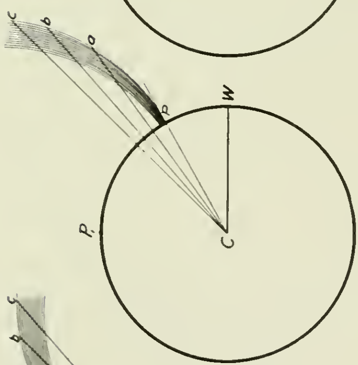


FIG. 3

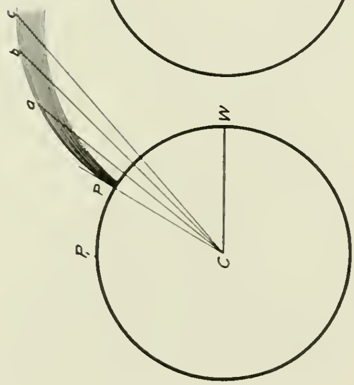


FIG. 2

sun, the point on the sun's surface from which the particle was ejected, the time it was ejected, and hence the position of the point of ejection at any time; we can also find the velocity with which the particle was ejected, the velocity of the particle at any time, the orbit of each particle in the stream, and the resultant of all forces acting on the particle, and, if we assume the law of the force, we can find the magnitude of any repulsive force, such as light or electric pressure, that may exist.

If we pass a plane through the line joining the center of the sun and the observer and through the west side of the sun; and another plane through the same line and a point on the *stream;* the vectorial angle to the corresponding point on the *streamer* measures the angle between these planes. This angle can be measured in the photograph.

We shall now make a mathematical statement of the problem.

Choose as

z-axis, the axis of the sun;

x-axis, the equatorial diameter passing through the west side of the sun;

y-axis, a line perpendicular to the xz-plane.

Let x, y, z be the rectangular heliocentric co-ordinates of any point;

Let ϕ_1 be its heliocentric latitude;

Let θ_1 be its heliocentric longitude, measured from the yz-plane, positive in the direction of the sun's rotation;

Let R_1 be its radius vector;

Let A_1 be the vectorial angle of the point on the streamer corresponding to this point;

Let λ be the angle between the z-axis and the line through the observer and the center of the sun;

Let the x- and the z-axis project into a ξ- and η-axis respectively;

Let A be the angle that the projection of a radial line makes with the ξ-axis.

Then,
$$\left.\begin{aligned} x&=R_1 \cos \phi_1 \sin \theta_1, \\ y&=R_1 \cos \phi_1 \cos \theta_1, \\ z&=R_1 \sin \phi_1 \end{aligned}\right\}$$

$$\xi=x,$$

$$\eta=z \sin \lambda-y \cos \lambda,$$

$$\tan A_1=\frac{\sin \lambda \tan \phi_1-\cos \lambda \cos \theta_1}{\sin \theta_1} \qquad (a)$$

We know that as we pass successively from one point of the streamer to another, starting at the base, θ_1 constantly decreases. Therefore if

$$\frac{d}{d\theta_1}(\tan A_1) \lesseqgtr 0$$

for one point on the streamer, it is for every point unless for some point on the stream the quantities θ_1 and ϕ_1 satisfy the equation

$$\frac{d}{d\theta_1}(\tan A_1) = 0 \qquad (b)$$

Differentiating (a) with regard to θ_1 we see that, since

$$\frac{d\phi_1}{d\theta_1} \mp \infty$$

$\frac{d}{d\theta_1}(\tan A_1)$ can be infinity if, and only if, $\phi_1 = \pm 90°$; or if $\theta_1 = 0°$ or $180°$. That is, the stream is either at the pole of the sun or directly behind or in front of the sun. In either case the vectorial angle is $90°$; and, except in rare cases, the stream would be invisible. We shall therefore leave this case out of consideration. Hence the streamer curves in one direction only, unless an equation (b) exists. Since (a) contains only two unknowns one would surmise that (b), if it exists, would be the additional equation necessary to solve for these quantities. This is indeed true, a statement I now propose to prove.

Let P be that point in the *stream* that is projected into that point on the *streamer* at which the radius vector (in the photograph) is tangent to the *streamer*.

Let P_0 be the point on the sun from which P was ejected;

Let P' be a particle at any point in the streamer;

Let P'_0 be the point of ejection of P';

Let t be the time at which the photograph was made;

Let t_0 and t'_0 be the time when P and P' left the sun respectively;

Let θ, ϕ, R, w = respectively, the heliocentric longitude, heliocentric latitude, radius vector, and true anomaly of P at the time t;

Let θ_0, ϕ_0, R_0, w_0 = respectively like values for P_0 at time t_0;

Let θ', ϕ', R', w' = respectively like values for P' at time t;

Let θ'_0, ϕ'_0, R'_0, w'_0 = respectively like values for P'_0 at time t'_0;

Let P and P' project respectively into π and π';

Let ρ and ρ' be the radius vector of π and π' respectively;
Let A and A' be the vectorial angles of π and π' respectively;
Let v_2 = velocity of ejection;
Let v_1 = velocity of the particle due to the sun's rotation;
Let V = velocity with which the particle left the sun;
Let e = eccentricity of the orbits;
Let a = semi-major axis of the orbits;
Let ω = angular velocity of the sun;
Let n = the mean motion in the orbit;
Let C^2 = the constant entering into the differential equations of motion;
Let ψ = angle between the normal to the sun and the initial velocity.

In the computations we have chosen as P' the point which projects into the base of the streamer. If we assume that P and P' have been acted on by the same forces, the particles are describing equal orbits. Hence a, e, and w_0 are the same for every particle of a given stream.

Let P_1 be the pole of the sun.

Project radially from the center of the sun the points P_1, P_0, P'_0, P, and P' into the points p_1, p_0, p'_0, p, and p' respectively on the surface of a sphere concentric with the sun. Then

$$\begin{aligned}
\text{arc } p_1p_0 &= 90^\circ - \phi_0, \\
\text{arc } p_1p &= 90^\circ - \phi, \\
\text{arc } p_1p' &= 90^\circ - \phi', \\
\text{arc } p_0p &= w - w_0, \\
\text{arc } p'_0p' &= w' - w_0.
\end{aligned}$$

We obtain from the spherical triangle p_0p_1p the following relations:

$$\sin (w - w_0) = \sin (\theta - \theta_0) \cos \phi \qquad (c)$$

$$\tan (w - w_0) = \tan (\theta - \theta_0) \cos \phi_0 \qquad (1)$$

$$\tan \phi = \cos (\theta - \theta_0) \tan \phi_0 \qquad (2)$$

$$\sin \phi = \cos (w - w_0) \sin \phi_0 \qquad (3)$$

There are four similar relations for the triangle p'_0p_1p'.

Projecting R and ρ on the x-axis we get

$$\rho \cos A = R \cos \phi \sin \theta \qquad (4)$$

Similarly,

$$\rho' \cos A' = R' \cos \phi' \sin \theta' \qquad (5)$$

Now θ and ϕ satisfy the equation

$$\tan A = \frac{\sin \lambda \tan \phi - \cos \lambda \cos \theta}{\sin \theta}$$

$$= \frac{\sin \lambda \cos (\theta - \theta_0) \tan \phi_0 - \cos \lambda \cos \theta}{\sin \theta} \qquad (6)$$

The co-ordinates of a second point on the stream nearer than P to the sun satisfy the equation

$$\tan A + \Delta \tan A = \frac{\sin \lambda \cos (\theta + \Delta\theta - \theta_0 - \Delta\theta_0) \tan \phi_0 - \cos (\theta + \Delta\theta) \cos \lambda}{\sin (\theta + \Delta\theta)}.$$

Subtracting these two equations, dividing by $\Delta\theta$, taking the limit and putting

$$\frac{d}{d\theta}(\tan A) = 0,$$

we obtain

$$\frac{d\theta_0}{d\theta} = \frac{\cos \theta_0 - \cot \lambda \cot \phi_0}{\sin \theta \sin (\theta - \theta_0)} \qquad (d)$$

Also the co-ordinates of P and the second point on the streamer nearer the sun satisfy respectively the equations

$$\tan (\theta - \theta_0) \cos \phi_0 = \tan (w - w_0),$$

and

$$\tan (\theta + \Delta\theta - \theta_0 - \Delta\theta_0) \cos \phi_0 = \tan (w - \Delta w - w_0).$$

Treating these equations in the same way we obtain

$$1 - \frac{d\theta_0}{d\theta} = \frac{-\sec^2 (w - w_0) \dfrac{dw}{d\theta}}{\cos \phi_0 \sec^2 (\theta - \theta_0)} = \frac{\sec^2 (w - w_0)}{\sec^2 (\theta - \theta_0) \cos \phi_0} \quad \frac{\dfrac{dw}{dt} \cdot \dfrac{d\theta_0}{d\theta}}{\omega}.$$

Now w is the true anomaly of a particle that has moved in its orbit for a time $(t - t_0)$, and its radius vector has described the angle $(w - w_0)$. If the particle had moved for the time $t - \Delta t - t_0$, it would have moved through an angle $w - \Delta w - w_0$. Now the second particle left the sun at a time $t_0 + \Delta t$. It has therefore moved in its orbit for a time $t - \Delta t - t_0$. Its true anomaly is therefore $w - \Delta w - w_0$. That is, the difference of the true anomaly of the two successive particles in a stream is the same as the difference of the true anomaly of two positions of a particle in its orbit at the time t and $t - \Delta t$. That is, as we trace a streamer, tracing toward the sun, the true

anomaly of the successive particles that we encounter changes at the same rate as the true anomaly of a particle moving in its orbit. Hence we may write

$$\frac{dw}{dt}=\frac{C^2\sqrt{a(1-e^2)}}{R^2}.$$

This is merely the numerical value of $\frac{dw}{dt}$, the sign being given to Δw.

Now,

$$n^2a^3=C^2$$

$$\tan\psi=\frac{v_1}{v_2}$$

$$4a^2e^2=R^2_0+(2a-R_0)^2+2R_0(2a-R_0)\cos 2\psi$$

$$v_1=\omega R_0\cos\phi_0$$

$$v_1^2+v_2^2=V^2$$

$$V^2=C^2\left[\frac{2}{R_0}-\frac{1}{a}\right]$$

We shall now put $R_0=1$.

Substituting from the above equations we obtain,

$$\frac{dw}{dt}=\frac{\cos\phi_0\,\omega}{R^2}$$

Reducing we obtain

$$\frac{d\theta_0}{d\theta}=\frac{\rho^2\cos^2 A}{\rho^2\cos^2 A-\sin^2\phi_0\cot^2\phi\sin^2\theta\cos^2(\theta-\theta_0)}\qquad(e)$$

Equating (d) and (e) and eliminating by the preceding equations, remembering the identity,

$$\cos\theta_0=\cos(\theta-\theta_0)\cos\theta+\sin(\theta-\theta_0)\sin\theta$$

we obtain

$$+\tan^2(\theta-\theta_0)\left[-P_1\tan^3\theta+g\tan^2\theta+k\tan\theta+\frac{P_3}{2}\right]$$
$$+\tan(\theta-\theta_0)[-P_7\tan^3\theta-P_8\tan^2\theta]+[-f\tan^3\theta$$
$$+h\tan^2\theta+j\tan\theta+m]=0\qquad(7)$$

where

$$f=P_1+P_{10}-P_8,\qquad j=2P_1-P_5-P_{10},$$
$$g=P_2-P_3,\qquad k=2P_1-P_5,$$
$$h=P_2+P_{13}-P_3-P_7,\qquad m=\frac{P_3}{2}+P_{13},$$

and where

$$P_1=\rho^2a^2b^2d, \qquad P_7=bc^2, \qquad a=\cos A,$$
$$P_2=\rho^2a^2b^3, \qquad P_8=c^2d, \qquad b=\tan A,$$
$$P_3=2\rho^2a^2bd^2, \qquad P_{10}=\rho^2a^2c^2d, \qquad c=\sin\lambda,$$
$$P_5=\rho^2a^2d^3, \qquad P_{13}=\rho^2a^2bc^2, \qquad d=\cos\lambda.$$

We have also

$$R=\frac{a(1-e^2)}{1+e\cos w} \tag{8}$$

$$R'=\frac{a(1-e^2)}{1+e\cos w'} \tag{9}$$

$$1=\frac{a(1-e^2)}{1+e\cos w_0} \tag{10}$$

$$\tan A'=\frac{\sin\lambda\tan\phi'-\cos\lambda\cos\theta'}{\sin\theta'} \tag{11}$$

$$n(t'_0-t_0)=\int_{R'}^{R}\frac{RdR}{a\sqrt{a^2e^2-(a-R)^2}} \tag{12}$$

$$\theta'_0-\theta_0=\omega(t'_0-t_0) \tag{13}$$

Now a large change in θ produces a small change in ϕ when $\theta-\theta_0$ is small; except in very high latitudes. Hence if we choose for P' the point in the streamer at the edge of the moon's shadow, we can without sensible error put $\phi'=\phi_0$.

The solution requires one other equation involving a linear quantity. I have chosen as this quantity the aphelion distance, $a(1+e)$. If we measure the greatest extension of the streamer, we know that $a(1+e)$ cannot be less than this measured distance. Its maximum value is much less definite. The stream may not have reached its greatest height, or the entire stream may not have been photographed. In the computations of the streamers discussed later there are certain limits beyond which, in a given case, $a(1+e)$ cannot go; and actual computations show that one can change $a(1+e)$ within rather large limits without effecting large changes in either θ or ϕ. However, a change in $a(1+e)$ does change $t-t_0$ and hence changes the position of the point of ejection at any time. This measured quantity, $a(1+e)$, completes the number of equations required to solve our problem.

Since, however, some of the equations are transcendental, it is not easy to solve them. Analytically, we should, in general, expect that more than one set of values would satisfy them; but in addition to these equations, the values admissible to our problem must satisfy the inequalities,

$$\left.\begin{array}{l} \theta<\theta' \\ \left.\begin{array}{l} |\phi'|<|\phi_0| \\ |\phi|<|\phi'| \end{array}\right\} \text{if } (\theta-\theta_0)<90^\circ \\ R'<R \\ \tan^2\phi'<\dfrac{\cos^2\lambda+\tan^2 A}{\sin^2\lambda} \end{array}\right\} \qquad (14)$$

The method that we adopted in the solution of these equations is as follows: We substituted in (7) a value of tan θ arbitrarily chosen and solved for tan $(\theta-\theta_0)$. To guide us in our choice of tan θ we formed the discriminant of equation (7), regarding tan θ as a part of the coefficients. Since no value of tan θ that made this discriminant negative was admissible, the range of our choice was thereby limited. Having found a value of tan θ that gave real values of tan $(\theta-\theta_0)$, we solved the equations *seriatim*, having care that none of the values thus found violates the inequalities (14). By virtue of this arbitrary choice of tan θ we have one more equation than is necessary for solution. Accordingly in our trial solution we did not make use of equation (13). The true solution must, however, satisfy this equation and therefore we used it as a check. That is, the test of the solution is that the *angular velocity of the sun multiplied by the difference between the time of ejection of two particles must equal the difference of the longitudes of the points of ejection of these particles.* If this check was not satisfied we rejected the solution and tried another value for tan θ.

This theory is, in some respects, the same as that given by the writer in the *Astrophysical Journal*, **27**, 286–295, 1908. However, equations (4) and (5) of that article are not true. They have been replaced by (7) of the present paper. I have also improved the method of the solution of these equations. For these reasons I have presented it here giving the details of the solutions.

APPLICATION OF THIS THEORY TO THE LICK PHOTOGRAPHS

This theory had been applied, with gratifying results, to the large-scale photographs made by Professor Cogshall and myself in 1905.[1] At the invitation of Director Campbell, I spent a part of the summer of 1909 at Lick Observatory measuring the very complete set of large-scale photographs made, since 1893, by eclipse parties of that observatory. This afforded a severe test of this theory, because (1) too much cannot be said commending the uniform excellence of these plates; (2) the series is very complete: it contains photographs of every total solar eclipse, except one, when it was cloudy, that has occurred since 1893, viz., those of 1893, 1898, 1900, 1901, 1905, and 1908; (3) the period covers more than one sun-spot cycle; (4) the different eclipses occurred when the axis of the sun was inclined at various angles to the line of sight, the inclination being approximately 82° in 1905; 95° in 1893; 91° in 1900, and for the other dates somewhere within those limits; (5) all the large-scale photographs were compared with others made with lenses of shorter focus. The plates made in 1905 by Professor Campbell's party in Spain were checked by those made by Professor Hussey in Egypt with an instrument that was a duplicate of the one used in Spain. They were also checked by the plates made by Professor Cogshall and myself also in Spain but with an instrument of different type—the Lick photographs all being made by the familiar ingenious method devised by Schaeberle. Our photographs were made with a horizontal telescope fed by a coelostat. The plates of 1900 were checked by those made by the Smithsonian Institution, loaned to me by the director.

The measures of the plates were necessarily approximate compared with measures of precision. I measured the original negatives on the glass side. The film of the negative rested on a ground glass behind which were electric lights whose intensity and position the operator could change at will. The angles were measured with a protractor reading (by vernier) to minutes. All the streamers except three were measured on three different days, and on each day each measure was repeated from three to seven times.

[1] *Astrophysical Journal*, 27, 286, 1908.

The equator and the center of the sun were determined by the orientation of the plate at the time of the exposure, and were determined once only.

From the entire series I selected 22 streamers that seemed to have the shape required. I afterward rejected six of these either because I was uncertain whether or not the streamers really reversed their direction of curvature, or because it was impossible to tell where their bases came to the margin of the moon's shadow. Most of these were verified either by Director Campbell, Dr. Albrecht, or Mr. Olivier. There are other streamers that might have been added to this list, but I regarded them as being too uncertain in one or more of the essential particulars. The sixteen streamers chosen are, in my judgment, all that should be included, and there is no doubt that all of these are of the type described in Fig. 4.

Below is a résumé of the measures. A and A' were measured from the western extremity of the sun's diameter, positive in a counter-clockwise direction, and ρ and ρ' were measured in terms of the radius of the sun.

TABLE I

Streamer	A	A'	λ	Log ρ	Greatest Extension of Streamer in Inches
I 1893....	193° 30′	197° 29′	95 21′ 29″	.18002	7.5
II 1893....	197 34	196 06	95 21 29	.31054	7.5
I 1898....	− 4 43	− 7 46	95 21 12	.17142	5.5
II 1898....	17 15	18 48	95 21 12	.29315	6.5
I 1900....	−23 30	−22 14	90 58 02	.04135	5.3
II 1900....	−27 32	−24 59	90 58 02	.04659	5.3
I 1901....	188 35	173 45	92 13 17	.25230	5.0
II 1905....	105 30	103 34	82 49 56	.10480	4.2
III 1905....	263 53	267 58	82 49 56	.10032	3.7
V 1905....	−20 21	−24 10	82 49 56	.24494	6.7
VI 1905....	−30 54	−36 48	82 49 56	.18842	6.7
I 1908....	253 00	251 55	93 21 48	.14768	5.0
II 1908....	244 25	232 31	93 21 48	.18638	8.5
III 1908....	241 11	227 45	93 21 48	.21249	8.5
IV 1908....	198 42	200 55	93 21 48	.18146	4.9
VI 1908....	190 26	191 35	93 21 48	.15335	5.5

Below I append some notes made while measuring these streamers.

1893

This eclipse lasted 186 seconds. Eight plates were exposed. I measured No. 4, which was exposed for eight seconds, beginning 50 seconds after totality began. Plate No. 5 and the plates made with the Dallmeyer lense were used as checks. The streamers on these plates were the most perplexing of the series. The corona around the entire edge of the moon was complex, being made up of a great number of interlacing streamers. It was oftentimes difficult, sometimes impossible, to decide certainly which of two streamers from their point of intersection proceeded in a given direction. Two streamers seemed to me to meet the conditions. They leave the moon's margin at nearly the same place and intersect each other twice. Both are sharp on the convex side of the streamer. Director Campbell expressed some doubt as to the existence of streamer No. II. It was my judgment, however, that it was more certain than streamer No. I. A reference to Table II shows that No. II could not be solved.

1898

This eclipse lasted 159 seconds. Twelve plates were exposed. I measured No. 5, exposed for eight seconds, beginning 43 seconds after totality began. The checks were made with No. 6, and the Floyd plates. Both streamers, No. I and No. II, are well defined. They are sharp on the convex side. The bases are well defined.

1900

This eclipse lasted 90 seconds. Ten plates were exposed. I measured No. 5, exposed for 16 seconds, beginning 42 seconds after totality began. The checks were with No. 2, the Floyd plates, and a plate made by the Smithsonian Institution with the 135-foot telescope. Streamers No. I and No. II are between two prominences, are definite, but are not smooth, being somewhat crinkly. They curve first toward the greater prominence, then away from it.

1901

This eclipse lasted 6 minutes, 9 seconds. Twelve plates were exposed. I measured No. 7, exposed for 150 seconds. This

streamer is well defined and very sharp on the convex side. It apparently originated about 12° south of the southern extremity of a large disturbed region.

1905

This eclipse lasted 3 minutes, 45 seconds. Ten plates were exposed. I measured No. 4, exposed for 8 seconds, beginning 24 seconds after totality began. Streamer No. I near No. II was very faint and was not measured. Streamer No. II is well defined though faint. It is sharpest on the concave side. Streamer No. III is a thin streamer well defined at the base and indeed to the point where the streamer reverses its direction of curvature; the top is less definite. Streamers No. V and No. VI are both very definite and are sharp on the convex side.

1908

This eclipse lasted 3 minutes, 52 seconds. Six plates were exposed. I measured plate No. 5, exposed for 64 seconds. Streamer No. I is fairly well defined. Streamer No. IV is of a short bushy type resembling the Bredichin type of comets' tails that are composed of the heavier metals. Streamers No. II, No. III, No. V, and No. VI are clearly defined. All are sharp on the convex side. Streamers No. II and No. III are on the southern extremity of a region that covers an arc of the edge of the shadow about 50° in length. Streamers No. IV, No. V, and No. VI are on the northern extremity of this region. This entire region is covered with complex, ill-defined coronal masses, while the streamers that bound it are clearly defined. Both groups of streamers curve first away from this region, then toward it, so that if the tops of the streamers were produced in the general direction that they are taking there, the two groups would cross each other.

In almost every instance the streamers are sharper on the convex side than on the concave side; that is, if the streamer curves first away from the pole and then toward it, the streamer is sharpest on the side away from the pole. There were two exceptions—noted above—to this statement. In many instances, the larger, well defined streamers were less dense along a line defining the middle of the streamer than on the edges parallel to this line.

TABLE II

Streamer	θ	ϕ	θ_0	ϕ_0	θ'	ϕ'	$\theta-\theta_0$	R	R'	a	e	F*	v_1†
I 1893...	243° 26′	− 9° 51′ 20″	174° 41′	−25° 36′ 35″	286° 50′	−18° 15′ 40″	68° 45′	1.6702	1.0617	5.488	0.822	0.9970	1.6
II 1893...	......	....	......	will not solve	......	....	.....	......	......	.....	.....	......	....
I 1898...	35 45	− 7 16 20	6 54	− 8 6 10	86 34	− 8 6 10	28 51	2.5509	1.0026	3.133	.915	.9937	1.89
II 1898...	99 21	17 54 10	78 2	19 8 10	117 10	19 8 10	21 25	1.9983	1.1504	2.612	.914	.9931	2.09
I 1900...	45 00	−17 42 50	10 00	−21 18 50	60 27	−20 00	35 00	1.4794	1.1066	2.353	.700	.9962	0.64
II 1900...	50 12	−22 21 40	8 2	−29 1 50	71 14	−24 4 10	42 10	1.3893	1.0484	2.431	.645	.9967	0.37
I 1901...	296 34	− 8 38 20	171 55	14 57	327 29	1 30	124 39	2.0000	1.8504	1.850	.459	.9964	very small
II 1905...	248 12	73 16 30	199 50	78 42 40	271 43	76 32 30	48 22	1.2731	1.0084	2.1978	.820	.9988	0.31
III 1905...	251 35	−83 38	191 44	−86 47 20	320 49	−86 47	59 50	1.2760	1.0453	1.5544	.930	.9994	0.16
V 1905...	92 52	−20 47	60 11	−24 17	121 37	−24 17	32 41	1.7649	1.2041	3.262	.84	.9958	0.97
VI 1905...	109 10	−31 25 28	82 40	−34 19	126 11	−34 19	26 30	1.6425	1.2307	2.1870	.829	.9955	1.41
I 1908...	243 26	−71 00	223 11	−72 5 50	254 54	−71 31 50	20 15	1.4107	1.0034	2.043	.958	.9966	1.06
II 1908...	......	....	......	will not solve	......	....	.....	......	......	......	....	.....	....
III 1908...	......	....	......	will not solve	......	....	.....	......	......	......	....	.....	....
IV 1908...	248 12	−16 19 50	199 56	−23 45 25	278 44	−21 10 20	48 16	1.6144	1.1006	2.402	.665	.9964	0.48
VI 1908...	251 34	− 7 37 20	208 27	−10 23 20	278 36	−10 23 20	43 07	1.4888	1.0837	4.444	.800	.9966	0.20

* F is the repulsive force acting on these particles. It is measured in terms of the attractive force of the sun.

† v_1 is expressed in miles per second.

This was more noticeable near the sun than at some distance from it. The phenomenon might be accounted for if the streams were hollow. A glance at Table I shows that there were several pairs of streamers.

The two groups of streamers mentioned in connection with the discussion of the streamers of 1908 led me to examine the plates with the view of determining if the type of streamers that I was investigating occurred only, or chiefly, near the disturbed regions of the corona, and therefore were probably the result of local disturbances. There are no streamers of this type in the neighborhood of the disturbed region in the southeast quadrant of the corona of 1908. Neither are there streamers of this type near the disturbed region in the corona of 1905. In the corona of 1901, there is a disturbed region. The one streamer of this year that I measured is about 12° from one edge of this region; on the other side of this region there is no evidence that the coronal streams are affected. It seems to me that the only instances in which we can even suspect that the shape of the stream is due to local disturbances are in the cases of 1900 and 1908. In the former case the streamer lies between the projection of two prominences. There is no way of knowing, however, whether the prominences are on the same side or opposite sides of the sun, and it is equally uncertain regarding the streamers of 1908.

The methods developed in the earlier part of this paper were applied to the sixteen streamers with the results shown in Table II.

The results are much as we should expect. Of the thirteen streamers that gave real solutions, ten originated in the sun-spot zones, three did not. Eight originated below the equator, five above. If this mechanical theory be true, we should, in general, expect that the farther from the equator of the sun the streamer originated, the larger would be the eccentricity of the orbits of the particles composing the stream. Computations bear this out. The average of the eccentricities of the orbits of the particles in the thirteen streams is 0.794, the average of the eccentricities of the orbits of the particles in the three streams nearest the poles is 0.903. The average of the aphelion distance of the particles is 5.1 radii of the sun.

There are three streamers that do not lend themselves to this treatment. One of these, streamer No. II, 1893, is very indefinite; the other two are well-defined streamers apparently meeting all the conditions. They are apparently near each other and resemble each other strikingly. It is possible that their shape may have been influenced by local causes. If this is not so, it would show that at least not all the streamers are caused in the way that we have assumed. However, I believe that the non-agreement of some streamers with theory is what we might naturally expect. The surprise is that so large a percentage of them conformed to the theory.

If this mechanical theory is true, we should expect some streamers to cross the equator. For example, when the eclipse of 1893 occurred, the north pole of the sun was inclined away from us. Hence, if the projections of any stream crossed the projection of the east end of the sun's equator, we should expect them to cross from north to south. That is, the part of the streamer nearest the sun would project into a point north of the equator and those farthest from the sun south of it. With this in view, I examined the plates of each eclipse (except 1905) with the following results:

1893. None crossed east side. On the west side, four crossed the equator from south to north; that is, the base of the streamer is south of the west end of the sun's equator and the top is north of it, as they should be. They are all well defined.

1898. None on the east side. Two streamers on the west side cross from south to north as they should.

1900. No streamers cross the equator.

1901. On the east side, one well defined streamer crosses the equator as it should. On the west side, there are possibly two ill-defined ones. They cross as they should *not*.

1908. Two on the east side cross as they should, none on the west. That is, nine well defined streamers cross the equator, qualitatively at least, as a mechanical theory of the corona would require, and two doubtful ones cross it in the opposite direction.

THE CONVERSE PROBLEM

As a further verification of this theory I attempted two other problems, or rather, two other phases of this problem. The first

may be stated thus: Having assumed for a, e, ϕ_0, and θ_0, the set of values obtained for one of the streamers in the foregoing computation, I computed the rectangular co-ordinates of a series of points, which, according to the theory, should be points of the streamer. If one drew a curve through these points, it should be an exact reproduction of the streamer measured on the photographs. The method I adopted is as follows:

Let T = time of the perihelion passage of a particle;
E = eccentric anomaly of the particle at time t.

Then, using the previous notation,

$$n=\frac{\omega \cos \phi_0}{a^2\sqrt{1-e^2}} \tag{15}$$

$$n(t-T)=E-e \sin E \tag{16}$$

$$\tan \frac{w}{2}=\sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \tag{17}$$

For a given value of $t-T$ we obtain E from (16); w from (17); w_0 from (10); ϕ and θ_0 from (1) and (3) respectively. By assigning a series of values to $t-T$ we obtain a series of values for each of the other quantities; θ_0 is given for one point, and hence a certain value of $\theta-\theta_0$ by the previous computation. The θ_0 for any other point may be obtained by multiplying ω by the interval that has elapsed from the time one of the particles left the sun until the other did and adding or subtracting the product to the given θ_0. We may now obtain a series of values for θ and hence for ξ and η.

I applied the above theory to several of the streamers the orbits of the particles of which we had computed; I found in every instance when the streamer was plotted that the reversal of curvature was exactly as I had measured it on the plate. That is, A, A', and ρ measured as they were on the original photograph had been reproduced in the drawing. There was one feature of the streamers I could not reproduce in this way. The shorter streamers, those that did not go beyond the inner corona, could be reproduced in every detail; but the very long ones would turn back before they reached the distance from the sun shown in the photograph. On Fig. 6 (see p. 327) I have plotted three streamers that are reproduced from the computed value. They are

Streamer No. 20 on Fig. 6 which is Streamer No. III, 1905.
“ “ 21 “ “ 6 “ “ “ “ II, 1898.
“ “ 23 “ “ 6 “ “ “ “ V, 1905.

Streamer No. III 1905 is plotted longer than it appeared on the photograph, but the other two are about three-tenths of a sun's radius shorter than they were on the photograph. Moreover it is impossible with the computed values of a, e, and ϕ_0 to produce a streamer that, when plotted, will not turn back toward the sun somewhere near the extremity of the streamer in the drawing. One may find a stream as long as he chooses, but this long stream will not always project into a long streamer.

This discrepancy may be due to one of many causes. It may be due to errors of measurement. For example, in the case of streamer No. V 1905, if the measures of A and A' had been such that $A-A'$ had been decreased by half a degree, the streamer would have reproduced in every detail. Or, if for a given A, ρ had been measured longer, the computed streamer would have been as long as the one on the photograph. Now A', and ρ also, are usually difficult to measure accurately; but it is hardly probable that one would have made a mistake in the wrong direction in the measurement of every long streamer. Or, this discrepancy may be due to the fact that the radiant pressure near the sun's surface does not vary inversely as the square of the distance from the sun's center. Or it may be due to the fact that the particles in the part of the stream farthest from the sun were ejected with a greater force than those near the base of the streamer; or it may be that the particles in the stream are moving in a resisting medium and that this is denser in the inner than in the outer corona. And it may be (and I think this the most likely) that the particles in the end of the stream are finer than those at its base and hence the radiant pressure on them is greater.

But if it be any of these causes, the method of finding the heliocentric position of these streamers is valid. It becomes necessary only to replace equation (8) by the equation of the true orbit. Moreover the fact that all the theoretical results agree so perfectly with the measured values, with the exception noted, shows that the orbits are not very different from an ellipse. It is also worthy

of remark that if a stream of particles, after being ejected from the sun's surface, is acted upon by the attractive force of the sun only, these particles cannot arrange themselves in streams of the shape we have measured.

The statement is susceptible of mathematical proof.

Let the equations of motion of the particles be

$$\frac{d^2x}{dt^2} = -C^2\frac{x}{R^3},$$

$$\frac{d^2y}{dt^2} = -\frac{C^2y}{R^3},$$

$$\frac{d^2z}{dt^2} = -\frac{C^2z}{R^3},$$

where the mass of the sun is unity. Having the elements of the orbit we can determine C, which should be, but never is, equal to the Gaussian Constant—in fact, not even equal approximately to that constant. The reason of course is this: In order that the particles may reach the distances from the sun's surface measured on the photographs, the particles must be ejected with velocity so great that the stream for distances such as we have photographed must be straight lines and hence the streamers will be straight lines. This has long been recognized. Ranyard[1] discussed this at some length in his very exhaustive report to the Royal Astronomical Society. He says:

> We have, therefore, evidence that many rays, especially those seen towards the edges of the synclinal groups, are inclined at considerable angles to the normal to the surface of the photograph. It is difficult to conclude how explosions within a gaseous body like the sun can give rise to oblique rays, but the evidence for the existence of such rays in many coronas, besides that visible during the eclipse of 1871, is overpowering. Some of these oblique rays are straight or nearly straight, while others show considerable curvature and others bend over in one direction in their lower parts and are again carried slightly in a contrary direction above. . . . .
>
> The existence of these curving forms and rays showing contrary flexure is a matter of considerable importance as they appear to indicate the existence of an atmosphere with currents carrying the matter of which the structures are composed with different velocities at different altitudes.

[1] *Memoirs Royal Astronomical Society*, 41, 487, 1879.

The second problem may be stated as follows:

I assumed arbitrary values of a, e, ϕ_0, and θ_0 and computed in the same way a series of streamers. Table III shows the values I assumed. I wished to see if, when the streamers were plotted, the *ensemble* would resemble a photographed corona. I did not attempt to represent any given corona, but rather chose such values as would produce various types of streamers: that is, long ones, short ones, those nearly straight, those curved, streamers originating in various latitudes, etc. I was guided somewhat in my choice of values by the results in Table II. To represent the actual corona, a greater number of streamers should have been plotted in low latitudes. Moreover there should have been a greater number of short streamers, which can be obtained by decreasing θ_0, or by decreasing e, or by increasing the major axis.

Fig. 5 and Fig. 6 show coronas resulting from plotting streamers computed from the data given in Table III. The circumference of the circle represents the sun. The points, the position of which we computed, are at the intersection of the streamers and the short lines crossing them. I plotted a good many points—more than necessary—that were projected into the sun to show the direction of the streamer at the margin of the sun.

In Fig. 5[1] λ is assumed to be 90°. Since this is not very different from λ at the time of the eclipse of 1900, the general features of this drawing strikingly resemble those of the corona of that year. The streamers in Fig. 6, except streamers Nos. 20, 21, 22, 23, which occur only on Fig. 6, are computed from the same data as those of Fig. 5; except λ in Fig. 6 is assumed equal 82° 49′ 56″. This is the value of λ at the time of the eclipse of 1905. The streamers which are computed from the same values of a, e, ϕ_0, and θ_0 bear the same numbers on the two plates. For example, viewed from the sun's center, streamer No. 3 on both plates would appear the same; that is, with reference to the sun, streamer No. 3 on the two plates is identical. Fig. 5 shows this streamer as it would appear *from the earth* when the axis of the sun was perpendicular

[1] In making the computations for these plates, I was most ably assisted by Mr. Thomas R. Taylor, a Junior in Swarthmore College. He assisted in all the computations, and made the drawing for Fig. 5.

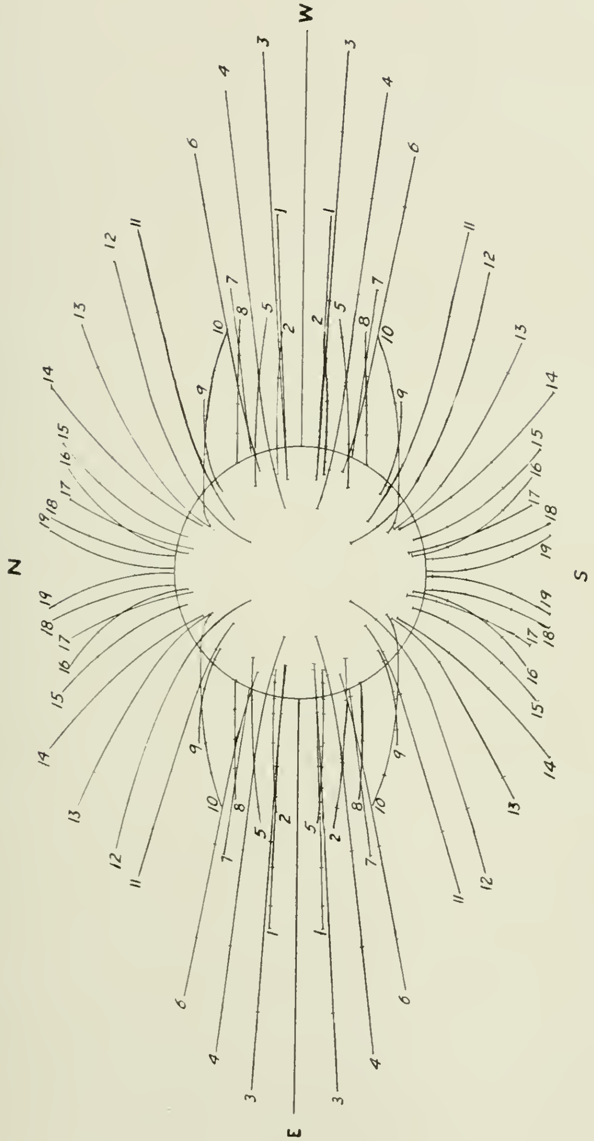


Fig. 5

to the line of sight; and Fig. 6 shows the same streamer as it appears from the earth, when the sun's axis is inclined $82^\circ\ 49'\ 56''$ to the line of sight. On Fig. 6 also, streamer No. 22 shows how this same streamer would appear from the earth if the angle between the sun's axis and the line of sight were $95^\circ\ 21'\ 12''$ (λ at time of eclipse of 1898). The streamers were numbered as follows. Those in the first quadrant were numbered *seriatim* from the west side of the sun's equator to its north pole. Those in the second quadrant

TABLE III

Streamer	ϕ_0	a	e	θ_0
No. 1.......	5°	3.00	0.915	$135^\circ 24'$
2.......	$10^\circ\ 23'\ 20''$	4.44	.800	125 24
3.......	5	6.00	.99	126 4
4.......	10	5.0	.99	109 8
5.......	20	5.0	.82	117 8
6.......	10	5.0	.99	137
7.......	15	3.0	.98	162 02
8.......	25	2.5	.7	105 24
9.......	40	2.5	.7	127 19
10.......	45	5.00	.82	157 52
11.......	25	2.0	.98	155 44
12.......	30	2.0	.98	143 12
13.......	40	2.0	.98	154 06
14.......	55	2.0	.98	140
15.......	65	2.0	.965	140 15
16.......	65	2.0	.965	160 15
17.......	75	2.0	.98	139 20
18.......	80	2.0	.98	139 31
19.......	80	2.0	.98	169 31
20.......	−86 47	1.554	.930	269
21.......	19 8 10	2.612	.914	105 54

were derived from those in the first quadrant by changing θ_0 in the expressions for ξ and η into $180^\circ + \theta_0$. The streamers of the third and fourth quadrants are derived by putting $-\phi$ for ϕ in the expressions for ξ and η in the second and first quadrants respectively. Accordingly the numerical values of ξ and η for corresponding points in the first and third quadrants are equal but their signs are different. The same is true for the corresponding points of the second and fourth quadrants. I have numbered all these streamers thus derived with the same number as the streamer bears from which they were derived. For example, there is a streamer No. 1 in each quadrant of each figure. Streamer No. 1

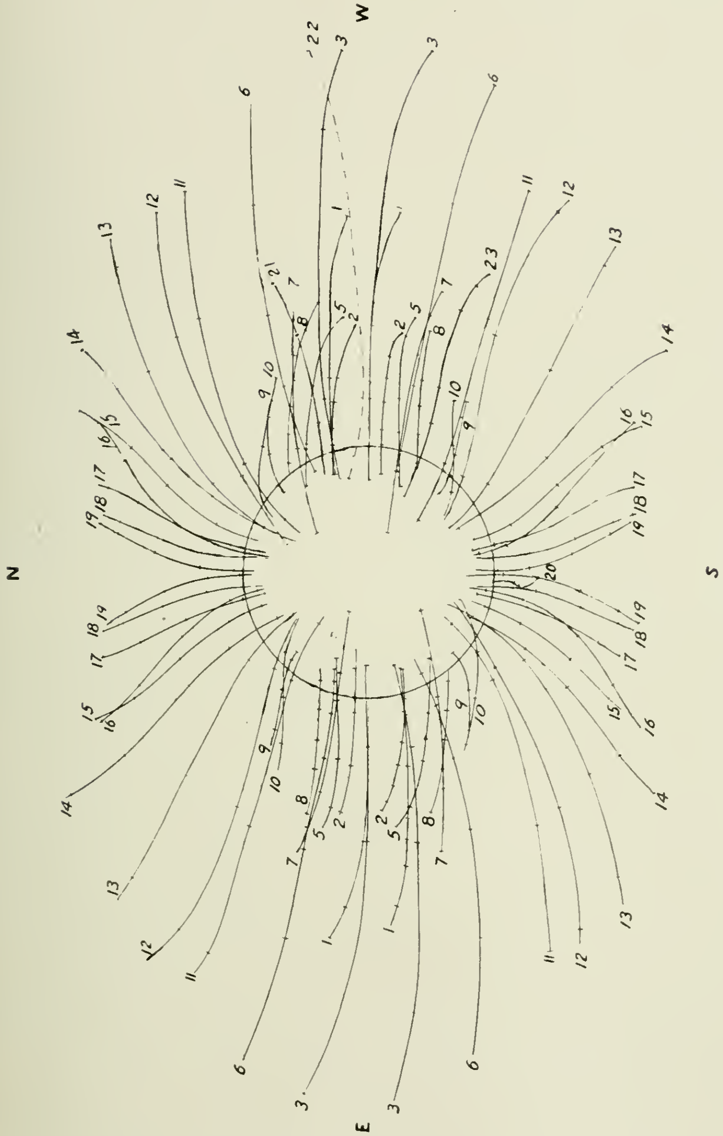


FIG. 6

in the first quadrant is computed from the data given in Table III. Streamer No. 1 in the second quadrant is computed with the same data except θ_0, which has been changed to $180+\theta_0$, and so on. As viewed from the center of the sun they are symmetrical with respect to the sun's equator and also with respect to a meridian of the sun passing through the line of sight.

As we should expect, the streamers in Fig. 5 are symmetrical with respect to the sun's axis, but those of Fig. 6 are not. Streamer No. 13 is typical. There are four points plotted on this streamer. It will be observed that for a given ξ the η of every point in the first quadrant is greater than the η for the corresponding point in the second quadrant and this is in general true for points on the streamers within three radii of the sun from the sun's center. The lack of symmetry with respect to the axis is not as striking as I had anticipated it would be, but would be more striking if the streamers were shorter. I believe the symmetry with respect to a line not coincident with the sun's axis discussed by Ranyard[1] may be accounted for in this way. But if so, the axis of symmetry should be on one side of the axis of the sun when $\lambda > 90°$ and on the other when $\lambda < 90°$.

It has occurred to me since making these computations that we should be able to find on our photographic plates some streamers that turn back toward the sun. By this I do not mean that the streams turn back (though they may do so) but that their projections turn back. If the chief source of the light received from these streamers is reflected sunlight, the long streamers on the west side of the sun should in general be brighter than the long streamers on the east side of the sun. The converse is true of the short streamers. Also one would be more likely to find streamers that turn back on the east side of the sun than on the west side. If the exposures on the photographs of large scale were long enough to photograph longer streamers, a very much greater number could be found that reverse their direction of curvature.

It would seem, from the foregoing results, that it is reasonable to conclude, or at least to assume as a working hypothesis, that the streams of the sun's corona are made of moving particles, obeying

[1] *Op. cit.*, pp. 488 ff.

mechanical laws; that other forces than the attractive force of the sun is acting on these particles; that the shape of the streamers is a function of the inclination of the sun's axis to the line of sight. It is of interest perhaps to note that if a sun-spot is formed from particles ejected above the solar surface, the spot will drift toward the equator unless the particles move in dense resisting media, and that of two such spots at the same height above the solar surface, that will show the shorter period of rotation which originated nearer the equator. The same remarks apply to faculae or any body of particles ejected from the solar surface.

Swarthmore College
Swarthmore, Pa.
January 14, 1911

ABOLITION OF TWO OF THE SPHERICAL ERRORS OF A THIN LENS-SYSTEM

By JAMES P. C. SOUTHALL

The essential requirement of an optical image is that it shall be a clear and faithful reproduction of the object to be depicted; but there are numerous and more or less insuperable difficulties in the way of the realization of this aim, which, from the standpoint of geometrical optics, may all be classed under two general heads, viz., aberrations on account of color-dispersion (chromatic aberrations) and aberrations due to sphericity, the so-called spherical aberrations which are encountered in every optical system composed of a series of lenses. So far as the corrections of the color-faults are concerned, the modern optician has an immense advantage over his predecessors on account of the variety of the new kinds of optical glass that are now at his disposal. The favorable circumstance that varieties of glass of equal refractivity but of different dispersion are now available, enables the designer of a system of lenses to postpone the correction of the color-faults until after he has effected the correction of the spherical errors. For our present purposes, therefore, we may leave out of account altogether the color-faults of the image.

The five spherical abberrations of the third order as originally distinguished by Seidel[1] (which are therefore often called the Seidel image-faults) may be enumerated in the following order:[2]

[1] "Zur Dioptrik. Ueber die Entwicklung der Glieder 3-ter Ordnung, welche den Weg eines ausserhalb der Ebene der Axe gelegenen Lichtstrahles durch ein System brechender Medien bestimmen," *Astronomische Nachrichten*, **43**, 289–332, 1856.

[2] For the analytical expressions of the spherical errors of a centered system of spherical refracting surfaces, the reader is referred to the writer's book, *The Principles and Methods of Geometrical Optics, Especially as Applied to the Theory of Optical Instruments* (New York, 1910). The theory of the spherical aberrations is treated in chap. xii. It is worth while to note that the symbols S^{I}, S^{II}, etc., which are used on p. 466 of this book, are not entirely identical with the symbols S_1, S_2, etc., which are employed in this paper. The connections between these two sets of symbols may be exhibited as follows: $S_1=S^{\mathrm{I}}$; $S_2=S^{\mathrm{II}}$; $S_3=S^{\mathrm{IV}}-3S^{\mathrm{III}}$; $S_4=S^{\mathrm{IV}}-S^{\mathrm{III}}$; $S'_3=S^{\mathrm{IV}}-2S^{\mathrm{III}}$; $S'_4=-S^{\mathrm{III}}$; $S_5=S^{\mathrm{V}}$.

1. *The Spherical Aberration* proper, or the aberration of the image of the point of the object which lies on the optical axis of the centered system of spherical refracting surfaces. This error will be denoted by S_1.

2. The defect called *Coma*, in consequence of which the definition of parts of the image not on the axis is impaired; which will be denoted by S_2. The equation $S_2=0$ is equivalent also to the famous *Sine-Condition*, which is so important in the optical theories of Helmholtz and Abbe. The so-called Fraunhofer-condition, discovered by Seidel, is included in the sine-condition.

3 and 4. *The Curvatures* of the primary and secondary image-surfaces in consequence of *Astigmatism*. This pair of spherical errors will be denoted by S_3 and S_4. If the two faults S_3, S_4 are both abolished, we obtain a flat, stigmatic image. It may be mentioned, however, that the curvatures of the image depend essentially on the refractivities of the lenses, so that with unsuitable kinds of glass it is impossible by any choice of the geometrical dimensions of the lens-system (radii, thicknesses, distances) to obtain a plane stigmatic image, as was first recognized by Petzval and more clearly still by Seidel.

In order to remedy as far as possible this defect of curvature, designers of optical instruments sometimes compromise on a kind of artificial flattening of the image-field; which consists in contriving so that the so-called "circles of least confusion" of the astigmatic bundles of image-rays shall all be made to fall in a definite transversal image-plane. The curvature of the surface which is the locus of the circles of least confusion is approximately equal to the arithmetical mean of the curvatures of the primary and secondary image-surfaces; accordingly, if the curvature of this surface is denoted by S'_3, we may write:

$$S'_3=\frac{S_3+S_4}{2}.$$

If $S'_3=0$, the image will be more or less plane, but in general not stigmatic.

On the other hand, again supposing that we cannot obtain an image which is both perfectly stigmatic and perfectly plane ($S_3=S_4=0$), we may be content to disregard the curvature-error and to

effect a compromise as to the *Astigmatism;* in which case we can endeavor to make vanish the expression:

$$S'_4 = \frac{S_3 - S_4}{2}.$$

5. *The Distortion* of the marginal parts of the image, which will be denoted by S_5. Only in case the five conditions

$$S_1 = S_2 = S_3 = S_4 = S_5 = 0$$

are satisfied simultaneously, will the image of a plane object placed at right angles to the optical axis of a centered system of spherical refracting surfaces be at once sharply defined, flat and true, so that it will indeed coincide completely with the so-called Gaussian or theoretical (collinear) image.

Although it is impossible to attain this degree of perfection, we may at least endeavor to design the optical system so as to abolish those faults, which, for the particular type of instrument in view (telescope, microscope, photographic objective, etc.), are the most detrimental to the image, and perhaps also to minimize the residual image-faults. The problem consists therefore in determining the radii, thicknesses, distances, etc., and the kinds of glass in such way that one or more of Seidel's five expressions for the spherical errors will be made to vanish.

On account of the algebraic difficulties involved in the solution of these equations, the method is applicable only to optical systems of comparatively simple structure. The analytical difficulties are very greatly reduced provided we can neglect the thicknesses of the lenses, and a still further simplification will be introduced if we may also disregard the intervals between each pair of successive lenses.

We propose, therefore, to consider here only this simple and more or less fictitious case of an *optical system of infinitely thin lenses all in contact*. If, as is frequently the case, the lenses are to be placed in close juxtaposition, and especially if they are to be cemented together, a preliminary calculation on the assumption that the entire thickness of the system is negligible will be justifiable and will afford the designer at least an accurate basis for further calculation.

We proceed now to give simple and convenient expressions for the conditions of the abolition of *two* of the spherical errors and for the calculation of the residual spherical errors of a system of thin lenses in contact.

If the positions of the optical center and the primary focal point of the i-th lens are designated by A_i and F_i, respectively, the primary focal length of the lens will be equal to F_iA_i. The power of the lens, denoted by ϕ_i, is the reciprocal of the primary focal length. If M_i, M_{i+1} designate the positions of the points where a paraxial ray, emanating originally from the axial object-point M_1, crosses the optical axis, before and after refraction through the i-th lens, and if we put

$$\frac{1}{x_i}=A_iM_i\,,\quad \frac{1}{x'_i}=A_iM_{i+1}\,;$$

and, similarly, if $\mathbf{M}_i$, $\mathbf{M}_{i+1}$ designate the positions of the points where a paraxial ray, going originally through the stop-center $\mathbf{M}_1$, crosses the optical axis before and after refraction through the i-th lens, and if we put

$$\frac{1}{\mathbf{x}_i}=A_i\mathbf{M}_i\,,\quad \frac{1}{\mathbf{x}'_i}=A_i\mathbf{M}_{i+1}\,;$$

and, finally, if the refractive index of this lens (supposed to be surrounded by air) and the curvatures of the first and second surfaces are denoted by n_i, c_i, c'_i, respectively; then

$$x'_i-x_i=\mathbf{x}'_i-\mathbf{x}_i=(n_i-1)(c_i-c'_i)=\phi_i.$$

In the case of a system of m infinitely thin lenses in contact, it is easy to show[1] that Seidel's expression for the spherical aberration along the axis can be put in the form:

$$S_1=\sum_{i=1}^{i=m}\left\{\left(\frac{n_i}{n_i-1}\right)^2\phi_i^3+\frac{3n_i+1}{n_i-1}x_i\phi_i^2+\frac{3n_i+2}{n_i}x_i^2\phi_i\right.$$
$$\left.-\frac{2n_i+1}{n_i-1}c_i\phi_i^2-\frac{4(n_i+1)}{n_i}c_ix_i\phi_i+\frac{n_i+2}{n_i}c_i^2\phi_i\right\}.$$

If, moreover, for the sake of brevity, we put

$$N=\sum_{i=1}^{i=m}\left\{\frac{n_i}{n_i-1}\phi_i^2+\frac{2n_i+1}{n_i}\phi_ix_i-\frac{n_i+1}{n_i}\phi_ic_i\right\},$$

[1] See, for example, *The Principles and Methods of Geometrical Optics*, secs. 268, 271.

and if also we remark that for a system of infinitely thin lenses in contact we have:

$$x_i - x_1 = \sum_{r=1}^{r=i-1} \phi_r = \mathbf{x}_i - \mathbf{x}_1 ,$$

and that, consequently we may write:

$$x_i - \mathbf{x}_i = x_1 - \mathbf{x}_1 = a \text{ (say)};$$

it will also not be difficult to show that the general expression of a spherical error of a centered optical system composed of a series of infinitely thin lenses in contact may be expressed as follows:

$$S = pS_1 + a \left\{ qX - a \sum_{i=1}^{i=m} L_i \phi_i \right\} ,$$

where p, q have certain integral values, positive or negative, and L_i denotes a function of n_i. The values of p, q, and L_i corresponding to each of the spherical errors S_1, S_2, etc., are exhibited in the subjoined table:

S	S_1	S_2	S_3	S_4	S'_3	S'_4	S_5
p	$+1$	$+1$	-3	-1	-2	-1	-1
q	0	-1	$+6$	$+2$	$+4$	$+2$	$+3$
L_i	0	0	$\frac{3n_i+1}{n_i}$	$\frac{n_i+1}{n_i}$	$\frac{2n_i+1}{n_i}$	$+1$	$\frac{3n_i+1}{n_i}$

Let

$$S = pS_1 + a \left\{ qX - a \sum_{i=1}^{i=m} L_i \phi_i \right\} ,$$

$$S' = p'S_1 + a \left\{ q'X - a \sum_{i=1}^{i=m} L'_i \phi_i \right\} ,$$

$$S'' = p''S_1 + a \left\{ q''X - a \sum_{i=1}^{i=m} L''_i \phi_i \right\}$$

be the expressions of three of the spherical errors of the lens-system. If the first error (S) has been corrected ($S = 0$), the condition of the simultaneous abolition of the second error (S'), that is, the condition of the fulfilment of the additional requirement $S' = 0$, will be given by the following convenient equation:

$$\sum_{i=1}^{i=m} \left\{ l\frac{n_i}{n_i-1}\phi_i^2+\left(l\frac{2n_i+1}{n_i}+M_i\right)\phi_i x_i - M_i\phi_i \mathbf{x}_i - l\frac{n_i+1}{n_i}\phi_i c_i \right\} = 0$$

where

$$l = p'q - pq',$$
$$M_i = pL'_i - p'L_i .$$

If two spherical errors have been abolished ($S=S'=0$), the magnitude of a third residual spherical error (S'') will depend only on the strengths of the lenses and their refractivities, and will, therefore, not be affected by any further bending of the lenses or by any alteration in the order of sequence of the lenses. Under these circumstances we find in fact:

$$S'' = a^2 \sum_{i=1}^{i=m} K_i \phi_i ,$$

where

$$K_i = \frac{(p''q'-p'q'')L_i + (pq''-p''q)L'_i - (pq'-p'q)L''_i}{pq'-p'q} .$$

The subjoined table exhibits the values of l and M_i for the simultaneous abolition of any pair of the spherical errors denoted by S, S' and also the expressions for K_i corresponding to each one of the residual spherical errors S''. For convenience of printing, the subscript i has been omitted from the letter n in this table. The numerals 1, 2, 3, 4, 3′, 4′, 5 refer to the spherical errors denoted by S_1, S_2, S_3, S_4, S'_3, S'_4, S_5, respectively. It will be observed that in each horizontal row in the table there are always two values $K_i=0$; the first or left-hand one of these corresponds to $S=0$ and the right-hand one to $S'=0$.

As an illustration of the method of procedure, let us suppose that we propose to design an ordinary telescope-objective composed of two thin lenses cemented together, so that $c'_1=c_2$. Since there are three radii to be determined, we can impose three other conditions, which would probably be:

1. A prescribed focal length ($f=1/\phi$);

2. Abolition of the spherical aberration in the center of the field of view ($S_1=0$);

3. Abolition of *coma* ($S_2=0$), whereby not only the center but the adjacent surrounding parts of the field of view will be portrayed distinctly.

l	M_i	K_i						
		1	2	3	4	3′	4′	5
$+1$	0	0	0	$-\frac{3n+1}{n}$	$-\frac{n+1}{n}$	$-\frac{2n+1}{n}$	-1	$-\frac{3n+1}{n}$
-6	$\frac{3n+1}{n}$	0	$-\frac{3n+1}{6n}$	0	$-\frac{2}{3n}$	$-\frac{1}{3n}$	$\frac{1}{3n}$	$-\frac{3n+1}{2n}$
-2	$\frac{n+1}{n}$	0	$-\frac{n+1}{2n}$	$\frac{2}{n}$	0	$\frac{n+1}{n}$	$\frac{1}{n}$	$-\frac{3n-1}{n}$
-4	$\frac{2n+1}{n}$	0	$-\frac{2n+1}{4n}$	$\frac{1}{2n}$	$-\frac{1}{2n}$	0	$\frac{1}{2n}$	$-\frac{6n+1}{4n}$
-2	$+1$	0	$-\frac{1}{2}$	$-\frac{1}{n}$	$-\frac{1}{n}$	$-\frac{1}{n}$	0	$-\frac{3n+2}{2n}$
-3	$\frac{3n+1}{n}$	0	$-\frac{3n+1}{3n}$	$\frac{3n+1}{n}$	$\frac{3n-1}{3n}$	$\frac{6n+1}{3n}$	$\frac{3n+2}{3n}$	0
-3	$\frac{3n+1}{n}$	$\frac{3n+1}{3n}$	0	0	$-\frac{2}{3n}$	$-\frac{1}{3n}$	$\frac{1}{3n}$	$-\frac{3n+1}{3n}$
-1	$\frac{n+1}{n}$	$\frac{n+1}{n}$	0	$\frac{2}{n}$	0	$\frac{1}{n}$	$\frac{1}{n}$	$-\frac{n-1}{n}$
-2	$\frac{2n+1}{n}$	$\frac{2n+1}{n}$	0	$\frac{1}{2n}$	$-\frac{1}{2n}$	0	$\frac{1}{2n}$	-1
-1	$+1$	$+1$	0	$-\frac{1}{n}$	$-\frac{1}{n}$	$-\frac{1}{n}$	0	$-\frac{n+1}{n}$
-2	$\frac{3n+1}{n}$	$\frac{3n+1}{2n}$	0	$\frac{3n+1}{2n}$	$\frac{n-1}{2n}$	$+1$	$\frac{n+1}{2n}$	0
$+3$	$-\frac{6n+2}{n}$	$\frac{3n+1}{n}$	$\frac{3n+1}{3n}$	0	$-\frac{2}{3n}$	$-\frac{1}{3n}$	$\frac{1}{3n}$	0
$+1$	-2	$\frac{3n-1}{n}$	$\frac{n-1}{n}$	$\frac{2}{n}$	0	$\frac{1}{n}$	$\frac{1}{n}$	0
$+2$	$-\frac{4n+1}{n}$	$\frac{6n+1}{2n}$	$+1$	$\frac{1}{2n}$	$-\frac{1}{2n}$	0	$\frac{1}{2n}$	0
$+1$	$-\frac{2n+1}{n}$	$\frac{3n+2}{n}$	$\frac{n+1}{n}$	$-\frac{1}{n}$	$-\frac{1}{n}$	$-\frac{1}{n}$	0	0

Accordingly, we have the following equations of condition:

$$\phi_1+\phi_2=\phi$$

$$\sum_{i=1}^{i=2}\left\{\left(\frac{n_i}{n_i-1}\right)^2\phi_i^3+\frac{3n_i+1}{n_i-1}x_i\phi_i^2+\frac{3n_i+2}{n_i}x_i^2\phi_i-\frac{2n_i+1}{n_i-1}c_i\phi_i^2 -\frac{4(n_i+1)}{n_i}c_ix_i\phi_i+\frac{n_i+2}{n_i}c_i^2\phi_i\right\}=0\,,$$

$$\sum_{i=1}^{i=2}\left\{\frac{n_i}{n_i-1}\phi_i^2+\frac{2n_i+1}{n_i}\phi_ix_i-\frac{n_i+1}{n_i}\phi_ic_i\right\}=0\,.$$

According to the very practical method of calculation given by E. von Höegh,[1] it is best to regard ϕ_1 as the unknown to be first determined: whence we can obtain, without difficulty, the required curvatures of the lens-surfaces. Thus, substituting $x_1+\phi_1$ in place of x_2, and eliminating both ϕ_2 and c_2, we shall obtain finally an equation of the fifth degree in ϕ_1, three of the roots of which will be found to be real; so that there will always be three possible combinations of a pair of cemented lenses of given refractivities, arranged in a given order of sequence, to produce the desired result. The residual aberrations will be given by the following formulae:

$$S_3 = S_5 = -a^2 \sum_{i=1}^{i=2} \frac{3n_i+1}{n_i} \phi_i ;$$

$$S_4 = -a^2 \sum_{i=1}^{i=2} \frac{n_i+1}{n_i} \phi_i ;$$

$$S'_3 = -a^2 \sum_{i=1}^{i=2} \frac{2n_i+1}{n_i} \phi_i ;$$

$$S'_4 = -a^2 \sum_{i=1}^{i=2} \phi_i .$$

Department of Physics
Alabama Polytechnic Institute
Auburn, Ala.
February 1911

[1] "Zur Theorie der zweitheiligen verkitteten Fernrohrobjective," *Zeitschrift für Instrumentenkunde*, **19**, 37–39, 1899.

ON THE EFFECT OF THE GROOVE FORM ON THE DISTRIBUTION OF LIGHT BY A GRATING

By J. A. ANDERSON and C. M. SPARROW

The following paper will aim to give a simple treatment of the theory of a plane reflecting grating—that is, a reflecting surface consisting of a great number of equidistant, similar, parallel grooves all lying in one plane—for the case in which each groove is bounded by a finite number of plane faces. Following the presentation of the theory will be given a short discussion of previous work on the subject, comparing such work with the results here obtained in order to bring out the points of agreement or difference. Finally the theory will be illustrated by application to a few numerical problems of practical interest. Experimental work will here be touched on only incidentally, it being the intention of the writers to elaborate this side of the question in a subsequent paper.

GENERAL THEORY

Notation.—We will consider a groove made up of plane portions $AB, BC, \ldots$ (Fig. 1) and will generally call such a plane portion a *face* in what follows. We will reckon all directions clockwise from the normal to the grating. We will call

$c_1, c_2 \ldots$ the widths of the faces AB, BC, ...
$\gamma_1, \gamma_2 \ldots$ the directions of the normals to these faces,
$\pi+\phi \ldots$ the direction of the incident light,
$\theta \ldots$ the direction of the diffracted light,
$\theta_k \ldots$ the direction of the spectrum of the k-th order, k being positive when $\theta_k-\theta_0$ is positive,
$a \ldots$ the grating interval.

Simple grating.—By a simple grating we mean a plane grating consisting of alternate strips of transparent and opaque, or of reflecting and non-reflecting, material. The theory of such a grating is given in all works on optics. If the disturbance in any direction θ due to a single slit is of amplitude R_θ and phase δ, we may represent it by a vector in a plane, and the resultant effect of the N slits is obtained by summing these elementary disturbances as vectors. We thus arrive at the result that, if N is large, all the

light is concentrated in the immediate neighborhood of directions θ_k such that $a(\sin\phi+\sin\theta_k)=k\lambda$, where k is the order of the spectrum. The maximum amplitude in these directions occurs when all these elementary disturbances are in phase; it is therefore NR, and hence the maximum intensity is proportional to N^2R^2. As the factor N is independent of the nature of the slit, it follows that the treatment will apply to any plane grating; and the problem thus becomes one of finding R_θ, the amplitude of the disturbance due to a single groove.

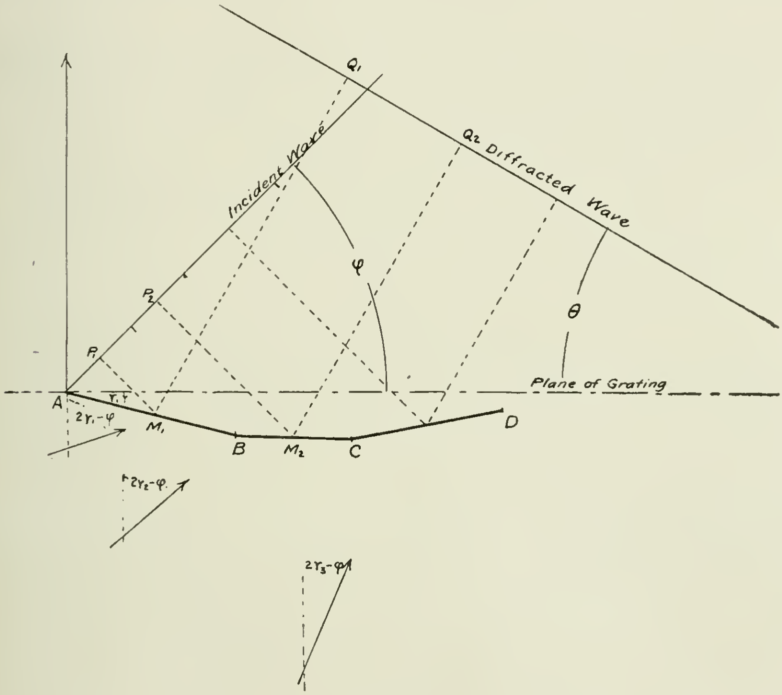


FIG. 1

Only one point needs further mention here. The quantity N^2R^2 is frequently spoken of as being proportional to the "intensity of the spectrum." The term is misleading, and has led to some inaccuracies in the textbooks.[1] The quantity N^2R^2 is the *maximum*

[1] See for example Schuster, *Theory of Optics* (2d ed.), p. 122, where a calculation is made of the efficiency of a grating, neglecting the factor given below.

value of the intensity of illumination in a spectrum line (corresponding to the greatest ordinate in the graph of $y=\sin^2 Nx/\sin^2 x$). But if the resolving power of the grating be sufficient to give us sharp lines, what is actually observed is the *total energy* of the line (corresponding to the area of the above-mentioned curve). To get a number proportional to the total energy we must therefore multiply N^2R^2 by a factor proportional to the width of the line.

Such a factor may be obtained as follows. The difference in path L between the two extreme lines of the grating is

$$L=Na\ (\sin\phi+\sin\theta).$$

If θ is the direction of a principal maximum we obtain the first minimum on either side by making a small change $d\theta$ such that the corresponding change of path is $\pm\lambda$. We thus have

$$dL=\pm\lambda=Na\cos\theta\, d\theta,$$

and the width of the line is $2\lambda/Na\cos\theta$; or for a given wave-length the width varies as $\sec\theta$. We thus get for J the "intensity of the spectrum"

$$J=AR_\theta^2\sec\theta. \tag{1}$$

It remains to calculate R_θ. If we have a single slit in an opaque screen, the usual formula for the amplitude R is

$$R=R_0\frac{\sin\alpha}{\alpha}\ ;\ \alpha=\frac{\pi e}{\lambda}(\sin j+\sin\psi), \tag{2}$$

where j is the angle of incidence, ψ the angle of diffraction, e the slit-width, and R_0 the amplitude in the direction of the incident light. The phase of the disturbance is that at the center of the slit. When $\frac{\pi e}{\lambda}$ is not large, or when we are dealing with directions of diffraction making a considerable angle with the wave normal, the formula is admittedly inadequate, for it takes no account of polarization, and rests on the further assumption that the effect of an element of the wave-front is the same in all directions. The results obtained below will require for their complete accuracy the same limitations; the defect, however, being in no way peculiar to the theory of the composite groove, but belonging equally to the theory of the simple grating as ordinarily given.

General case.—Our method of treating the more general case is a rather obvious extension of the simple theory outlined above. Suppose we have given a groove $ABCD$ . . . . consisting of plane faces AB, BC, CD, ... and that lpane waves are incident on it at an angle ϕ. Each face, such as AB, gives rise to a disturbance whose form we may determine as follows: We construct the direction of regular reflection $2\gamma_1-\phi$ (Fig. 1), and regard AB as a slit in a screen in the plane of AB illuminated by plane waves incident from behind in a direction $2\gamma_1-\phi$. The amplitude P_θ in any direction θ is obtained from (2) by putting $j=\phi-\gamma_1$, $\psi=\theta-\gamma_1$, namely,

$$P=P_0\frac{\sin a}{a}\,;\quad a=\frac{\pi c_1}{\lambda}[\sin(\phi-\gamma_1)+\sin(\theta-\gamma_1)]. \tag{3}$$

The intensity P_0 at the center of the pattern is proportional to the reflecting power of the surface and to the area of the wave-front falling on AB, so that we may write

$$P=Ar\cos(\phi-\gamma_1)c_1\frac{\sin a}{a} \tag{4}$$

where A depends only on the intensity of illumination, and is therefore the same for all the faces AB, BC, ...

The disturbance due to any other face is given by an expression of the same form as (4). Each of these disturbances will have a definite phase and the resultant effect of the whole groove in the direction θ will be found by adding these separate disturbances as vectors. The phase of any one of these component disturbances may be found by taking the sum of the perpendiculars M_1P_1, M_1Q_1 from the center of the groove face to an arbitrary fixed wave-front and an arbitrary fixed plane normal to the direction θ. The phase is then $\frac{2\pi}{\lambda}$ times this distance, plus the phase-change due to reflection. We thus see that the phase-differences consist of two parts, one depending on the angle of incidence and the other on the angle of diffraction, the two terms being independent of each other. A case of particular interest is offered by the simple triangular groove, with normal illumination on the grating. In this case the difference of path in the direction θ of the disturbances due to the two faces is simply $a/2\cdot\sin\theta$. But the spectra are diffracted in directions θ such that $a\sin\theta=k\lambda$, so that the phase-relation between

the two parts of the groove changes by half a wave-length as we pass from one order to the next; or if β is the phase-difference between the disturbances in the central image, it will be the same in all the even orders, while in the odd orders it will be $\beta+\pi$.

In case the direction of the incident light is such that it does not reach to the bottom of an angle, we must take as our slit the effective portion of the groove-face. If again the direction of reflection is such that the light meets another face of the groove before emerging, we should consider it as reflected from this second face, and take this face as our aperture. Strictly speaking this treatment is only approximate; we should rather regard the second face as illuminated, by the first, considering each element of the first face as a source, so that we would have to deal at the second face with light *diffracted* by the first face, not with light regularly reflected. Since, however, the direction of regular reflection coincides with the diffraction maximum, the above method gives an approximate solution. The difficulty should nevertheless be borne clearly in mind, as we shall see that many of the difficulties of numerical computation find their explanation in the complications introduced by multiple reflection.

In the case of metallic reflection the phase-change due to reflection is a function, not only of the angle of incidence, but also of the plane of polarization. The same is true of the reflecting power of the groove-face. As the diffraction formula used here takes no account of polarization, it would be useless labor in applying the theory to take account of these refinements. In the numerical examples given below we have assumed perfect (or at least uniform) reflecting power, and have neglected the phase-change due to reflection. The practical effect of these simplifications can be seen best in a consideration of the detailed results.

To sum up the preceding: The energy of a spectrum which is diffracted in a direction θ is proportional to

$$R^2 \sec \theta ,$$

where R is the amplitude of the disturbance due to a single groove. If the groove consist of a number of plane faces, R will be given

by the vector sum of the disturbances due to these faces. If the width c of a face is such that $\frac{\pi c}{\lambda}$ is not small, and if $\phi-\gamma$ is not large, the amplitude of the disturbance due to this face will be given by

$$P=Ar\cos(\phi-\gamma)c\frac{\sin\alpha}{\alpha}\ ;\ \alpha=\frac{\pi c}{\lambda}[\sin(\phi-\gamma)+\sin(\theta-\gamma)]\ ;$$

and the phase will be a sum of two independent parts, one a function of the angle of incidence, and the other of the angle of diffraction.

PREVIOUS WORK

We have next to consider the relation of the foregoing to previous treatments of the subject. Schuster in his *Theory of Optics* (2d ed., p. 122) considers briefly the question of groove-form in discussing gratings with predominant spectra. He considers a plane face of a groove as giving rise to plane waves of finite extent, and concludes that if these plane waves differ in path by a whole number of wave-lengths they will build up again into a single plane wave, all the light reflected from these faces of the grooves being concentrated into one order. The same point of view is adopted by Trowbridge and Wood[1] in a recent experimental paper, but only as an approximation. Such reasoning ignores the process of diffraction altogether, and is not in our opinion valid. For the only case in which we can consider the groove-face as giving rise to plane waves is when the grating interval is large compared to the wave-length, and in this case the spectra are correspondingly close together, so that the energy is still distributed among several spectra. (The echelon grating furnishes an illustration of this state of things.)

A theory of the effect of groove-form based on the general theory of diffraction is given in a paper by Rowland.[2] The general method used in this paper is sound, but there are errors in the calculation, so that Rowland's final formula is incorrect, as well as

[1] *Phil. Mag.* (6), **20**, 886, 1910.

[2] *Astronomy and Astrophysics*, **12**, 129, 1893; *Phil. Mag.* (5), **35**, 397, 1893; *Rowland's Physical Papers*, p. 525.

a number of his conclusions. When corrected, Rowland's method leads to a formula which, when applied to grooves with plane faces, is equivalent to that obtained above, so far as the value of R is concerned. He likewise neglects, however, the distinction between the intensity of illumination and the energy of the spectrum. As the paper is one which has been rather extensively quoted and referred to, and as the errors seem to have hitherto passed unnoticed, it seems worth while to indicate briefly his general method, showing how his results can be made to agree with ours. The general equations will be simplified to conform to the limitations of the present investigation. Taking a grating in the $x\,z$ plane, with lines parallel to the z axis, and with illumination perpendicular to these lines, we may ignore the z co-ordinate altogether. If x', y' are the co-ordinates of the source A, x, y the co-ordinates of a point P of the grating surface, and x'', y'' the co-ordinates of a point B of the screen, we may represent the disturbance at x'', y'' due to an element dS of the grating surface by an expression of the form

$$Ae^{-\frac{2\pi i}{\lambda}[\overline{AP}+\overline{PB}]}$$

where A is the amplitude, and the imaginary exponent gives the phase. If we are dealing with parallel light, A will not involve the distance. If χ be the angle between dS and AP, and if we ignore variations in the reflecting power of the surface, we may take A equal to $dS \cos \chi$. In Rowland's paper the factor $\cos \chi$ is overlooked, and A is taken as simply equal to dS—an evident error, for the effect of dS must be proportional to the area of the wave-front which it intercepts. With the notation used above, $\tan \phi = \frac{x'-x}{y'-y}$ and $\tan \theta = \frac{x''-x}{y''-y}$. Also, if the grating is near the origin as compared with the source and screen, we have approximately

$$\overline{AP} = \sqrt{(x'-x)^2+(y'-y)^2} = \sqrt{x'^2+y'^2} - \frac{xx'+yy'}{\sqrt{x'^2+y'^2}},$$

$$\overline{PB} = \sqrt{(x''-x)^2+(y''-y)^2} = \sqrt{x''^2+y''^2} - \frac{xx''+yy''}{\sqrt{x''^2+y''^2}}.$$

Writing $R=\sqrt{x'^2+y'^2}+\sqrt{x''^2+y''^2}$,

$$l=\sin\phi+\sin\theta=\frac{x'}{\sqrt{x'^2+y'^2}}+\frac{x''}{\sqrt{x''^2+y''^2}},$$

$$m=\cos\phi+\cos\theta=\frac{y'}{\sqrt{x'^2+y'^2}}+\frac{y''}{\sqrt{x''^2+y''^2}},$$

we have $\overline{AP}+\overline{PB}=R-(lx+my)$, an equation which becomes exact for parallel light. We have therefore to integrate

$$\int e^{-\frac{2\pi i}{\lambda}[R-lx-my]}\cos\chi\,ds=B\int e^{\frac{2\pi i}{\lambda}(lx+my)}\cos\chi\,dS$$

over the surface of the grating. As the ruling is periodic this breaks up into a number of equal integrals, one over each groove; so that we have only to consider a single groove. If further the groove be made up of a number of plane faces, we may take one of these faces to be

$$x=-y\cot\gamma+b\ ;$$

whence

$$lx+my=(-l\cot\gamma+m)y+lb=$$
$$\frac{-(\sin\phi+\sin\theta)\cos\gamma+(\cos\phi+\cos\theta)\sin\gamma}{\sin\gamma}y+lb=-\frac{\lambda a}{\pi c\sin\gamma}y+b\ ,$$

where $a=\frac{\pi c}{\lambda}[\sin(\phi-\gamma)+\sin(\theta-\gamma)]$ has the same value as in (3),

$$dx=dy\cot\gamma\ ,\quad ds=dy\csc\gamma\ ,\quad \cos\chi=\cos(\phi-\gamma)\ .$$

The integral thus becomes

$$e^{\frac{2\pi ibl}{\lambda}}\cos(\phi-\gamma)\csc\gamma\int_{u_1}^{u_2}e^{-\frac{2ai}{c\sin\gamma}y}\,dy\ ,$$

where u_1 and u_2 are the ordinates of the edges of the face. The integration gives at once

$$\frac{e^{\frac{2\pi ibl}{\lambda}}c\cos(\phi-\gamma)}{-2ia}\left[e^{-\frac{2ia}{c\sin\gamma}u_2}-e^{-\frac{2ia}{c\sin\gamma}u_1}\right].$$

The factor in parentheses can be written

$$-2i\sin\left[\frac{a}{c\sin\gamma}(u_1-u_2)\right]e^{-\frac{a}{c\sin\gamma}(u_1+u_2)}\ ,$$

but $u_1-u_2=c\sin\gamma$, so that we get for the final value of the integral

$$c\cos(\phi-\gamma)\frac{\sin a}{a}\,e^{i\left[-\frac{a(u_1+u_2)}{c\sin\gamma}+\frac{2\pi bl}{\lambda}\right]}\ . \qquad (5)$$

The real part of this expression gives the amplitude, which is that found above (4). To show that the phase is the same, we take a wave-front through the origin, and a plane through the origin perpendicular to the direction of diffraction. The co-ordinates of the center of the face are

$$\bar{x}=b-\frac{u_1+u_2}{2}\cot\gamma\,,\quad \bar{y}=\frac{u_1+u_2}{2}\,,$$

and the distances of this point from the planes just mentioned are

$$\left(b-\frac{u_1+u_2}{2}\cot\gamma\right)\sin\phi+\frac{u_1+u_2}{2}\cos\phi\,,$$

$$\left(b-\frac{u_1+u_2}{2}\cot\gamma\right)\sin\theta+\frac{u_1+u_2}{2}\cos\theta\,.$$

Adding these two distances we have

$$bl+\frac{u_1+u_2}{2}[-l\cot\gamma+m]=bl-\frac{u_1+u_2}{2}\,\frac{\lambda}{\pi c}\,\frac{a}{\sin\gamma}\,,$$

and multiplying by $\frac{2\pi}{\lambda}$ to reduce to angle, we get $\frac{2\pi bl}{\lambda}-\frac{(u_1+u_2)a}{c\sin\gamma}$ as found above (5).

NUMERICAL EXAMPLES

We give below a few examples of how the theory works out in simple cases. As a typical groove we will take a simple triangular groove, consisting of two plane faces which make angles of 15° and -30° respectively with the plane of the grating (Fig. 2 *a*). We will take no account of loss of light by reflection or of phase-change on reflection.

Variation of distribution with wave-length.—In Table I are given the percentages of the total incident energy in the various orders for normal illumination, and for different wave-lengths. The results were obtained by a combination of graphical and numerical methods, the estimated probable error of the computation being about 1 per cent. The wave-lengths are expressed in fractions of the grating interval, the numbers in parentheses below them are the values in $\mu\mu$ which would correspond to a ruling of 10,000 lines to the inch. Three columns are given under each wave-length. The third of these gives the percentage of the total light in the various orders—a quantity which we shall refer to as the

Order	0.10 (254)			0.30 (762)			0.40 (1016)			0.50 (1270)		
	AB	BC	Tota	B	BC	Total	AB	BC	Total	AB	BC	Total
−9	0.0	19.7	19.									
−8	0.1	9.9	12.									
−7	0.0	0.1	0.									
−6	0.1	0.6	1.									
−5	0.0	0.1	0.									
−4	0.0	0.1	0.									
−3	0.1	0.1	0.	0.1	23.1	19.9						
−2	0.0	0.0	0.	0.9	5.8	11.4	0.2	16.5	19.7			
−1	0.2	0.1	0.	0.2	0.2	0.0	2.1	3.4	0.4	2.5	7.3	1.3
0	0.1	0.0	0.	1.6	0.3	3.3	0.1	0.1	0.4	4.0	1.0	8.9
1	0.0	0.0	0.	9.3	0.4	13.8	45.5	0.3	38.6	54.5	0.0	54.5
2	0.5	0.0	0.	6.4	0.2	52.8	16.7	0.8	24.6	61.0	8.3	64.7
3	2.1	0.1	1.	2.1	0.0	2.1	64.6	21.1	83.7			
4	3.7	0.0	4.	0.6	30.0	103.3						
5	54.5	0.0	54.									
6	3.2	0.0	3.									
7	2.0	0.0	1.									
8	1.2	0.1	1.									
9	0.8	0.0	0.									
	68.6	30.9	101.4									

TABLE I

Distribution of Energy as a Function of the Wave-Length

Order	0.10 (254)			0.11 (279)			0.12 (305)			0.13 (330)			0.14 (356)			0.15 (381)		
	AB	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total
−9	0.0	19.7	19.4	0.0	0.2	0.2												
−8	0.1	9.9	12.0	0.0	20.7	21.9	0.2	9.8	13.2									
−7	0.0	0.1	0.0	0.1	6.3	4.6	0.0	17.8	16.6	0.1	20.7	17.6	0.0	9.3	9.3			
−6	0.1	0.6	1.1	0.0	0.0	0.0	0.1	2.2	3.3	0.2	9.6	12.2	0.0	18.0	19.3	0.1	21.8	24.5
−5	0.0	0.1	0.0	0.1	-0.5	0.1	0.0	0.2	0.1	0.0	0.2	0.1	0.2	2.5	1.4	0.2	7.7	5.3
−4	0.0	0.1	0.3	0.0	0.0	0.1	0.1	0.4	1.0	0.1	0.5	1.2	0.0	0.2	0.3	0.0	0.1	0.1
−3	0.1	0.1	0.0	0.1	0.1	0.0	0.0	0.0	0.0	0.1	0.2	0.0	0.2	0.5	0.0	0.2	0.5	0.1
−2	0.0	0.0	0.0	0.1	0.1	0.6	0.2	0.1	0.7	0.0	0.1	0.2	0.0	0.0	0.1	0.2	0.2	0.7
−1	0.2	0.1	0.0	0.0	0.0	0.0	0.2	0.1	0.0	0.4	0.2	0.0	0.2	0.2	0.0	0.0	0.0	0.0
0	0.1	0.0	0.2	0.3	0.1	0.7	0.0	0.0	0.0	0.2	0.0	0.4	0.6	0.1	1.3	0.7	0.2	1.6
1	0.0	0.0	0.0	0.5	0.1	0.2	0.7	0.0	0.3	0.1	0.0	0.0	0.1	0.0	0.1	0.9	0.1	0.5
2	0.5	0.0	0.8	0.1	0.0	0.1	1.5	0.0	2.2	2.3	0.1	3.2	1.1	0.0	1.6	0.0	0.0	0.0
3	2.1	0.1	1.6	2.1	0.0	1.7	0.6	0.0	0.5	9.4	0.1	8.0	25.5	0.1	22.5	41.5	0.1	37.9
4	3.7	0.0	4.2	27.0	0.1	29.5	50.0	0.0	52.0	51.7	0.0	52.7	37.6	0.1	40.7	20.3	0.1	23.4
5	54.5	0.0	54.0	35.5	0.0	33.1	9.9	0.1	8.4	0.3	0.0	0.2	1.3	0.0	1.0	3.2	0.1	2.3
6	3.2	0.0	3.7	1.4	0.0	1.7	3.0	0.1	3.9	0.8	0.1	1.2	0.1	0.0	0.1	1.5	0.0	1.5
7	2.0	0.0	1.5	0.0	0.0	0.0	1.6	0.1	1.0	0.6	0.0	0.6	0.8	0.0	0.8	68.8	30.9	97.9
8	1.2	0.1	1.8	0.0	0.0	0.0	1.4	0.0	1.4	66.3	31.8	97.6	67.7	31.0	98.5			
9	0.8	0.0	0.8	0.0	0.0	0.0	69.5	30.9	104.6									
	68.6	30.9	101.4	67.3	28.2	94.5												

Order	0.175 (445)			0.20 (508)			0.25 (635)			0.30 (762)			0.40 (1016)			0.50 (1270)		
	AB	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total
−5	0.3	20.7	16.7															
−4	0.3	5.2	8.0	0.1	14.9	17.7												
−3	0.0	0.0	0.0	0.1	1.7	1.0	0.3	12.2	8.6	0.1	23.1	19.9						
−2	0.4	0.5	1.7	0.1	0.1	0.4	0.4	1.2	2.9	0.9	5.8	11.4	0.2	16.5	19.7			
−1	0.3	0.2	0.0	0.8	0.5	0.0	0.1	0.1	0.0	0.2	0.2	0.0	2.1	3.4	0.4	2.5	7.3	1.3
0	0.1	0.0	0.2	0.1	0.1	0.4	2.0	0.5	4.5	1.6	0.3	3.3	0.1	0.1	0.4	4.0	1.0	8.9
1	2.2	0.1	1.2	0.6	0.1	0.3	4.1	0.1	2.8	19.3	0.4	13.8	45.5	0.3	38.6	54.5	0.0	54.5
2	8.9	0.1	11.1	30.7	0.2	35.5	53.6	0.0	53.6	46.4	0.2	52.8	16.7	0.8	24.6	61.0	8.3	64.7
3	52.6	0.0	50.8	33.7	0.2	29.3	2.7	0.1	1.7	2.1	0.0	2.1	64.6	21.1	83.7			
4	0.3	0.0	0.4	2.7	0.1	3.9	63.2	14.2	74.1	70.6	30.0	103.3						
5	0.5	0.0	0.5	68.9	17.9	88.5												
6	65.9	26.8	90.6															

efficiency of the grating. The first and second columns give the percentages which we should have if the faces BC and AB were blackened successively, all the light being reflected in turn from the $15°$ and $30°$ faces respectively. These figures will serve to show in a general way how the phase-relations of the two faces affect the distribution.

A check on the accuracy of the computation is furnished by the condition that the sum of the figures in any column should equal the theoretical total energy. This value is 100 for the columns headed "Total" and 68.3 and 31.7 for the columns headed AB and BC respectively. The agreement is as good as is to be expected from the rather rough methods employed, as far as $\lambda = 0.15a$. Beyond this value there are irregularities, which we will consider briefly. In the first place the values for BC begin to fall short of the theoretical values by considerable amounts. This is due to the fact that the direction of regular reflection from this face of the groove makes such a large angle with the normal to the grating that the center of the diffraction pattern due to this face lies outside the highest order spectrum on that side. The result is that we have light diffracted by this face of the groove in such directions that the energy must be redistributed, by reflection in the other face, among other spectra before it can escape from the grating surface. We are not able to take account of these secondary diffraction phenomena in this elementary treatment, so that this energy is lost to calculation. A second reason for shortage, in the case of the wave-lengths $0.20a$ and $0.25a$, is found in those spectra which escape just at grazing incidence, as this energy must likewise appear in other orders. Finally when the wave-length becomes greater than $0.3a$ the formula (2) becomes quite inaccurate because of the size of the quantity $\frac{\pi c}{\lambda}$.

The effect of the phase-relations of the two faces is greatest, relatively, in the weaker spectra. The phase-relations in a metallic grating would be much more complicated than is assumed here, but the same sort of effect is to be expected in any case, the details of the distribution being of course different.

Another point which is seen immediately is that while the effi-

ciency in the brightest spectrum is a function of the wave-length, it does not increase or decrease continuously with the wave-length, but is periodic. Thus the fifth order for $\lambda = 0.10a$ contains about the same percentage of the total energy as the second order for $\lambda = 0.25a$. The presence of color in the central image is likewise seen to be a property of gratings of this type, it being unnecessary to assume a square groove to explain this phenomenon. It is extremely probable that every form of groove would give such color. The statement made by Rowland (*op. cit.*, p. 6) that in general a simple groove will tend to give a bright first order is not corroborated by this investigation.

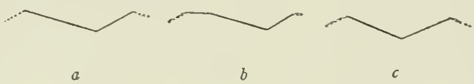


FIG. 2

The range of wave-lengths over which a grating will be bright in any one order is a question of some practical interest. Take, for instance, the fourth order in the above table. We find a fairly high efficiency between $0.11a$ and $0.15a$, or between 279 $\mu\mu$ and 381 $\mu\mu$; this spectrum would be very bright in the ultra-violet. With increasing wave-length the range of high efficiency becomes greater; thus the first order is bright from 762 $\mu\mu$ out to the limit of performance of the grating, the exact figures being, however, not obtainable.

Variation of distribution with angle of illumination.—We will now keep a fixed wave-length, and study the effect of varying the angle of incidence. The results of such a calculation are given in Table II, the wave-length chosen being $0.2a$, or green light for a 10,000 ruling.

The figures at the bottom of the table give the theoretical totals for the columns above them. We see that, for all except the first two angles of incidence, the calculated total for the face BC falls far short of the theoretical value. The obliqueness of the center of the diffraction pattern is the principal cause of this shortage, to which, however, the small effective width of the face contributes largely. For the angles 30° and 45° the light falling on

DISTRIBUTION OF ENERGY AS A FUNCTION OF THE ANGLE OF INCIDENCE

Order	−30°			−15°			0°			15°			30°			45°			60°		
	AB	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total	*AB*	*BC*	Total
−7.......	0.0	0.1	0.1																		
−6.......	0.1	10.4	10.5	0.3	4.9	6.6															
−5.......	0.0	17.8	17.8	0.0	18.8	18.0															
−4.......	0.1	9.3	9.4	0.2	9.2	11.3	0.1	14.9	17.7												
−3.......	0.1	1.4	1.5	0.1	0.8	0.5	0.1	1.7	1.0	0.8	7.7	5.1									
−2.......	0.0	0.1	0.1	0.1	0.2	0.4	0.1	0.1	0.4	0.0	0.0	0.1	1.8	8.7	8.5						
−1.......	0.0	0.7	0.7	0.6	0.6	0.4	0.8	0.5	0.0	0.5	0.4	0.2	0.3	3.4	4.1	2.7	1.2	6.9			
0.......	1.2	0.6	1.8	0.4	0.2	0.9	0.1	0.1	0.4	0.8	0.1	1.3	2.3	0.0	2.2	2.9	1.1	0.6	0.5	—	0.5
1.......	3.2	0.3	3.5	0.0	0.0	0.0	0.6	0.1	0.3	0.5	0.0	0.4	0.0	0.2	0.2	5.4	0.9	10.1	32.0	—	32.0
2.......	59.2	0.0	59.2	44.7	0.1	48.1	30.7	0.2	35.5	27.9	0.1	30.5	35.0	0.0	34.7	51.6	0.7	41.9	54.7	—	54.7
3.......	64.9	40.7	105.4	10.5	0.3	8.4	33.7	0.2	29.3	40.2	0.1	37.9	35.9	0.0	36.3	22.3	0.5	26.0	15.9	—	15.9
4.......				56.9	35.2	94.6	2.7	0.1	3.9	0.3	0.0	0.4	0.2	0.0	0.2	1.0	0.3	0.7	2.2	—	2.2
5.......							68.9	17.9	88.5	0.2	0.0	0.1	0.5	0.0	0.5	0.2	0.1	0.6	0.0	—	0.0
6.......										0.2	0.0	0.2	0.8	0.0	0.8	0.7	0.0	0.5	0.3	—	0.3
7.......										71.4	8.4	76.2	0.8	0.0	0.9	0.6	0.0	0.6	0.3	—	0.3
8.......													77.6	12.3	88.4	0.6	0.0	0.3	0.3	—	0.3
9.......																88.0	5.8	88.2	0.4	—	0.4
																			106.6		106.6
	57.7	42.2		63.4	36.6		68.3	31.7		73.2	26.8		78.8	21.2		86.6	12.0		100.0		

BC undergoes a second reflection before escaping from the grating. This sends the center of the pattern off somewhat less obliquely, and the figures show a slight improvement. The second of the causes just mentioned has become more important, however.

Two things strike one at once on examining this table. In the first place, we see that over the entire range of angles considered, the orders 2 and 3 are both bright; so that a casual observer looking at the grating would say that the second and third orders were both bright, regardless of the particular angle at which he might happen to hold the grating. On the other hand, the fluctuations of intensity of these two spectra are by no means less noticeable, so that a measurement of the intensities of the spectra obtained by keeping the bolometer or other energy-measuring device fixed, and rotating the grating, would give results which were in no way comparable with those obtained by keeping the grating fixed and moving the bolometer. Measurements of spectral intensities which involve the former procedure do not therefore give true measurements of the *distribution* of energy.[1]

It seems worth while to consider briefly the effect of certain variations in the groove-form. If we imagine the groove considered above to have its angle determined by the shape of the ruling point, there are two ways by which with a given point we can alter the character of the ruling. In the first place we may alter the depth of the ruling. In the case considered above the entire original surface of the grating plate has been ruled away; if this is not done there will remain flat portions *CD* (Fig. 2 *b*) of the original surface between each groove, and we must now regard the groove as including one of these flat portions. The general effect of this change should be clear in the light of what has been said above. The directions of the diffraction maxima of the oblique faces will be the same as before, but these faces being now smaller, the intensities of the disturbances due to them will be cut down. We shall have, besides, a new disturbance with its maximum in the direction of the central image. The general effect will thus be to

[1] See for instance Wood, *Physical Optics*, p. 179; and Trowbridge and Wood, *op. cit.*, p. 6. The inapplicability of the method is evident when we consider that by rotating the grating we get in all twice as many spectra as the grating can give in any one position.

increase the brightness of the central image at the expense of the other spectra. Phase-relations will no doubt complicate this, but not in such a way as to alter the general character of the effect.

A second way of changing the ruling is by tilting the point in the plane of incidence. The general effect of this should likewise be clear—the direction of maximum efficiency will be shifted to other orders, and in addition the orders on one side will be enhanced at the expense of the others owing to the change in the relative area of the two faces (Fig. 2 *c*).

Practical application.—In conclusion we may inquire how far the results obtained above should be applicable to the case of actual gratings. The limitations of the formula for the diffraction of a single slit are obvious; it takes no account of polarization, and is, moreover, strictly applicable only to the case of wide slits. That it gives reasonably consistent results is itself no criterion for its applicability; an examination of the question from the standpoint of electromagnetic theory could alone answer this question. We have at present, however, no better way of treating diffraction phenomena, save in the case of a single diffracting edge, the complete theory of which has been worked out by Sommerfeld. The phase-changes on reflection assumed above are undoubtedly too simple, but correction on this head seems to offer no insuperable difficulties. The foregoing theory is believed to be at least more complete than any heretofore advanced, though undoubtedly capable of much further improvement. The question may be asked finally, how far the assumption of grooves with plane faces is realized in practice. The pictures of a grating groove given in the textbooks—curved and irregular—may perhaps be taken as indications of current ideas on the subject, so that the triangular groove would appear to many too simple. It is certainly true that no matter how irregular the shape of the grating groove, the grating will be good provided the grooves are identical and equally spaced. The experience of one of the writers, however, who has had charge of the ruling engines in this laboratory during the past year, enables us to state with some assurance the conviction that such gratings have seldom if ever been ruled. In the first place the ruling is done, not with a *point*, as is commonly supposed, but with the

edge of the natural diamond; and it is found that unless the edge is so sharp as to possess no visible width under the highest magnifying power the diamond will not rule; further, that a diamond, after continued use, will cease to rule without the edge showing visible bluntness. The metal of the grating is thus probably not gouged or ploughed out, but pressed aside by the edge, and follows in general the shape of this edge. This view is also borne out by the fact that tilting the diamond in the plane of incidence changes the direction of maximum intensity in the manner to be expected on the above theory. If the groove were simply an irregular scratch this were not to be expected.

Physical Laboratory
Johns Hopkins University
March 3, 1911

PHOTOGRAPHIC DETERMINATIONS OF STELLAR PARALLAX MADE WITH THE YERKES REFRACTOR. V

By FRANK SCHLESINGER

Lalande 25372 ($13^h 41^m, +15° 26'$)

This is a star of the ninth magnitude having an annual proper motion of 2″.3. The eleven plates secured are described in Table 1.

TABLE 1

Plates of *Lalande 25372*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
200 ...	1904 Jan. 26	$+0^h.2$	S	Poor	
229 ...	Feb. 14	−1.6	S, S	Good	
242 ...	Mar. 3	−0.1	S, Su, S	Fair	
342 ...	May 26	+1.0	S, Su, S	Good	
351 ...	June 4	+1.2	S, F, S	Poor	
581 ...	1905 Jan. 15	−0.4	S, Su, S	Good	
592 ...	Jan. 29	+0.3	S, Su, S	Fair	Second exposure poor
609 ...	Feb. 17	+0.2	S, Su, S	Poor	
618 ...	Feb. 25	−0.1	S, Su, S	Good	
653 ...	Apr. 22	+0.4	F, Su, F	Poor	
683 ...	May 22	+1.2	F, Su, F	Poor	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence Computed	Dependence Adopted
2	0.67	+250	−372	−.006	.00
3	0.57	+278	0	+.586	+.58
19	0.82	−160	+165	+.303	+.30
30	0.68	−368	+207	+.117	+.12
Parallax star.	1.03	+ 70.6	+ 75.3		

The parallax star and all the comparison stars except No. 2 are nearly in the same straight line. The dependence on No. 2 under these circumstances is necessarily small; this comparison star was not used in the reductions.

Plates 229, 242, 342, and 351 (two of which have negative parallax factors and two positive) were measured by both Miss Ware and the writer; the others by Miss Ware alone.

TABLE 2

REDUCTIONS FOR *Lalande 25372*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p} \cdot v$ in Arc
200	0.006	0.2	+0.932	−294	+.014	+″02
229	0.026	0.8	+0.778	−275	+ 1	.00
242	0.046	0.8	+0.554	−257	− 7	−.02
342	0.166	1.0	−0.755	−173	0	.00
351	0.182	0.5	−0.850	−164	0	.00
581	0.801	0.9	+0.973	+ 61	+ 12	+.03
592	0.810	0.6	+0.909	+ 75	− 7	−.01
609	0.839	0.4	+0.736	+ 94	− 11	−.02
618	0.856	0.9	+0.638	+102	− 5	−.01
653	0.932	0.4	−0.263	+158	− 4	−.01
683	0.989	0.4	−0.705	+188	+ 11	+.02

The normal equations are:

$$
\begin{aligned}
+3.991\pi - 0.286\mu + 1.973c &= +1.340 \\
+21.776 \quad -3.717 &= +2.646 \\
+6.900 &= +3.398
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.597 \\
\mu &= +0.2241 = +0''.596 \\
\pi &= +0.0568 = +0''.152 \pm 0''.007
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0046 = \pm 0''.012$

Other determinations of this parallax are:

Flint (transit circle)	+0″43	±0″065
Elkin (heliometer)	+ .17	55
Russell (photography)	+ .22	19

Fedorenko 2544 ($14^h\ 52^m$, $+54°\ 4'$)

This 8th-magnitude star has an annual proper motion of 1″1. The twelve plates secured were measured in both right ascension

and in declination, and independent values of the parallax derived from the displacements in these two directions.

TABLE 1

PLATES OF *Fedorenko 2544*

No.	Date		Hour Angle	Observers	Quality of Images	Remarks
293 ...	1904	Apr. 26	-2^h0	S, Su, S	Poor	
298 ...		Apr. 28	-0.9	S, Su, S	Poor	
310 ...		May 1	-0.2	S, Su, S	Fair	
320 ...		May 15	-0.4	S, Su, S	Good	
341 ...		May 26	-0.7	S, Su, S	Fair	
350 ...		June 4	-0.4	S, F, S	Poor	
366 ...		June 19	-0.1	Su, S	Poor	
593 ...	1905	Jan. 29	-0.3	S, Su, S	Fair	Star (40) lacking on first exposure
610 ...		Feb. 17	-0.4	S, Su, S	Fair	
619 ...		Feb. 25	-0.8	S, Su, S	Poor	
654 ...		Apr. 22	-0.2	F, Su, F	Fair	
692 ...		June 24	-0.1	F, Su, F	Fair	

COMPARISON STARS

No.	Diameter	X (right ascension)	Y (declination)	Dependence	
				Computed	Adopted
6	1.06	+384	+ 49	+.127	+.125
9	0.93	+242	−129	+.042	+.04
21	0.71	− 37	− 19	+.162	+.165
24	0.77	− 78	−196	+.059	+.06
27	0.56	− 98	+258	+.345	+.343
40	0.94	−413	+ 37	+.266	+.267
Parallax star.	1.28	− 95	+ 85		

Plates 310, 320, and 366 were measured by both Miss Ware and the writer; the others by Miss Ware alone. Had Miss Ware's measures alone been used for these three plates, none of the quantities m and l would have been changed by more than $0.006 = 0\overset{''}{.}016$, from those given in Tables 2 and 3.

TABLE 2

REDUCTIONS IN RIGHT ASCENSION FOR *Fedorenko 2544*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p} \cdot v$ in Arc
293	0.536	0.3	+0.152	−103	+.014	+.″02
298	0.575	0.3	+0.119	−101	+ 56	+.08
310	0.521	0.8	+0.069	− 98	+ 6	+.01
320	0.476	1.0	−0.157	− 84	− 21	−.06
341	0.487	0.7	−0.329	− 73	+ 6	+.01
350	0.449	0.4	−0.463	− 64	− 20	−.03
366	0.444	0.4	−0.660	− 49	+ 6	+.01
593	0.266	0.7	+0.938	+175	+ 10	+.02
610	0.230	0.7	+0.919	+194	− 5	−.01
619	0.204	0.4	+0.881	+202	− 23	−.04
654	0.154	0.7	+0.218	+258	− 2	.00
692	0.080	0.7	−0.715	+321	+ 8	+.02

The normal equations are:

$$\begin{aligned} +2.284\pi + \ 2.307\mu + 0.606c &= +0.064 \\ +21.014 \ \ +4.245 \ &= -0.338 \\ +7.100 \ &= +2.518 \end{aligned}$$

These yield

$$\begin{aligned} c &= +0.414 \\ \mu &= -0.1020 = -0.''271 \\ \pi &= +0.0212 = +0.''056 \pm 0.019 \end{aligned}$$

Probable error corresponding to unit weight, $\pm 0.0101 = \pm 0.''027$

As will be seen from the next to last column in Table 2, the residual for Plate 298 is unusually large. A solution without this plate yields $+0.''050$ for the parallax; this is only slightly different from the first value, as we might have anticipated from the low weight and the small parallax factor for this plate. But in this second solution the probable error of the parallax is reduced to $0.''014$. Not to overestimate the accuracy of the result, the original solution is regarded as the definitive one.

TABLE 3

REDUCTIONS IN DECLINATION FOR *Fedorenko 2544*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
293........	0.037	0.3	+0.922	−103	−.004	−.″01
298........	0.050	0.3	+0.930	−101	+ 8	+.01
310........	0.068	0.8	+0.941	− 98	+ 24	+.06
320........	0.027	1.0	+0.956	− 84	− 24	−.06
341........	0.064	0.7	+0.932	− 73	+ 8	+.02
350........	0.066	0.4	+0.888	− 64	+ 6	+.01
366........	0.056	0.4	+0.770	− 49	− 8	−.01
593........	0.167	0.7	−0.186	+175	− 2	.00
610........	0.178	0.7	+0.126	+194	− 3	−.01
619........	0.200	0.4	+0.254	+202	+ 13	+.02
654........	0.213	0.7	+0.902	+258	− 8	−.02
692........	0.258	0.7	+0.723	+321	+ 6	+.01

The normal equations are:

$$
\begin{aligned}
+4.292\pi + 0.439\mu + 4.777c &= +0.463 \\
+21.014 \quad +4.245 \quad &= +1.411 \\
+7.100 \quad &= +0.853
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.083 \\
\mu &= +0.0501 = +0''.133 \\
\pi &= +0.0105 = +0''.028 \pm 0''.025
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0081 = \pm 0''.022$

Combining the two determinations in accordance with their probable errors we have for the final value

$$\pi = +0''.046 \pm 0''.015$$

Other determinations of this parallax are:

Peter (heliometer)..........	+0″.08	±0″.022
Flint (transit circle)........	− .04	36
Chase (heliometer).........	+ .07	40

Weisse I, 17^h 322 (17^h 21^m, $+2°$ 14′)

This 8th-magnitude star has a proper motion of 1″4 per annum. Sixteen plates were obtained as described in Table 1; they were measured by Miss Ware alone.

TABLE 1

PLATES OF *Weisse I*, 17^h322

No.	Date		Hour Angle	Observers	Quality of Images	Remarks
68...	1903	Aug. 9	$+1^h7$	S, Su, S	Fair	Telescope East
86...		Sept. 11	+2.2	S, Su, S	Poor	Telescope East
93...		Sept. 20	+2.1	S, S	Fair	Telescope East
259...	1904	Mar. 22	−0.6	S, Su, S	Good	
270...		Mar. 27	−0.7	S, Su, S	Fair	
280...		Apr. 3	−0.4	S, S	Good	
285...		Apr. 17	−0.9	S, Su	Poor	
288...		Apr. 19	−0.4	S, Su, S	Fair	
322...		May 15	+0.6	S, Su, S	Good	
402...		July 28	+1.3	S, Su, S	Fair	
658...	1905	Apr. 22	+0.3	F, Su, F	Poor	
665...		Apr. 28	+0.4	F, Su, F	Fair	
668...		May 6	0.0	F, Su, F	Fair	
694...		June 24	+0.5	F, Su, F	Good	
728...		July 29	+1.8	F, Su, F	Fair	
731...		Aug. 4	+1.4	F, Su, F	Good	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
13..........	0.70	−231	+201	+.272	+.275
15..........	0.80	− 58	−186	+.451	+.45
18..........	1.10	+ 36	−191	+.379	+.375
22..........	1.20	+253	+176	−.102	−.10
Parallax star.	1.48	−101	−120		

TABLE 2

REDUCTIONS FOR *Weisse I, 17h322*

Plate	Solution (*m*)	Weight (*p*)	Parallax Factor (*P*)	Time in Days (*t*)	First Solution Residual (*v*)	Second Solution Residual (*v*)
68........	0.509	0.7	−0.852	−364	+.035	......
86........	0.453	0.4	−1.006	−331	+ 4	......
93........	0.427	0.5	−0.995	−322	− 18	......
259........	0.404	0.9	+0.970	−138	− 24	−.020
270........	0.444	0.7	+0.948	−133	+ 20	+ 23
280........	0.402	0.6	+0.906	−126	− 16	− 11
285........	0.384	0.3	+0.783	−112	− 22	− 16
288........	0.425	0.7	+0.761	−110	+ 21	+ 27
322........	0.366	0.9	+0.413	− 84	− 10	+ 1
402........	0.258	0.7	−0.735	− 10	− 28	− 6
658........	0.204	0.4	+0.731	+258	+ 1	− 11
665........	0.201	0.7	+0.658	+264	+ 4	− 7
668........	0.191	0.7	+0.550	+272	+ 4	− 7
694........	0.160	0.8	−0.242	+321	+ 32	+ 31
728........	0.079	0.7	−0.745	+356	− 8	− 5
731........	0.069	0.9	−0.809	+362	− 12	− 8

The normal equations are:

$$
\begin{aligned}
+6.253\pi - 0.877\mu + 0.904c &= +0.594 \\
+66.240 + 2.759 &= -2.776 \\
+10.600 &= +3.192
\end{aligned}
$$

These yield

$$c = +0.312$$
$$\mu = -0.0543 = -0\overset{''}{.}144$$
$$\pi = +0.0424 = +0\overset{''}{.}113 \pm 0\overset{''}{.}013$$

Probable error corresponding to unit weight, $\pm 0.0119 = \pm 0\overset{''}{.}032$

We see in Table 1 that the three earliest plates were secured with the telescope east of the pier, while for the others the telescope was (as usual) to the west of the pier. The hour angles for these early plates also differ widely from the others. A second least-

squares solution was therefore undertaken from which these plates were omitted. This yielded

$$c=+0.301$$
$$\mu=-0.0493=-0''.131$$
$$\pi=+0.0566=+0''.150\pm 0''.017$$

Probable error corresponding to unit weight, $\pm 0''.028$

This value of the parallax differs considerably from the former, being 0".037 greater. Similar computations in the case of other regions in this list indicate that the reversal of the telescope and the hour-angle error combined have only a slight effect upon the relative position of the parallax star. Nevertheless, as the latter of the two values is entirely free from any such effects, it is probably nearer the truth, in spite of its somewhat greater probable error. As the best value that the present data afford, I therefore adopt

$$+0''.14\pm 0''.020$$

Other determinations of this parallax are:

Russell (photography)	+0".095	±0".012
Chase (heliometer)	+ .134	14
Flint (transit circle, first series)	+ .17	55
Flint (transit circle, second series)	+ .194	27

The last of these has been kindly communicated by Professor Flint in advance of publication.

Positiones Mediae 2164 ($18^h\,42^m$, $+59°\,29'$)

This well-known double star, otherwise designated as *Struve 2398*, is a system of unusually great interest. The two components are now separated by about 17", this distance having increased

nearly $5''$ since it was first measured by Struve in 1832. The preceding component is of the 8.7 magnitude and has a spectrum of the K type. The following component emits about half as much visual light: so far as I know its spectrum has never been investigated, but observers report that it is whiter than the other component. The whole system is in rapid motion, $2''.3$ per annum.

In 1904[1] I derived a preliminary parallax for this binary from eleven plates taken with the 40-inch telescope. These plates were not competent to determine both the proper motion and the parallax, so that the former was assumed. In this connection it was necessary to allow for the relative motion of the two components. In comparing the position of the fainter component as derived from the plates themselves, with previous micrometric determinations, I noticed that the relative motion must have recently changed its direction by nearly 90°. Measures kindly made at my request by Professor Barnard and Professor Burnham confirmed this, and showed that the pair has comparatively rapid orbital motion, when we consider the great separation and the faintness of the components.

The present determination of the parallax depends upon the twenty-three plates described in Table 1. The distribution in time is excellent, so that the parallax and the proper motions may be determined from them with very small probable errors. This region is close to the pole of the ecliptic, so that the displacements in declination are considerable. The plates were therefore measured in this co-ordinate as well as in right ascension. Furthermore, both components are well measurable and we may therefore derive four nearly independent values of the parallax.

[1] *Astrophysical Journal*, 20, 129, 1904. This computation is superseded by the present work, in which the same plates are definitively discussed in connection with others secured later.

TABLE 1

PLATES OF *Positiones Mediae 2164*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
84....	1903 Aug. 20	$+1^h7$	S, Su, S	Good	Telescope East
87....	Sept. 11	+1.6	S, Su, S	Good	Telescope East. Star (28) lacking on first exposure
90....	Sept. 17	+1.8	S. S, S	Good	Telescope East
94....	Sept. 20	+1.4	S, Su, S	Fair	Telescope East
106....	Sept. 27	+0.6	S, Su, S	Good	Telescope East
150....	Nov. 1	+2.1	S, Su, S	Poor	Telescope East
281....	1904 Apr. 3	−1.2	S, Su	Good	
286....	Apr. 17	−1.5	S	Fair	
289....	Apr. 19	−1.2	S, Su, S	Poor	
323....	May 15	−0.3	S, Su, S	Good	
327....	May 17	−0.6	S, Su, S	Good	
335....	May 20	−0.9	S, Su, S	Good	
353....	June 4	−2.0	S, F, S	Good	
427....	Aug. 25	−0.2	S, Su, S	Good	
438....	Aug. 27	−0.4	S, Su, S	Good	Images slightly triangular
443....	Sept. 4	−0.2	S, Su	Poor	Images triangular
659....	1905 Apr. 22	−0.6	F, Su, F	Fair	
664....	Apr. 28	−1.5	F, Su, F	Good	
669....	May 6	−0.9	F, Su, F	Fair	
675....	May 20	−0.5	F, Su, F	Good	
701....	July 15	−0.1	F, Su, F	Good	
749....	Sept. 2	−0.3	Su, Su, Su	Fair	
753....	Sept. 10	0.0	Su, Su, Su	Good	

COMPARISON STARS

No.	Diameter	X (right ascension)	Y (declination)	Dependence	
				Computed	Adopted
1...........	0.72	−378	+ 62	+.248	+.25
2...........	0.82	− 8	+200	+.480	+.48
3...........	0.72	+ 84	−130	+.115	+.11
4...........	1.58	+302	−132	+.156	+.16
Parallax stars	+1.14 +0.68	− 41.8 − 38.8	+ 78.4 + 72.7		

These dependences were computed with the mean position of the two parallax stars, and apply alike to the reductions for both.

Plates 84 to 106 and 281 to 443 were measured by both Miss Ware and the writer, the other eight by Miss Ware alone.

TABLE 2

REDUCTIONS IN RIGHT ASCENSION FOR THE PRECEDING STAR OF *P.M. 2164*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
84........	1.086	1.0	−0.742	−351	+.005	+″01
87........	1.028	1.0	−0.933	−331	− 7	−.02
90........	1.030	1.0	−0.966	−325	+ 7	+.02
94........	1.020	0.8	−0.977	−322	+ 2	.00
106........	1.003	0.8	−0.995	−315	− 4	−.01
150........	0.964	0.4	−0.869	−280	− 10	−.02
281........	0.964	0.6	+0.994	−126	− 6	−.01
286........	0.944	0.4	+0.951	−112	− 2	.00
289........	0.941	0.5	+0.940	−110	− 2	.00
323........	0.897	1.0	+0.708	− 84	+ 12	+.03
327........	0.877	1.0	+0.684	− 82	− 2	−.01
335........	0.863	1.0	+0.647	− 79	− 8	−.02
353........	0.829	1.0	+0.436	− 64	− 1	.00
427........	0.596	0.9	−0.803	+ 18	+ 5	+.01
438........	0.595	0.8	−0.822	+ 20	+ 8	+.02
443........	0.564	0.4	−0.891	+ 28	− 5	−.01
659........	0.461	0.7	+0.923	+258	+ 1	.00
664*.......	0.450	0.9	+0.880	+264	+ 2	+.01
669........	0.430	0.7	+0.808	+272	+ 1	.00
675........	0.399	0.9	+0.648	+286	+ 5	+.01
701........	0.228	0.9	−0.235	+342	0	.00
749........	0.071	0.7	−0.874	+391	− 26	−.06
753........	0.093	0.9	−0.931	+399	+ 12	+.03

* Plate 664 was used to illustrate the method of reduction. As a matter of arithmetical convenience, 1.700 has been added to each m.

The normal equations are:

$$
\begin{aligned}
+12.117\pi + 11.335\mu - 1.457c &= -1.225 \\
+116.801 - 2.630 &= -15.916 \\
+18.300 &= +13.005
\end{aligned}
$$

These yield

$$c = +0.700$$

$$\mu = -0.1307 = -0\rlap{.}''348$$

$$\pi = +0.1055 = +0\rlap{.}''281 \pm 0\rlap{.}''004$$

Probable error corresponding to unit weight, $\pm 0.0051 = \pm 0\rlap{.}''014$

TABLE 3

REDUCTIONS IN RIGHT ASCENSION FOR THE FOLLOWING STAR OF *P.M. 2164*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
84........	1.100	1.0	−0.742	−351	+.004	+″.01
87........	1.060	1.0	−0.933	−331	+ 11	+.03
90........	1.044	1.0	−0.966	−325	+ 8	+.02
94........	1.030	0.8	−0.977	−322	− 1	.00
106........	1.016	0.8	−0.995	−315	− 4	−.01
150........	0.928	0.4	−0.869	−280	− 57	−.10
281........	0.980	0.6	+0.994	−126	+ 6	+.01
286........	0.948	0.4	+0.951	−112	− 2	.00
289........	0.932	0.5	+0.940	−110	− 16	−.03
323........	0.884	1.0	+0.708	− 84	− 3	−.01
327........	0.882	1.0	+0.684	− 82	+ 1	.00
335........	0.880	1.0	+0.647	− 79	+ 7	+.02
353........	0.825	1.0	+0.436	− 64	− 6	−.02
427........	0.593	0.9	−0.803	+ 18	+ 4	+.01
438........	0.600	0.8	−0.822	+ 20	+ 16	+.04
443........	0.572	0.4	−0.891	+ 28	+ 6	+.01
659........	0.455	0.7	+0.923	+258	+ 10	+.02
664........	0.437	0.9	+0.880	+264	+ 4	+.01
669........	0.409	0.7	+0.808	+272	− 6	−.01
675........	0.379	0.9	+0.648	+286	0	.00
701........	0.203	0.9	−0.235	+342	− 6	−.02
749........	0.069	0.7	−0.874	+391	− 7	−.02
753........	0.062	0.9	−0.931	+399	+ 3	+.01

The normal equations are:

$$\begin{aligned} +12.117\pi + 11.335\mu - 1.457c &= -1.282 \\ +116.801 - 2.630 &= -16.476 \\ +18.300 &= +12.985 \end{aligned}$$

These yield

$$c = +0.698$$
$$\mu = -0.1355 = -0.''360$$
$$\pi = +0.1051 = +0.''280 \pm .''006$$

Probable error corresponding to unit weight, $\pm 0.0070 = \pm 0.''019$

TABLE 4

REDUCTIONS IN DECLINATION FOR THE PRECEDING STAR OF *P.M. 2164*

Plate	Solution (l)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
84........	0.008	1.0	+0.672	−351	−.011	−″03
87........	0.020	1.0	+0.355	−331	− 3	−.01
90........	0.031	1.0	+0.258	−325	+ 6	+.02
94........	0.026	0.8	+0.210	−322	+ 1	.00
106........	0.027	0.8	+0.092	−315	+ 1	.00
150........	0.036	0.4	−0.482	−280	+ 4	+.01
281........	0.383	0.6	+0.090	−126	− 12	−.02
286........	0.456	0.4	+0.323	−112	+ 9	+.02
289........	0.434	0.5	+0.355	−110	− 20	−.04
323........	0.546	1.0	+0.722	− 84	+ 1	.00
327........	0.553	1.0	+0.744	− 82	+ 2	+.01
335........	0.570	1.0	+0.777	− 79	+ 9	+.02
353........	0.606	1.0	+0.908	− 64	+ 1	.00
427........	0.735	0.9	+0.599	+ 18	+ 4	+.01
438........	0.737	0.8	+0.572	+ 20	+ 5	+.01
443........	0.754	0.4	+0.454	+ 28	+ 17	+.03
659........	1.181	0.7	+0.398	+258	+ 3	+.01
664........	1.192	0.9	+0.489	+264	− 7	−.02
669........	1.215	0.7	+0.603	+272	− 12	−.03
675........	1.269	0.9	+0.775	+286	− 4	−.01
749........	1.443	0.7	+0.487	+391	− 4	−.01
753........	1.459	0.9	+0.365	+399	+ 10	+.03

The normal equations are:

$$\begin{aligned} +5.467\pi + \ 0.480\mu + \ 8.532c &= + \ 6.068 \\ +106.311 \ - \ 5.708 \ &= +17.204 \\ +17.400 \ &= +10.789 \end{aligned}$$

These yield

$$c = +0.631$$
$$\mu = +0.1952 = +0.''519$$
$$\pi = +0.1076 = +0.''286 \pm 0.''012$$

Probable error corresponding to unit weight, $\pm 0.0049 = \pm 0.''013$

TABLE 5

REDUCTIONS IN DECLINATION FOR THE FOLLOWING STAR OF *P.M. 2164*

Plate	Solution (l)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
84	0.104	1.0	+0.672	−351	−.010	−.″03
87	0.120	1.0	+0.355	−331	+ 7	+.02
90	0.120	1.0	+0.258	−325	+ 6	+.02
94	0.120	0.8	+0.210	−322	+ 7	+.02
106	0.113	0.8	+0.092	−315	0	.00
150	0.087	0.4	−0.482	−280	− 24	−.04
281	0.482	0.6	+0.090	−126	+ 12	+.02
286	0.533	0.4	+0.323	−112	+ 10	+.02
289	0.512	0.5	+0.355	−110	− 19	−.04
323	0.636	1.0	+0.722	− 84	+ 12	+.03
327	0.620	1.0	+0.744	− 82	− 11	−.03
335	0.663	1.0	+0.777	− 79	+ 23	+.06
353	0.673	1.0	+0.908	− 64	− 11	−.03
427	0.784	0.9	+0.599	+ 18	− 18	−.05
438	0.799	0.8	+0.572	+ 20	− 4	−.01
443	0.822	0.4	+0.454	+ 28	+ 18	+.03
659	1.233	0.7	+0.398	+258	+ 1	.00
664	1.253	0.9	+0.489	+264	− 1	.00
669	1.287	0.7	+0.603	+272	+ 4	+.01
675	1.319	0.9	+0.775	+286	− 11	−.03
749	1.486	0.7	+0.487	+391	− 8	−.02
753	1.510	0.9	+0.365	+399	+ 16	+.04

The normal equations are:

$$
\begin{aligned}
+5.467\pi + 0.480\mu + 8.532c &= +6.685\\
+106.311 - 5.708 &= +16.161\\
+17.400 &= +12.067
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.697\\
\mu &= -0.1889 = +0.''503\\
\pi &= +0.1186 = +0.''314 \pm 0.''019
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0077 = \pm 0.''020$

Collecting the results for the parallax we have,

From the right ascensions of the preceding star . . .	+0.″281	±0.004
From the right ascensions of the following star . . .	+ .280	6
From the declinations of the preceding star.	+ .286	12
From the declinations of the following star	+ .314	19

The four determinations agree fully as well as we should expect from their probable errors. Combining them in accordance with these errors we have as the definitive parallax of *P.M. 2164*,

$$+0.''282 \pm 0.''003$$

Other determinations are:

Lamp (equatorial micrometer)......	+0.″353	±0.″014
Kostinsky (photography)..........	+ .29	21
Russell (photography)............	+ .296	31
Flint (transit circle)...............	+ .32	43
Bohlin (photography).............	+ .25	

These results are unusually accordant and there are few stars whose distances are known with a smaller percentage of probable error than this.

If we examine the residuals for our four solutions in the same way that we did those for *Fedorenko 1457–8* ($9^h\ 8^m$, $+53° 7'$) we obtain from the right ascensions

$$\epsilon_1 = \pm 0.0053 = \pm 0.''014$$
$$\epsilon = \pm 0.0000 = \pm 0.''000 \text{ (from Table 2)}$$
$$\epsilon = \pm 0.0046 = \pm 0.012 \text{ (from Table 3)}$$

Here ϵ_1 is the purely accidental error of measurement, and ϵ is a systematic error that tends to shift both parallax stars in the same direction. A similar calculation for the declinations yields,

$$\epsilon_1 = \pm 0.0052 = \pm 0.''014$$
$$\epsilon = \pm 0.0000 = \pm 0.000 \text{ (from Table 4)}$$
$$\epsilon = \pm 0.0058 = \pm 0.015 \text{ (from Table 5)}$$

From these figures we conclude that there may be present a slight tendency to residuals of the same sign for the two parallax stars; but the effect is very small and can be completely accounted for by distortions of the film and the fact that the same comparison stars were used in both cases.

The remarks in Table 1 inform us that the six earliest plates were secured with the telescope east of the pier, whereas all the later ones were secured with the telescope in the usual position, west of the pier. The hour angles for these plates also differ considerably from the others. As the plates secured with the telescope in the usual position cover four parallax maxima, we have

here an excellent opportunity for testing the effect of reversing and the hour-angle error. The first six plates are accordingly omitted and four new least-squares solutions carried out. From the displacements in right ascension of the preceding (brighter) star, I obtain:

$$c=+0.700$$
$$\mu=-0.1308=-0''.348$$
$$\pi=+0.1050=+0''.279\pm0''.006$$

Probable error corresponding to unit weight, $\pm0''.015$

From the displacements in right ascension of the following star:

$$c=+0.700$$
$$\mu=-0.1362=-0''.362$$
$$\pi=+0.1028=+0''.273\pm0''.0045$$

Probable error corresponding to unit weight, $\pm0''.011$

From the displacements in declination of the preceding star:

$$c=+0.629$$
$$\mu=+0.1949=+0''.518$$
$$\pi=+0.1136=+0''.302\pm0''.020$$

Probable error corresponding to unit weight, ±0.014

From the displacements in declination of the following star:

$$c=+0.707$$
$$\mu=+0.1883=+0''.501$$
$$\pi=+0.1033=+0''.275\pm0''.032$$

Probable error corresponding to unit weight, $\pm0''.022$

The accordance of these results with those that flow from the use of all the plates is remarkable. The weighted mean value of the parallax is now $+0''.276\pm0''.004$, which is $0''.006$ less than before. In right ascension, the new weighted mean is less by just this amount, $0''.006$; while that in declination (which has much lower weight) remains unchanged. An equally good agreement exists between the two sets of proper motions. We therefore conclude that for this region at least, the reversal of the telescope and the hour-angle error have had no appreciable effect upon the relative positions of the parallax stars. We shall return to this subject later in connection with other stars and in a general discussion.

Lamont 18180 ($18^h\ 53^m$, $+5°\ 48'$)

This 9th-magnitude star has a proper motion of 1″.2 per annum. Twenty-one plates were secured, as described in Table 1; these were all measured by Miss Ware.

TABLE 1

PLATES OF *Lamont 18180*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
19....	1903 June 14	$-1^h.0$	S, S, S	Poor	
20....	June 14	+0.2	S, S, S, S	Fair	
57....	Aug. 2	+0.8	S	Poor	Telescope East
76....	Aug. 16	+1.7	S, Su, S	Good	Telescope East. Images elongated on second exposure
107....	Sept. 27	+0.3	S, Su, S	Good	Telescope East
111....	Oct. 11	+2.6	S, Su, S	Good	Telescope East
136....	Oct. 25	+1.7	S, Su, S	Good	Telescope East
324....	1904 May 15	0.0	S, Su, S	Good	
328....	May 17	−0.4	S, Su, S	Good	
336....	May 20	−0.6	S, Su, S	Poor	
346....	May 26	−0.7	S, Su, S	Fair	
428....	Aug. 25	+0.2	S, Su, S	Good	
439....	Aug. 27	0.0	S, Su, S	Good	
455....	Sept. 11	+0.2	S, Su, S	Good	
666....	1905 Apr. 28	−0.7	F, Su, F	Good	
670....	May 6	−0.6	F, Su, F	Good	
676....	May 20	0.0	F, Su, F	Good	
686....	May 30	+0.2	F, Su, F	Good	
732....	Aug. 4	+0.4	F, Su, F	Good	
745....	Aug. 26	0.0	Su	Good	
754....	Sept. 10	+0.7	Su, Su	Fair	

COMPARISON STARS

No.	Diameter	*X* (longitude)	*Y* (latitude)	Dependence	
				Computed	Adopted
1...........	1.65	−162	− 52	−.277	−.25
3...........	0.61	− 45	+ 93	+.329	$+.333=\frac{1}{3}$
4...........	0.66	+ 6	+ 78	+.403	+.40
5...........	0.80	+ 80	−144	+.023	$+.017=\frac{1}{60}$
7...........	0.72	+121	+ 25	+.521	+.50
Parallax star.	0.88	+ 97	+ 90		

TABLE 2

REDUCTIONS FOR *Lamont 1818o*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	First Solution Residual (v)	Second Solution Residual (v)
19........	0.320	0.4	+0.386	−420	+.002	+.011
20........	0.288	0.8	+0.385	−420	− 30	− 20
57........	0.318	0.2	−0.422	−371	+ 30	......
76........	0.281	0.8	−0.622	−357	+ 1	......
107........	0.250	0.9	−0.982	−315	− 10	......
111........	0.277	0.9	−0.997	−308	+ 19	......
136........	0.269	0.8	−0.954	−287	+ 17	......
324........	0.216	0.9	+0.775	− 84	− 7	− 4
328........	0.227	0.8	+0.753	− 82	+ 6	+ 7
336........	0.202	0.4	+0.720	− 79	− 18	− 16
346........	0.238	0.7	+0.644	− 73	+ 22	+ 24
428........	0.145	0.9	−0.742	+ 18	− 17	− 9
439........	0.147	0.9	−0.766	+ 20	− 14	− 5
455........	0.170	0.9	−0.902	+ 35	+ 16	+ 25
666........	0.107	0.9	+0.925	+264	− 12	− 16
670........	0.134	0.9	+0.865	+272	+ 19	+ 15
676........	0.129	0.8	+0.721	+286	+ 21	+ 17
686........	0.088	0.9	+0.595	+296	− 14	− 18
732........	0.061	0.9	−0.460	+362	− 1	0
745........	0.031	0.4	−0.751	+384	− 19	− 17
754........	0.032	0.5	−0.893	+399	− 10	− 7

The normal equations are:

$$\begin{aligned} +9.250\pi + \ 8.820\mu - \ 1.234c &= -0.320 \\ +113.158 \ - \ 1.558 \ &= -3.607 \\ +15.600 \ &= +2.859 \end{aligned}$$

These yield

$$\begin{aligned} c &= +0.182 \\ \mu &= -0.0308 = -0\overset{''}{.}082 \\ \pi &= +0.0191 = +0\overset{''}{.}051 \pm 0\overset{''}{.}009 \end{aligned}$$

Probable error corresponding to unit weight, $\pm 0.0100 = \pm 0\overset{''}{.}027$

The five plates from 57 to 136 inclusive were taken with the telescope east of the pier. A second solution was carried out from which these plates were omitted and the following values resulted:

$$\begin{aligned} c &= +0.177 \\ \mu &= -0.0290 = -0\overset{''}{.}077 \\ \pi &= +0.0246 = +0\overset{''}{.}065 \pm 0\overset{''}{.}010 \end{aligned}$$

Probable error corresponding to unit weight, $\pm 0\overset{''}{.}026$

This value of the parallax is 0."014 greater than the former. We may adopt

$$+0.''06 \pm 0.''012$$

as being the best that can be derived from the present data.

From plates secured at Cambridge (England) Russell has recently obtained $+0.''076 \pm 0.''065$ for this parallax.

Lamont 18816 ($19^h\ 2^m$, $+7°\ 29'$)

This 9th-magnitude star has a proper motion of 0."8 per annum. The eleven plates secured were all measured by Miss Ware.

TABLE I

PLATES OF *Lamont 18816*

No.	Date	Hour Angle	Observers	Quality of Images
429	1904 Aug. 25	$+0^h.6$	S, Su, S	Fair
440	Aug. 27	+0.4	S, Su, S	Fair
456	Sept. 11	+0.6	S, Su, S	Good
667	1905 Apr. 28	−0.5	F, Su	Poor
671	May 6	−0.2	F, Su, F	Poor
733	Aug. 4	+0.8	F, Su, F	Fair
737	Aug. 19	+1.4	F, Su, F	Fair
773	Sept. 17	0.0	Su, Su, Su	Fair
932	1906 May 8	−1.1	Su, J, Su	Poor
933	May 8	−0.7	Su, J, Su	Poor
935	June 5	−0.8	Su, J, Su	Fair

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
7	0.48	+193	− 43	+.137	$+.143=\frac{1}{7}$
10	0.66	+110	+201	+.274	$+.286=\frac{2}{7}$
27	1.18	−133	− 66	+.290	$+.286=\frac{2}{7}$
29	1.36	−170	− 92	+.299	$+.286=\frac{2}{7}$
Parallax star	1.33	− 33.3	+ 2.5		

TABLE 2

REDUCTIONS FOR *Lamont 18816*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
429........	0.364	0.7	−0.709	−282	+.006	+″01
440........	0.354	0.7	−0.731	−280	− 3	−.01
456........	0.344	0.9	−0.879	−265	− 4	−.01
667........	0.263	0.3	+0.943	− 36	− 3	.00
671........	0.262	0.4	+0.889	− 28	+ 1	.00
733........	0.207	0.7	−0.415	+ 62	+ 6	+.01
737........	0.193	0.7	−0.630	+ 77	+ 2	.00
773........	0.169	0.7	−0.922	+106	− 4	−.01
932........	0.072	0.4	+0.876	+339	− 18	−.03
933........	0.102	0.4	+0.875	+339	+ 12	+.02
935........	0.073	0.7	+0.557	+367	0	.00

The normal equations are:

$$
\begin{aligned}
+3.828\pi + 7.328\mu - 1.447c &= -0.632\\
+37.538 \quad +0.457 &= -1.549\\
+6.600 &= +1.516
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.236\\
\mu &= -0.0467 = -0.''124\\
\pi &= +0.0137 = +0.''036 \pm 0.''007
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0039 = \pm 0.''010$

No other determination of this parallax has been published.

31 b Aquilae ($19^h\ 20^m$, $+11°\ 44'$)

This star has a proper motion of about 1″ a year. It is of the 5th magnitude and accordingly the rotating disk was used to reduce its brightness to the mean for the comparison stars. The ten plates secured were measured by Miss Ware.

TABLE I

PLATES OF *31 b Aquilae*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
467....	1904 Sept. 22	$+0^h.2$	S, Su, S	Fair	Star (4) partly occulted on first exposure and lacking on second
469....	Sept. 24	+0.5	S, Su, S	Fair	Star (4) lacking on third exposure
493....	Oct. 6	+1.0	S, S, S	Fair	Star (4) lacking on first exposure
695....	1905 June 24	+0.8	F, Su, F	Fair	Star (1) lacking on second exposure
734....	Aug. 4	+1.2	F, Su, F	Good	
738....	Aug. 19	+1.9	F, Su, F	Fair	
774....	Sept. 17	+0.3	Su, Su, Su	Fair	
936....	1906 June 5	−0.4	Su, J, Su	Fair	
937....	June 5	0.0	Su, J, Su	Fair	
938....	June 5	+0.4	Su, J, Su	Good	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
2...........	0.72	−231	−254	+.135	+.125
3...........	0.63	−156	+ 48	+.200	+.20
4...........	0.92	− 49	− 82	+.181	+.20
6...........	0.53	+312	−134	+.192	+.20
7...........	0.98	+124	+422	+.292	+.275
Parallax star.	0.69	+ 25	+ 58		

TABLE 2

REDUCTIONS FOR *31 b Aquilae*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
467........	0.019	0.6	−0.916	−254	.000	″00
469........	0.018	0.7	−0.929	−252	− 2	.00
493........	0.042	0.8	−0.997	−230	+ 7	+.02
695........	0.283	0.8	+0.360	+ 21	− 7	−.02
734........	0.305	0.9	−0.319	+ 62	+ 8	+.02
738........	0.292	0.7	−0.547	+ 77	− 8	−.02
774........	0.304	0.7	−0.876	+106	− 8	−.02
936........	0.577	0.7	+0.641	+367	− 3	−.01
937........	0.561	0.7	+0.640	+367	− 19	−.04
938........	0.603	0.9	+0.640	+367	+ 23	+.06

The normal equations are:

$$
\begin{aligned}
+3.791\pi + 9.213\mu - 1.520c &= +0.497 \\
+45.109 \quad +5.320 &= +5.379 \\
+7.500 &= +2.315
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.259 \\
\mu &= +0.0808 = +0''.215 \\
\pi &= +0.0380 = +0''.101 \pm 0''.022
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0080 = \pm 0''.021$

Other determinations of this parallax are:

Peter (heliometer)..........	+0″.06	±0″.010
Flint (transit circle)........	+0.01	30
Chase (heliometer).........	+0.02	37

[*To be continued*]

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XXXIII JUNE 1911 NUMBER 5

ON DOPPLER'S PRINCIPLE, IN CONNECTION WITH THE STUDY OF THE RADIAL VELOCITIES ON THE SUN[1]

By A. COTTON

It is by means of the spectroscope, as is well known, and in making use of the Doppler principle, that astronomers are enabled to determine radial velocities. I was led to make some remarks on this subject at the meeting of the International Union for Co-operation in Solar Research, in September, 1910, at Mount Wilson, and I am desirous of publishing them here, together with some additional statements. They apply to the case where the source of light of which the displacements are to be determined is a star surrounded, as is the sun, by an atmosphere.

It seems to me that sufficient attention has not been given to the paper by W. Michelson, of Moscow, published in this Journal (**13**, 192, 1901), "On Doppler's Principle," where the author points out that the relative motion of the source and of the observer is not the only cause which could produce a change of wave-length in the spectral lines. If the observer is at rest and the source stationary, but if the thickness or index of a refracting medium intervening between them is rapidly changed as the light passes through it, the spectrum will be changed. The time that it takes the light to travel from the source to the observer depends, in fact, not on the simple geometrical thickness of the media passed through, but upon

[1] Translated from author's proofs of an article appearing in *Le Radium*.

their *optical* thickness (product of the thickness by the index of the medium for the light which passes). If the index of this medium is changed (for instance, if it is a gas, by compressing it), or its thickness, the luminous vibrations will not reach the observer separated by the same interval of time as if the medium remained constant, and the spectral lines will be slightly displaced while this medium varies.

The small variation $d\lambda$ of the wave-length λ (in a vacuum) of a given monochromatic ray is easily computed if the variations at each instant of the total *optical length* of the path traveled are known. This total optical length L is the sum of a series of terms n_1l_1, n_2l_2, n_3l_3, ... where l_1, l_2, l_3 . . . . are the paths traveled in the different media, n_1, n_2, n_3 . . . . being their indices (referred to a vacuum),

$$L = n_1l_1 + n_2l_2 + n_3l_3 \; . \; . \; . \; . = \Sigma nl.$$

If we let dL/dt represent the derivative of this expression with respect to the time, we have the following general formula for the Doppler-Fizeau effect,

$$\frac{d\lambda}{\lambda} = \frac{l}{V} \cdot \frac{dL}{dt},$$

where V is the velocity of the light in a vacuum.

If the light everywhere passes through a vacuum, we find that the formula usually employed in practice for the computation of this effect, dl/dt, reduces then to the velocity v of the relative motion of the source and of the observer. Otherwise, it is necessary to consider the different terms of the sum; then we have the expression given by Michelson

$$\frac{d\lambda}{\lambda} = \frac{1}{V}\Sigma\left(l\frac{dn}{dt} + n\frac{dl}{dt}\right).$$

To be specific, I will discuss the two following simple cases where the distances l vary.

First case.—The source S and the observer O are separated by a constant distance D, but rays from S pass first through a thickness e of a uniformly refracting atmosphere of index n surrounding the source. Let us assume that this atmosphere is bounded by a

transparent screen perpendicular to SO. This limiting screen is displaced parallel to itself, along SO, with a velocity de/dt. We find, therefore,

$$V\frac{d\lambda}{\lambda}=\frac{d}{dt}(ne+D-e)=(n-1)\frac{de}{dt}.$$

Second case.—In this case, the observer O is again at rest, but the source S moves through the *motionless* atmosphere. Let de/dt be the velocity of the source. Then,

$$V\frac{d\lambda}{\lambda}=n'\frac{de}{dt},$$

where n' is the index of the atmosphere for the light which passes through it, already affected by the motion of the source, while in the preceding case n was the initial index. If the medium is dispersive, these two indices are not precisely identical: in fact, they will differ generally very little, but we shall see that this is not always the case, and for this reason I have designated them by different symbols.

Applications.—W. Michelson himself observed that the foregoing statement was applicable to various astronomical phenomena, particularly to certain peculiarities presented by the sun. He refers indeed to the abnormal displacements, very large in some cases, observed for certain lines near spots and prominences. The observers who discovered them, and who sought to explain them solely by the motion of the gases producing these lines, were led sometimes to admit enormous velocities—as great as 500 km per second. W. Michelson remarks that we need not assume in all cases these velocities which he considers exaggerated. It appears to him more plausible that these changes are due, for a considerable part at least, to motions in the non-luminous gases of higher level through which the rays pass in succession. Such motions, even directed almost perpendicularly to the rays, can in fact modify rapidly the optical path.

I will add that in these gaseous masses through which the rays pass there can be produced not only motions, but physical changes, notably condensations, accompanying the motions themselves. If, for example, a mass of vapors cools and passes to the state of

small liquid drops, or even fine solid particles (of which the influence on the propagation of light may be neglected), the rays which pass through it ought to have their wave-lengths displaced toward the violet. The inverse would be the result in the case of volatilization. If we consider the vast distances traversed we can conceive how such phenomena can persist, without appreciable changes, throughout the duration of the observations.

Case of the solar rotation.—W. Michelson considers in his memoir only those accidental phenomena observed in studying the sun; the question may be raised whether the same considerations ought not to be involved in the spectroscopic investigation of the regular rotational motion of the sun itself. It is evident indeed that *rigorously* this influence of the variations in the optical path ought to alter, first, the law of distribution of radial velocities with the distance to the center, and then the manner in which the displacements estimated for the lines change with the wave-length.

To seek to estimate exactly what these alterations are is really impossible, for it presupposes data about the solar atmosphere which we do not possess. The rigorous solution would involve, in addition, the consideration of the impulse given the waves by a medium in motion. But it is possible to gain at least an idea of the way in which they operate in the purely theoretical case of a sphere S surrounded by a homogeneous atmosphere which rotates with it and is limited, for example, by a thin transparent envelope.

We will assume that this rotating sphere has on its surface a point emitting a monochromatic[1] radiation, and that we follow with the spectroscope the corresponding line, moving the spectroscope as the sphere rotates. In reality, the observations are made differently: we compare the positions of lines corresponding to different points of the solar image, that is, at any point on the sun which we are observing the luminous sources are renewed at each instant. But, as the light emitted by each of these sources behaves as that of a luminous point considered separately, and as the variations of the optical path affect it in the same way, the comparison is legitimate.

[1] This sphere S will correspond then to the periphery, compared to a definite surface, of the reversing layer beginnin g with which the spectrum of the emitted light contains the dark-line standards.

To simplify this problem further, let us assume provisionally that the index n is not altered by the motion, and draw Fig. 1, neglecting the refractions on leaving the assumed atmosphere. It is thus possible to compute, for each position of the point under examination, the value of the optical path and the corresponding expression of the Doppler-Fizeau effect. Omitting the details of the solution of this problem, I indicate only this result: the Doppler-Fizeau effect is greater,[1] and increases a little more rapidly with the distance from the center, than if there were no atmosphere.

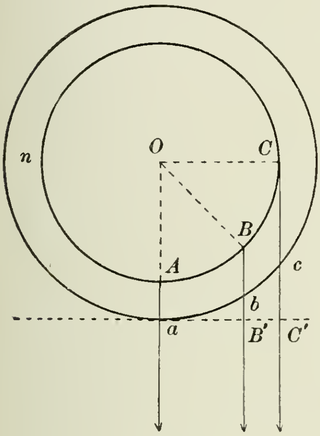


FIG. 1

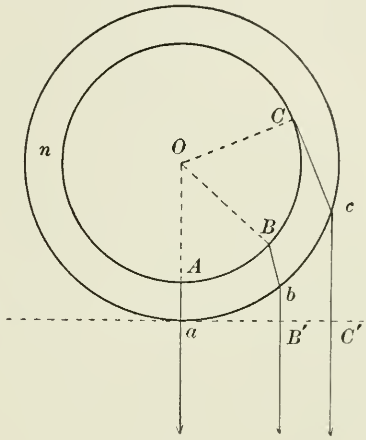


FIG. 2

At the edge of the image it is equal to n times the value it would have if there were no atmosphere. If the index differs only slightly from 1, the correction due to the atmosphere would be insignificant.

We have neglected the refractions. We could consider them in the case theoretically assumed: it is possible indeed, if not analytically, at least graphically, to seek to find how the optical path varies, as a function of the time, or, what amounts to the same thing, the value of $nBb+bB'$ (Fig. 2). But in reality, the atmosphere not being homogeneous, and the index of refraction decreasing on passing from the sphere S, the actual problem will be more complicated. Suffice it to say that we can expect to find for certain regions of the solar image, where the curvilinear part of the course

[1] I assume here the index greater than 1.

has great relative importance, a Doppler effect greater than that computed above without considering the refractions.

If the refractions enter and play an important rôle, as Julius in particular has assumed, we would see with respect to the sun a veritable anamorphosis, a deformed image, especially near the limbs, where we would see points which without refraction would be invisible from the earth. The rotation period—assuming that it could be determined with precision (if there were, for example, a standard, such as a spot, sufficiently well defined)—would vary with its distance from the solar equator; and would vary even with the wave-length, if the atmosphere is dispersive.

We do not actually know whether these refractions exist and play an appreciable rôle. If they do exist, there results from them, from the point of view of the Doppler effect, a first consequence, pointed out recently by M. Perot,[1] which is found from a purely geometrical point of view: M. Perot remarks that the direction of the ray having changed between the point of departure of its curved path and the rectilinear part which it then follows to reach the earth, the Doppler effect gives us the component of the velocity of the sun according to the direction of the luminous ray *near the source*. But this is not all: we must take into account also the progressive alterations which the vibratory periods undergo *later* by reason of the continual variation in the optical path. There is no decisive reason, it seems to me, for neglecting this second effect. To admit that the rays deviate notably from propagation in a straight line is to admit that the values of the indices differ from 1 by appreciable amounts; it is accordingly necessary to take into account the effect which the refracting media passed through have *themselves* produced on the observed wave-length. This will contribute to a modification of the laws of the Doppler effect; for this further reason (granted that the indices are not the same for the different colors) the changes undergone by lines of the same origin will no longer vary proportionally to the length of the initial wave.

The question accordingly is: Do the indices differ appreciably from unity? At this point intervenes the anomalous dispersion

[1] *Comptes Rendus*, **151**, 848, 1910.

on which Julius has insisted, first in pointing out that it could increase the assumed refractions, and more recently[1] in endeavoring to show that *diffusion* can intervene, which could widen the lines in an unsymmetrical fashion.

Many astronomers feel that, at least in the first form that he gave to his theory, Julius exaggerated the importance of the rôle played by the anomalous dispersion; it is not less true that this rôle ought to be examined a priori, since the rays from the sun pass through some media in which there are at least some of the gases of the layer which causes the reversal of the lines. As soon as the dispersion of these media becomes anomalous in the region of certain Fraunhofer lines, then we may inquire whether this anomalous dispersion intervenes in the Doppler effect itself.

Doppler effect and anomalous dispersion.—It will be recalled that the anomalous dispersion of the gases in the different cases studied always presents the same characteristics, namely, those studied in the region of the D-lines by Henri Becquerel. We are led to liken the curve of dispersion in the region of the line to the two branches of a hyperbola having one asymptote parallel to the n-axis. On the red side of the maximum of absorption the curve rises rapidly, on the violet side it falls, as shown in Fig. 3, where it has been assumed, for simplicity, that the index, at a distance from the line on either side, was equal to 1. Observations have up to this time been impossible in the central region of the line. It seems to me that it would be difficult to assume infinite and *negative* values for the index, and I would more willingly believe that the curve resembles, in the narrow portion heretofore unknown, the curve for liquids or absorbing solids, i.e., that between a maximum and minimum value the index returns to normal values. But from our point of view, it is sufficient to admit what is an observed fact, namely, that if the center of the line is approached closely enough from the red we find large values

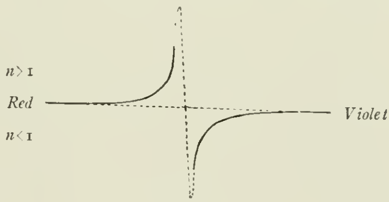


FIG. 3

[1] *Le Radium*, 7, 281, 1910.

for the index, and that the reverse is true when we enter the ray from the violet side.

We can accordingly return to the two simple cases, where we deal with a simple motion of recession of a limiting surface or of the source, which I discussed at the beginning. Both are interesting in that they correspond to possible experiments,[1] but I will discuss only the second, because it seems to correspond to an experiment that has been made.

Let us suppose that the source emits a ray, that the surrounding gas at rest absorbs this line and introduces the anomalous dispersion. If the source moves toward the observer, the index n', which occurs in the formula given above,

$$\frac{d\lambda}{\lambda}=n'\frac{v}{V},$$

corresponds to an apparent period shorter than the initial period; it is accordingly sensibly less than 1. The Doppler effect will accordingly be diminished by the presence of the absorbing medium proportionally, up to a certain limit at least, as the value of the velocity will be less. Contrariwise, if the source recedes from the observer with the same velocity, the index not being notably more than 1, the Doppler effect is increased.

It is precisely in this way that Dufour,[2] who has drawn attention to this intervention of the anomalous dispersion, explains the observations which he has made in rotating a mercury-arc suitably arranged in a magnetic field which makes it turn very rapidly. The displacement of the lines (or rather of the fringes) obtained is about twice as great when the side of the arc which recedes from the observer is studied: this is what Dufour is considering when he assumes that the moving portion of the arc is surrounded by an absorbing layer at rest.

It would be well to be assured that the mercury vapor indeed

[1] The first case could be realized by an experiment analogous to that of Belopolsky. Belopolsky was the first to seek to demonstrate the Doppler effect by an optical experiment in a laboratory: he varied rapidly by reflections the path in air between the source and the observer (see this Journal, **13**, 15, 1901). The air might be replaced, for the variable portion of the course, by a refracting medium.

[2] *Comptes Rendus*, **151**, 62, 1910.

exhibits anomalous dispersion in the region of the spectrum under discussion; to determine also whether such a dissymmetry in the Doppler effect, and an influence analogous to anomalous dispersion, do not enter into the experiments recently made on the Doppler effect in the case of canal rays or anode rays.

In short, we may say that the *rapid and irregular* motions of a source in the midst of an atmosphere, with regard to anomalous dispersion, ought to cause a displacement *toward the red* of the "center of gravity" of the corresponding line transmitted by this atmosphere. We might even be tempted to explain, on this basis, by the molecular motions themselves, the displacement of the lines toward the red, produced by an increase of pressure, which Humphreys has discovered. This displacement has quite the characteristics of a dissymmetrical broadening; it is accomplished quite as would be expected, but it is difficult thus to interpret the fact that it is the *total* pressure which is taken into account and not the pressure of the gas solely which gives the lines studied.

If we pass now to the case of the solar rotation, discussing the possible influence of the anomalous dispersion, we find that the problem is more complicated, because rigorously it is not legitimate to make the foregoing assumption, namely, that the index is not altered by the motion of the atmosphere. Without doubt the atmosphere is drawn along by the rotational motion, but even if there is not a relative sliding of the superposed layers, the effect is not exactly the same as if there were a simple motion of translation: the velocities at different points are not the same, and their components with respect to the ray are different. When a ray which the observer will receive later passes a point M of the atmosphere, this point M receives vibrations which no longer have the initial period, but an apparent period which differs from it.[1] But

[1] It should be added that to treat the problem completely it would be necessary to take into account the Fizeau effect (impulse of the waves) which occurs when the medium where the light is propagated is in motion with respect to the source. The velocity *of the waves* (Lord Rayleigh, Gouy) in a medium which moves with a velocity of which the component perpendicular to the waves is v is given (see Drude, *Lehrbuch der Optik* [1st ed.], p. 427; or Wood, *Physical Optics*, p. 526) by the equation

$$\omega' = \frac{V}{n} + v\left(\frac{n^2-1}{n^2} - \frac{\lambda}{n}\frac{dn}{d\lambda}\right) .$$

in the medium assumed a very small variation of period may produce a considerable change in the index.

I do not believe, in view of these considerations, that it would be useful to take up again even the theoretical problems indicated above, and to investigate a priori whether one can expect effects of which the order of magnitude should be acceptable.

It is more interesting, it seems to me, to note that there is an experimental way of determining whether the anomalous dispersion intervenes, in a general way, in astronomical observations; of learning whether it explains certain paradoxical facts ascertained sometimes in the study of radial velocities.[1]

This would consist of systematically determining whether these established anomalies are observed precisely on the lines near which the anomalous dispersion is the most evident. The researches (still much too few) on anomalous dispersion show, in fact, that its influence is not equally distinct in the vicinity of different lines of the same body; it is thus that Ladenburg and Loria[2] have succeeded in demonstrating it clearly for hydrogen near the red line, but they have not been able under the same conditions to establish it definitely in the region of the blue line. Here also the comparison of astronomical observations with researches in the laboratory will be fruitful.

Laboratoire de Physique
École Normale Supérieure
Université de Paris
December 1910

[1] Different lines, even of the same origin, lead to different computed velocities.

[2] *Berichte der deutschen physikalischen Gesellschaft*, **10**, 858, 1908.

ON THE MAGNETIC SEPARATION OF THE SPECTRAL LINES OF CALCIUM AND STRONTIUM

By B. E. MOORE

I. INTRODUCTION AND METHOD

A few of the lines of these substances have been investigated by Runge and Paschen.[1] William Miller[2] has studied a few more of the lines.

The method and notation[3] are the same as used by these experimenters and by myself in previous contributions.[4] The grating has been kindly loaned by Professor Hale of Mount Wilson Solar Observatory. It is a concave grating of 21 feet focal length and 14,438 lines to the inch. The camera is of the circular type. The field-strength varied from 28,000 to 31,000 gauss. All measurements have been reduced to 24,450 gauss to compare with my previous measurements. The field-strength was estimated from the separation of lines recorded by Runge and Paschen. These strong lines occurred as reversals upon my plates when the salt was introduced into the spark. They were of the usual type, when they came from the impurities present in the carbon electrodes. Calcium is more prominent in the carbon than strontium. Consequently there were more good lines to determine the field-strength with the strontium plates than with the calcium. The *p*- and *s*-components were separated by calcite and photographed at different times. Consequently there has been no attempt to obtain the relative intensities of the *p*- and *s*-components.

In order to make this investigation more complete than previous experiments, it has been necessary to take photograms varying

[1] *Astrophysical Journal*, **16**, 123, 1902.

[2] *Annalen der Physik*, **24**, 105, 1907.

[3] The components of the vibrations parallel to the lines of force are designated by *p*- and those perpendicular to the lines of force by *s*-. Wave-lengths are designated by λ; and change of wave-frequency per centimeter with a field-intensity of 24,450 gauss is represented by $\frac{\Delta\lambda}{\lambda^2}$. Intensity of components, when recorded, is designated by *i*.

[4] *Astrophysical Journal*, **28**, 8, 1908, and **30**, 143, 1909.

in time of exposure from 15 minutes to 10 hours. For these long exposures it was necessary to maintain the grating room at constant temperature and to insure the grating and carriage from any sudden shock. This could be done satisfactorily only at night. Fortunately the required field-strength could be obtained with the magnet excited with as small a current as six amperes, which permitted running the magnet continuously without excessive heating.

II. OBSERVATIONS AND RESULTS UPON CALCIUM

Table I contains some representatives of the first subordinate series. Miller designates 4956.1 as a quadruplet. The character of the p-component cannot be distinguished upon my plates. There are at least two p-components, and it is safe to call it quadruplex.

TABLE I

λ	$\frac{\Delta\lambda}{\lambda^2}$		λ	$\frac{\Delta\lambda}{\lambda^2}$	Remarks
	Moore	Miller			
4956.8	±1.27	....	....	....	Satellite
56.1	1.29s	....	3644.9	....	Satellite
55.0	1.29	±1.24	44.5	±1.24	Principal line
4435.0	−1.40s	1.13s	3631.1	....	
	+0.78s	1.03p			Satellite
35.1	1.14	1.06	30.8	1.10	Principal line
34.12	1.11				Satellite
4425.6	0.70	0.57	3624.1	(0.60)	Principal line
24.83	1.13	....	....	....	Satellite

Its companion line 3644.9 has only one weak red component visible; the blue components are doubtless lost in 3644.5. It is, therefore, presumably quadruplex also. The next pair, 4455.0 and 3644.5, are duplicates. Line 4435.9 has a red s-component of intensity 5 and a blue s-component of intensity 10. I think this line is unsymmetrical, having the s (red) separation twice the magnitude of the s (blue). Measured symmetrically it agrees well with Miller's value. The same difficulty is met with the p-components of this line as with 4956.1. Line 3631.1 is the hypothetical duplicate of 4435.9. It has both its p- and s- blue components lost in 3630.8, and its red components are weak and diffuse. The

position of the components makes the line at least a quadruplet. The next pair, 4435.1 and 3630.8, are the same. 4435.1 has a blue satellite 4434.12, which I have not been able to trace to an impurity. Lines 4425.6 and 3624.1 are probably duplicates. It is difficult to get sharp measurements for these close pairs. Line 4425.6 has also a blue satellite 4424.83.

In the second subordinate series of triplets, Kayser and Runge give six terms. Only the first two terms are well defined upon my plates, the third term is weak and the remainder are not present. The lines and their separations are given in Table II. The corresponding members of the two terms are plainly duplicates. For the quadruplet pair *s*- equals 3*p*-.

TABLE II

λ	$\frac{\Delta\lambda}{\lambda^2}$	λ	$\frac{\Delta\lambda}{\lambda^2}$
6162.5	±1.34	3973.9	±1.34
22.5	1.82*s*	57.2	1.78*s*
	.60*p*	.....	(0.57*p*)
03.0	2.09	49.1	2.08

TABLE III

λ	λ	λ	λ	λ
4586.12	4098.82	3876.2	3754.2	3678.5
81.66	5.25	2.9	50.9	5.5
78.82	2.93	0.9	49.0	4.4

Fowler gives another triplet series as recorded in Table III. The first three terms of this series were noticed by Kayser and Runge. Only the first two terms are well enough defined upon my plates for reasonable determination. The third term is confused by the presence of the carbon band lines. The fourth term is very weak, and although it looks like the first two terms, it needs a much stronger exposure. This series of lines is altogether abnormal in behavior. The lines are uniformly broad upon *s*-, *p*-, and "no field" plates. They are diffused over a space as wide as the normal triplet. If these lines were sharp, a separation of one-third normal could readily be measured. It was thought that

the widening of the lines (a method originally adopted by Zeeman and more recently by Hale in sun-spot study) might be taken as evidence of small separation. However, photograms of the "no field" lines of the same intensity as the s- and p-components showed no appreciable difference in their width. Line 4586.12 was the only exception in the six cases. It seemed somewhat wider on the s-plates. From this evidence I am disposed to say that in these lines we have series which show no Zeeman effect, although the possibility of a small separation must be conceded.

Table IV contains all the quadruplets observed outside of the principal series and those above recorded. These four lines con-

TABLE IV

λ	i	$\frac{\Delta\lambda}{\lambda^2}$
6499.9	2	$1.04s=10\times0.104$
	1	$0.84p=8$
94.0	10	$0.93s=9$
	5	$0.73p=7$
5264.4	8	$2.03s=20$
	3	$1.03p=10$
61.6	2	$1.48s=14$
	1+	$1.27p=12$

sist of two close pairs. They are all different but are all related. The separations can all be represented by multiples of a small factor, 0.104. This small factor has in numerous cases been found to be an aliquot part of a separation designated as normal. The normal value, "a," for the present field-strength is 1.105. The value 0.104 lies between $a/11$ and $a/10$, and too far from either of these values to be accepted as a rational part of the normal, or to state that the separations are multiples of that rational part.

Table V contains a list of the triplets measured. Generally the intensity of the p-component is about twice that of the s-pair of components. The intensities of the s-components are the only ones recorded. With reference to the Zeeman effect, the group of six lines between 4318.8 and 4283.2 is one of the most prominent of the calcium spectrum. The lines have nearly the same intensity and they differ but little in sharpness. The p-components are only one-third to one-half stronger than the s-components. But

the most prominent feature is the separation itself, which is the same for all of the lines. The separation seems also related to the normal, "*a*," and is one-half larger. No other magnitudes of separation in the triplet list are conspicuous.

TABLE V

λ	i	$\frac{\Delta\lambda}{\lambda^2}$	λ	i	$\frac{\Delta\lambda}{\lambda^2}$
6713.0	1	±0.99	5270.5	15	1.24
471.9	3	1.24	262.5‡	5	....
62.8	15	1.05	189.0	2	1.08
50.0	3	1.08	041.9	1	(0.92)
39.4	20	1.19	4878.3	2	0.99
6169.9*	..	....	318.8	12	1.66
0.4*	..	....	307.9	12	1.65
5857.8	10	1.03	302.7	15	1.66
603.0	3	2.05	299.1	12	1.66
1.5	5	1.67	89.5	12	1.65
5598.7†	..	0.78	83.2	15	1.66
94.7	15	1.19	3961.2§	1+	(1.10)
90.3	4	(1.87)			
89.0	20	1.43			
82.2	8	1.50			

* These lines overlap. Separation is large.

† Unsymmetrical in intensity and probably also in separation. Red intensity 8, blue intensity 12.

‡ Has very narrow separation.

§ Possibly aluminum line 3961.7.

III. OBSERVATIONS AND RESULTS UPON STRONTIUM

Table VI contains the lines studied belonging to the second subordinate series of triplets. Three more terms of this triplet series are known, but they do not appear upon my plates. It is at once seen that the pairs in these two terms agree. The middle pair is a magnetic quadruplet whose separations are in the ratio of 4 to 1.

TABLE VI

λ	$\frac{\Delta\lambda}{\lambda^2}$	λ	$\frac{\Delta\lambda}{\lambda^2}$
7070.7	±1.50	4438.22	±1.48
6878.8	1.90*s*	361.87	1.95*s*
	0.50*p*	......	0.48*p*
6791.4	2.22	326.60	2.10

Table VII contains all the observable lines belonging to the first subordinate series. There are three strong lines in each term

called principal lines. These are accompanied by weaker lines designated satellites. In all there are six and possibly seven members in each term. The first principal line is the only one which can be measured for three terms, and in the third case it is probably overlapped by its first satellite (3706.2). These three representatives agree. The satellites 4968.11 and 4032.51 agree in their s-components. It is difficult to tell the real character of the p-components. The p-plates show that both lines are either unsymmetrical in magnetic separation, or that each line is a close natural doublet. If they are unsymmetrical in separation, one component

TABLE VII

λ	$\frac{\Delta\lambda}{\lambda^2}$	λ	$\frac{\Delta\lambda}{\lambda^2}$	λ	$\frac{\Delta\lambda}{\lambda^2}$	Remarks
4971.85	±2.73	4033.25	....	....	....	Satellite
68.11	1.63s	2.51	±1.58s	....	....	Satellite
	0.81p	....	... p	....	....	
62.45	1.29	0.45	1.30	3705.88	±1.29	Principal line
4876.35	1.61	3970.15	....	....	....	Satellite
2.66	1.03	69.42	(1.59?)	....	....	Principal line
4832.23	1.65	3940.88	0.00?	....	....	Principal line
0.15	0.49	....	....	....	....	Satellite

is about three times as far removed from the zero position as the other; and the total separation equals the value for the s-components. For line 4968.11 the blue p-component is three times as bright as the red and one-third as far from the "no field" line. For the companion line, 4032.51, the phenomena are reversed. The appearance of the "no field" line favors regarding these lines as opposite and equal dissymmetries. If the first line has really a close blue companion, then the second one must have a close red companion. These two weaker lines would have equal p-doublets. But if this be true, it is difficult to reconcile the small displacement upon the "no field" plate relative to the stronger p-component. Line 3970.15 is a diffuse triplet and too weak to measure. It may be identical in separation with its companion 4876.35. These lines are not alike, however, in general appearance. Line 3969.42 is broad and diffuse, and only the red component is visible. The

blue component is overlapped by the strong calcium line 3968.63, which occurs as an impurity in the electrodes. However, the separation is roughly the same as 4876.35 and not the same as the line 4872.66, which precedes it in the series. Line 4832.23 is the first representative of the third principal series. Its separation is three halves times the normal triplet and identical with the first line of the second series. Line 4832.23 is further characterized by having a satellite upon its more refrangible side whose wave-length is 4832.15. This line was first noted by Miller, and its components lie entirely within those of the principal line. The next line in the series with 4832.23 is 3940.88. Its peculiar diffuseness suggests a small paration, together with the presence of an overlapping line, as if there were a close satellite present as in line 4832.23. However, the separation here, if there be any separation at all, must be much smaller than obtained for 4832.23. So far as these series have been studied, then, it is seen that Preston's law is confirmed only in the first three terms of the first principal line.

Table VIII contains the separations of three series of lines given by Fowler. It is apparent that there is no uniformity in the

TABLE VIII

Series 1		Series 2		Series 3	
λ	$\frac{\Delta\lambda}{\lambda^2}$	λ	$\frac{\Delta\lambda}{\lambda^2}$	λ	$\frac{\Delta\lambda}{\lambda_2}$
5535.01	±1.66	5504.48	±1.31	5486.37	2.04*s*
257.12	1.31	229.52	1.55*s*	....	1.32*s*
4892.20	1.32	....	0.61*p*	....	0.70*p*
4338.05	0.00	4868.92	1.21	....	0.00*p*
4087.67	....	4319.39	0.00?	5213.23	....
3950.96	....	4071.01	....	4855.27	1.41
		3935.33	....	4308.49	0.00
				4061.21	....
				3926.27	....

separations of these lines in either of the three series. The separation 1.66, which is three halves times the normal, occurs four times, three times in triplets and once in a seven component line, but it occurs only twice in one series. There are no further duplications. These series are characterized by having a constant difference on

the scale of vibration between corresponding members. That is, between F_{11} (Fowler's series 1, line 1) and F_{21}, the frequency-difference is the same as between F_{12} and F_{22}. Designate this frequency-difference by $\nu_1=100$ (about) and the difference between F_{21} and F_{31} by $\nu_2=59$ (about). They are, then, similar to the principal lines in the two previous series of triplets. However, none of the types of separation, except 1.66, in these series occurs in the previous series. But there are some lines outside of the known series which agree in type with some of the lines in the Fowler series. The most prominent case is the seven component lines 5540.3 and 5486.4 (F_{31}). Each of these two lines has two other lines similarly situated with respect to each other on the scale of vibration frequency as shown in Table IX. However, in this new

TABLE IX

5540.3	$17.4=\nu_3$	5486.4 F_{31}	$17.6=\nu_3$
35.0 F_{11}		81.1	
04.5 F_{21}	$100.=\nu_1$	51.1	$100.=\nu_1$

grouping, it is 5451.1 which is the magnetic duplicate of 5535.0 and not 5481.1. The value ν_3 corresponds closely to the distance of the first satellite from the first principal line in the first subordinate series. This value is repeated in the distance between the lines 5229.5 (F_{22}) and 5225.4. However, F_{22} has four instead of seven components; and 5225.4 duplicates neither 5535 nor 5481.1, but is probably a duplicate of 5522. Both 5225.4 and 5522 are unsymmetrical, and the total distance between their respective components is probably the same.

Fowler's series do not conform to Preston's law in any way whatsoever. Judged by the Zeeman effect, there are some lines not included in the series more intimately associated with some of the series lines than these series lines are with each other. It has not been possible to group either the lines which do agree into a series or to take the corresponding lines outside of the series and group them into other co-ordinate series.

Table X contains two quadruplets, the only ones which occur outside of the series already given. These separations are multiples of the same small value, 0.104, found for the four quadruplets in

calcium, Table IV. However, the multiples of this small value differ for all six lines. Therefore, although related, they are of different types.

TABLE X

λ	$\frac{\Delta\lambda}{\lambda^2}$
6641.4	$0.95s = 9\times0.104$
	$0.94p = 9\times .104$
6547.0	$1.35p = 13\times .104$
	$0.73p = 7\times .104$

Table XI contains a list of all the triplets studied which are not already included in the series Tables VI to VIII. A few of the stronger lines have a separation of three halves times the normal "*a*." However, there is no defined tendency of the separations to cluster about any given magnitude.

TABLE XI

λ	i	$\frac{\Delta\lambda}{\lambda^2}$	λ	i	$\frac{\Delta\lambda}{\lambda^2}$
6550.5	8	±1.12	4812.0	25	±1.64
504.2*	10	1.14	784.4	15	1.71
466.2	1	(0.95)	70.6‡	2	(1.12)
46.2	1+	1.73	42.1	10	1.68
08.6	15	1.23	22.4	25	1.64
6390.7	1+	(1.16)	4678.4‡	2	(1.03)
86.8	1+	(1.41)	607.5§	..	1.10
80.9	2	1.53	531.5	8	1.46
5543.5	2	1.16	480.7¶	2	0.00
522.0	8	0.60	54.8¶	3	1.30
481.1	15	1.47	51.9¶	1+	0.87
451.1	10	1.65	12.9	5	1.72
330.0†	2	....	4031.9	1+	(1.50)
5238.8	8	1.09	3963.3	5	1.26
225.4	3	0.63	44.3	1+	(1.50)
156.4	2	1.47	3307.6	3	1.78

* *p*-component has the same intensity as the *s*-pair of components.

† Not identified, like 5522 and 5225.4.

‡ These two lines are alike in diffuseness.

§ Reversal.

¶ Lines not identified.

IV. COMPARISON OF CALCIUM AND STRONTIUM

The principal and second subordinate series of doublets show the same uniform separation both in magnitude of separation and in number of components in all substances in which the types have

been found. In calcium and strontium we have two other types of series called triplet series. However, in the first subordinate series the principal lines are accompanied by weak companions or satellites. From Preston's law we should expect not only that the lines in successive terms of the series should correspond, but also that there should be a similar agreement for the same series in passing from one substance to another. Now the first principal line in the first subordinate series in calcium is represented by separation (1.24) and 1.29, whereas the same series in strontium is represented by the values 1.29, 1.30, and 1.29; or the type is the same in the two substances. However, in the satellites there is no correspondence. In the second principal line the strontium representatives have separations in the ratio of three to two; and the separations of the calcium lines are 10 per cent larger than the smaller strontium value. The third principal line in calcium agrees in type with a close blue satellite of the third principal line of strontium. The remaining satellites neither agree in number of components nor in magnitude of separation. In the second subordinate series Preston's law is followed very well in either of the substances taken separately. But in passing from one substance to another the only similarity we find is that in each case we have two triplets and one quadruplet. The magnitudes of these separations are not comparable.

GENERAL CONCLUSION

The lines in the series sometimes follow Preston's law, but more frequently they fail to do so.

Brace Laboratory of Physics
Lincoln, Nebraska
March 1911

ON THE RADIATION OF THE COMPANION OF *ALGOL*

By JOEL STEBBINS

In a previous study of the *Algol* system by the writer,[1] it was found that the form of the light-curve between minima is best explained by assuming that the companion keeps one side always toward the primary, and is brighter on that side, either because of reflection, or because of the heating effect of the radiation received from *Algol*. On page 212 of my article the reflection theory was dismissed as not sufficient to account for the observations. I am indebted to Father J. Stein of Amsterdam for calling my attention to a paper by Wilsing[2] on the reflection of the light of an *Algol* star by its companion, and the considerations in the present note have come from the correspondence between Father Stein and myself. We reached the agreement that while my reasoning in regard to the untenability of the reflection theory was insufficient, the main conclusion was correct. Although Wilsing's results are slightly in error, his method is perfectly suited to the case in hand, and from the importance of his formula it seems worth while to derive it anew. Pannekoek[3] has already pointed out Wilsing's mistake, but he also has committed a trivial error in one numerical coefficient.

Let us proceed then with Wilsing and find, on the basis of Lambert's law, the amount of light reflected to the earth by the companion. Let the radius of *Algol* be taken as unity, and let J be the measure of the radiation emitted per unit surface of *Algol* in a direction perpendicular to the surface. Then the total radiation from the bright disk in the direction of the earth is given by

$$J_1 = \pi J. \tag{1}$$

Consider the companion to be at superior conjunction, and we have for the total radiation, J_2, which the companion reflects toward the earth,

$$J_2 = \frac{\mu}{\pi} \int\int \frac{J \cos\theta \cos\theta' \cos\phi \, ds \, ds'}{\rho^2} \tag{2}$$

[1] *Astrophysical Journal*, **32**, 185, 1910.

[2] *Astronomische Nachrichten*, **124**, 121, 1890.

[3] *Untersuchungen über den Lichtwechsel Algols*, Leipzig, 1902.

where μ is the albedo of the companion's surface, ds and ds' are surface elements of the two spheres, ρ the distance of the elements, θ, θ' the angles between the normals of the elements and the line joining them, ϕ the angle between the normal of ds' and the line joining the centers of the spheres.

Let κ be the radius of the companion, a the distance between centers, 2δ the apparent diameter of the primary seen from ds'. If the bodies are not too close together we may put

$$\int \frac{\cos\theta\, ds}{\rho^2} = \pi \sin^2\delta\,.$$

Putting also $ds' = \kappa^2 \sin\phi\, d\phi\, d\psi$, we find

$$J_2 = J\mu\, \kappa^2 \int_0^{2\pi} d\psi \int_0^{\frac{\pi}{2}} \sin^2\delta \cos\theta' \cos\phi \sin\phi\, d\phi$$

or

$$J_2 = 2\pi J\mu\kappa^2 \int_0^{\frac{\pi}{2}} \sin^2\delta \cos\theta' \cos\phi \sin\phi\, d\phi\,.$$

Substituting

$$\sin\delta = \frac{1}{\sqrt{a^2+\kappa^2-2a\kappa\cos\phi}}\,, \qquad \cos\theta' = \frac{a\cos\phi-\kappa}{\sqrt{a^2+\kappa^2-2a\kappa\cos\phi}}\,,$$

there follows

$$J_2 = 2\pi J\mu\kappa^2 \int_0^{\frac{\pi}{2}} \frac{\sin\phi\cos\phi\,(a\cos\phi-\kappa)}{(a^2+\kappa^2-2a\kappa\cos\phi)^{3/2}}\,d\phi\,,$$

and carrying out the integration,

$$J_2 = \tfrac{2}{3}\pi J\mu \cdot \frac{2a^3+\kappa^3-(2a^2-\kappa^2)\sqrt{a^2+\kappa^2}}{a^2\kappa}\,.$$

We can now put $\frac{\kappa}{a} = \gamma$, and obtain with sufficient approximation[1]

$$J_2 = \tfrac{2}{3}\pi J\mu(\gamma^2 + \tfrac{3}{4}\gamma^3 + \;.\;.\;.\;.\;). \tag{3}$$

[1] Equation (3) is the same as Wilsing's corrected by Pannekoek, except for the coefficient of γ^3 which he gives as $\frac{1}{2}$.

In my previous paper, the increase of surface intensity on the brighter side of the companion was called λ_1, the total increase of radiation being $\kappa^2\lambda_1$, and from (1) and (3)

$$\kappa^2\lambda_1=\frac{J_2}{J_1}=\tfrac{2}{3}\mu(\gamma^2+\tfrac{3}{4}\gamma^3)$$

or

$$\mu=\frac{3}{2}\cdot\frac{\kappa^2\lambda_1}{\gamma^2+\frac{3}{4}\gamma^3}. \tag{4}$$

The numerical values of the elements are

$$\kappa=1.14\ \pm0.05$$
$$a=4.77\ \pm0.05$$
$$\kappa^2\lambda_1=0.049\pm0.007$$

which substituted in (4) give $\mu=1.09$. As pointed out by Wilsing, the integration is extended over too great an area of each sphere, but even limiting ϕ to $\frac{\pi}{3}$ I find $\mu<1.15$, and we may therefore adopt

$$\mu=1.10\pm0.20$$

where the probable error depends upon the probable errors of κ, a, and λ_1, almost wholly the last. From the size of the probable error it is evident that μ may be less than unity, but in any event the reflection theory is untenable, which agrees with the conclusion in my paper.

The interpretation to be given to this high value of μ is that the companion of *Algol* receives and emits nearly equal amounts of the radiation to which selenium is sensitive. It is probable that the spectral energy-curves of the two bodies are entirely different, but the integrated effect is practically the same as though the companion reflected all of the light which it receives. At present we know nothing of the spectral type of the companion, but we do know that it is a body which of itself is probably more intense than the sun. Dr. W. J. Humphreys has suggested to me that we should expect the companion to have a low albedo, because of its extremely small density, and that its coefficient of absorption may be something like that of a black body. If this is true, then the absorbed radiation is no doubt re-emitted in quite different form. It is

evident that the observational data are not sufficient to lead us to any final conclusion as to the radiation of the companion, except to state definitely that only a small portion of the extra light from one side can be due to reflection.

It should be noted that *Algol* is not the only case where the companion is known to be brighter on one side, for Dr. R. S. Dugan of Princeton has found the same phenomenon in two other stars, *RT Persei* and *Z Draconis*.[1] Preliminary announcements of the character of the variation of these stars were made by Dr. Dugan in 1908 and 1909, and it is my understanding that the work will soon be published in full.[2]

University of Illinois Observatory
Urbana Illinois
April 6, 1911

[1] *Publications of the Astronomical and Astrophysical Society of America*, **1**, 311 and 320, 1910.

[2] See *Contributions from the Princeton University Observatory*, No. 1: "The Algol-System *RT PERSEI*," by Raymond Smith Dugan.—Ed.

ON REGULARITIES IN THE SPECTRUM OF NEON

By HERBERT EDMESTON WATSON

In 1908 the author published a paper on the "Spectrum of the Lighter Constituents of the Air,"[1] in which were given the wave-lengths of a number of lines ascribed to neon. As the primary object of the research was to detect, if possible, the presence of a new gas, long exposures were given to the plates, and the accuracy of the measurements was consequently somewhat impaired. It was considered, however, that the values given for the wave-lengths were usually correct to 0.03 Å.U., and this was borne out by the agreement with the previous measurements by Baly.[2]

The existence of some 200 new lines was determined by these experiments, making the total 321, and it seemed worth while to examine the spectrum with a view to discovering a possible regularity. For this purpose, the wave-lengths were first converted into the corresponding oscillation-frequencies *in vacuo*.

Since the helium spectrum had been found to be capable of resolution into six series of lines with gradually diminishing intensities,[3] it was thought at first that such series might also exist in neon, and as two of the helium series consisted of doublets, it was hoped that the detection of similar doublets in neon might indicate lines which belonged to the same series. In the case of helium the constant difference of oscillation-frequency between members of a pair is 1.00, and consequently, if this difference is proportional to the square of the atomic weight, the corresponding value for neon might be about 25. Now there is, as it happens, an unusually large number of oscillation-frequency differences between 25 and 26, but the total number is only 13, and there appears to be no connection between the intensities of the pairs of lines. Consequently, this hypothesis was, for the time being, abandoned.

[1] *Proc. Roy. Soc.*, A, **81**, 181, 1908.

[2] *Phil. Trans.*, A, **202**, 183, 1903.

[3] Runge and Paschen, *Astrophysical Journal*, **4**, 91, 1896.

The next step was to plot the positions and relative intensities of all the lines on a large scale, and an examination of the map of the spectrum thus produced showed that the lines appeared to be divided into three groups with considerable gaps between them. The first of these groups extends fron the extreme red to λ 4071, and consists of a great mass of lines, 252 in all, among which are many weak ones, especially at the blue end. The second group extends from λ 3754 to λ 3370, and contains 29 lines only, but all of these, with the exception of 5 very weak and doubtful ones, are of considerable strength. The third group reaches from λ 3167 to λ 2736, there being 40 lines in all, though a subdivision is possible owing to the fact that there is an obvious gap between λ 2911 and λ 2872. Also the lines on the less refrangible side of this gap are considerably stronger than those on the other, and among the former no very weak ones are to be found.

This arrangement suggested that the groups might be in some way repetitions of each other, that is to say, there might be constant frequency-differences between certain corresponding lines in them, and consequently these differences were calculated between each line of the second group and all those of the third. The results were indefinite.

After this, the difference between each pair of lines in the second group was determined, and so also for the third group, and it was then found that a distinct regularity existed, inasmuch as many of the lines could be grouped into triplets with constant frequency-differences, the difference between the first and second lines being approximately 1429, and that between the first and third 1847. A search was then made in the rest of the spectrum for more of these triplets, and several were found in the extreme red, but no others seemed to exist. There were, it is true, several other pairs of lines with one of the above differences between them but this is to be expected from considerations of probability, and in no case was the triplet complete.

In addition, on examining the second group, it was found that there were three pairs of lines with a frequency-difference of 1070, the first line in each case being the first member of a triplet. In view of the small number of lines in this group, this could hardly

be accidental, especially as three similar pairs occurred in the third group, and three in the first, and the difference in question did not appear again. It seems, therefore, that in the whole spectrum there are nine quadruplets, and a number of triplets with the same frequency-differences, but with the second line absent. Moreover, it was found that these triplets and quadruplets were not merely groups of three or four lines occurring among a number of others, but that practically all the lines in certain regions of the spectrum were capable of arrangement into such groups. Thus, in the second large division of the spectrum, which practically consists of 24 bright lines, there are three quadruplets and four triplets, in one of each of which a line is missing, and only two lines do not fit into the scheme. The first of these is the strongest line in the whole group, and the second one is brighter than nearly all the others. In the third group, or rather in the first subdivision of it previously mentioned, there are also only two lines which are not members of triplets or quadruplets, while all the rest, 25 in number, fall into these natural groups. It is interesting to note, however, that one line λ 2992 occurs twice. Also, the members of the last two quadruplets are so close together that they appear to have been resolved only in two cases. When the wave-lengths were first published, the line λ 2949 was marked "broad," and is doubtless double, a fact which is also indicated by the abnormal frequency-difference, and it is quite possible that the line λ 2913 is double as well.

On turning to the first group of lines, it is found that not all of them belong to triplets or quadruplets, although most of the stronger ones do. It must be remembered, however, that the numbers representing the intensity according to the usual convention merely indicate which lines are brighter than others, and which are of approximately equal intensity. Actually, the energy corresponding to a line of intensity 10 is probably many thousand times that of one of intensity 1, and this difference is very apparent on visual observation, especially in the case of the present spectrum, for on looking at the region in which the regularities occur with an ordinary spectroscope, 23 very bright lines are seen and no others, unless special precautions are taken. Of these lines, all are mem-

bers of triplets or quadruplets except two, λ 6402 and λ 6074. In addition, the former is the brightest line between the wave-lengths under consideration. It is also interesting to note that if the pressure of the gas is fairly high, say above 5 mm, these 23 lines and two others, λ 7245 and λ 5852, constitute practically the whole of the visible spectrum; and it is remarkable that the latter line, which is the brightest in the entire spectrum, is just outside the region of triplets and quadruplets, and does not take part in their composition. It may also be mentioned that while examining these bright lines with a Hilger constant-deviation spectroscope, an additional very weak one was observed in the extreme red, the wave-length as given by the instrument being about λ 7445.

The following tables show how the lines have been arranged.

QUADRUPLETS

$\frac{10^8}{\lambda}$	λ	Int.	Diff.	λ	Int.	Diff.	λ	Int.	Diff.	λ	Int.
14232.22	7024.38	2	1070.33	6533.08	6	1429.78	6383.14	9	1847.20	6217.44	6
14883.08	6717.22	7	69.97	6266.69	6	9.34	6128.63	5	6.62	5975.76	5
15149.28	6599.18	9	70.24	6163.73	6	9.41	6030.20	5	6.95	5882.06	5
26628.6	3754.32	3	69.7	3609.33	3				6.4	3510.87	4
27767.6	3600.32	6	70.2	3466.70	5	9.1	3424.08	3	7.3	3375.74	2
27818.8	3593.69	8	69.7	3460.61	5	9.6	3418.03	5	6.3	3370.02	5
31560.6	3167.62	2	69.0	3063.83	2	8.5	3030.44	2	6.0	2992.57	3
32465.7	3079.31	1	70.0	2981.06	1	30.8					
32468.7	3079.02	1	69.8	2980.81	1	27.8	2949.32	1	7.1	2913.28	2

TRIPLETS

$\frac{10^8}{\lambda}$	λ	Int.	Diff.	λ	Int.	Diff.	λ	Int.
13934.95	7174.25	2	1429.67	6506.69	9	1846.96	6334.65	9
14426.56	6929.78	6	9.64	6304.97	6	6.89	6143.31	7
14969.37	6678.50	9	9.40	6096.36	6	6.85	5945.02	6
27009.9	3701.31	5	9.1	3515.32	5	6.6	3464.46	4
27123.2	3685.86	4	9.3	3501.34	5	7.4	3450.88	4
27148.9	3682.37	4	9.4	3498.19	5	6.8	3447.83	5
27511.8	3633.80	5	9.5	3454.31	5	....	....	
31701.7	3153.51	2	9.3	3017.47	3	6.6	2979.94	2
31750.2	3148.70	2	9.0	3013.09	3	6.4	2975.65	1
31759.0	3147.82	1	9.4	3012.25	3	6.2	2974.89	3
31977.4	3126.33	2	9.2	?2992.57	3	....	....	
32489.2	3077.08	2	8.9	2947.44	3	7.0	2911.55	1
32697.2	3057.51	3	9.0	2929.47	1	....	....	

In column 1 is given the oscillation-frequency *in vacuo* of the first line, followed by its wave-length in air and its intensity. The fourth column shows the difference between the *oscillation-frequencies* of the first and second lines, the wave-length and intensity of which follow. The next column gives the difference between the oscillation-frequencies of the first and third lines, and so on. The actual oscillation-frequencies are not given except for the first line, but of course may be calculated from the differences.

With regard to the agreement between the different values for the constant frequency-differences, it may be noted that if the error in the wave-length of a single line is 0.03, that of the difference of two may be 0.06, and the maximum variation is 0.12. This, in the scale of oscillation-frequencies, corresponds roughly to 0.3 for the first group of lines, and 1.0 for the other two, and it will be observed that this limit is rarely exceeded, though it may be remarked that the actual values of the differences in the first group seem slightly larger than the others, and also that those between the triplets are more constant than those between the quadruplets; possibly the lines composing the former are slightly sharper than the others, although no actual difference is apparent. The author's values for the wave-lengths and intensities have been used throughout, as Baly's figures, though possibly rather more accurate, are not complete.

There appears to be no very definite relation between the intensities. Generally speaking, the lines in the first group are brighter than those in the second, and these in turn brighter than those in the third. Also, corresponding lines in the same group have often the same intensities, this being especially well marked in the case of the triplets. The figures representing the intensities in the extreme red are probably far too small, as they show only the extent to which a photographic plate is affected, and in this portion of the spectrum the total energy is, no doubt, considerably greater than that indicated in this way.

It is not proposed to discuss fully a number of interesting points which are raised by the existence of these regularities. The main facts so far ascertained may, however, be outlined. First, if the bright lines only are considered, the spectrum falls naturally

into three groups of lines which diminish in general intensity on approaching the ultra-violet end. The first group consists of one very bright line, one weaker line, three quadruplets, and three triplets; the second of one very bright line, one weaker line, three quadruplets, and four triplets; while the third group contains also two bright lines, three quadruplets, and six, or possibly only five. triplets.

This arrangement naturally suggests a principal series and two subordinate ones, the second, or both of the latter, consisting of triplets or quadruplets, or possibly a principal and two subordinate series of quadruplets alone, and others of triplets alone, especially as a grouping in threes seems to predominate; but even if some such connection does exist, it would be exceedingly difficult to determine, since there are at present only three members of each series.

The whole arrangement also strikingly resembles the blue portion of the red argon spectrum investigated by Rydberg.[1] This author found that all the lines between certain limits, except a few very weak and doubtful ones, could be arranged in a scheme of 7 quadruplets, 6 triplets, and 10 doublets, in a very similar manner to the neon lines above, but the mean frequency-differences from the first line were in this case 846.5, 1649.7, and 2256.7. Two lines, however, were not included, and one of these was the brightest of all those under consideration. It seems hardly possible that such a coincidence can be accidental.

With regard to the other and weaker lines of the spectrum, which consist of a number in the first group, and all those in the second subdivision of the third group, a regularity is not at present apparent. A partial search has so far failed to reveal any constant frequency-differences, but it by no means excludes their existence. As, however, there are in all some 25,000 possible differences, the work is necessarily slow. Another possibility is that these lines are the components of a number of series similar to those in helium, a supposition which is strengthened by the fact that there are a great many weak lines close together at the blue end of the first group. From the point of view of intensity alone,

[1] *Astrophysical Journal*, 6, 338, 1897.

this portion of the spectrum is somewhat analogous to the secondary spectrum of hydrogen, and while there can be no doubt that there is some relation between the lines composing it, the solution of the problem is not easy. The investigation both of these lines and of the brighter ones is being continued, and it is hoped that by means of methods which are not solely mathematical, some further light may be thrown on the true constitution of this remarkable spectrum.

Trinity College
Cambridge

APPLICATION OF THE INTERFERENCE METHOD TO THE STUDY OF NEBULAE

By CH. FABRY and H. BUISSON

The spectroscopic study of the sidereal universe has been made almost solely with a single form of apparatus, the prism spectroscope, the general form of which has remained unchanged. It is only in very exceptional cases that gratings have been employed. It is doubtless necessary to conclude that the prism spectroscope is the apparatus best adapted to that class of researches and although in the future it will probably continue to play a leading part, it is possible now to consider other methods which may be able to contribute, at least in certain cases, to the progress of our knowledge of stellar astronomy.

Interference methods in varied forms have shown their efficiency in the analysis of light both in laboratory researches and in the study of the sun. Our present work has for its single purpose to show that it is neither impossible nor difficult to apply these methods to certain problems of sidereal astronomy.

Interference methods may be employed with special convenience when the source of light under investigation emits a small number of monochromatic radiations; the case of continuous spectra having dark lines is less simple. We have applied our method to the study of nebulae whose spectra consist of a small number of bright lines.

The interference apparatus consists of a film of air between two plane-parallel surfaces covered with a thin coat of silver. The fringes produced are rings situated at infinity. The observing apparatus, visual or photographic, should give a sharp image of those rings. It is desirable that there should be no mixing of the radiations emitted by the different parts of the object, and consequently that the sharp image of the nebula should be formed in the same plane as that of the rings. The most simple manner of arriving at this result would be to place the interferential apparatus in front of the entire observing apparatus; but then we should be limited as to the diameter of the objective by the size of the

silvered film; and further, the apparent diameter of the rings would be very large with respect to that of the nebula.

The following arrangement overcomes both these difficulties. The interference apparatus is placed at the end of the telescope, set upon the nebula for an eye focused for infinity and consequently forming an afocal system; on looking through this, if the eye is focused for infinity, it will see a sharp image of the nebula on which the interference rings will be projected. It is not possible to use more than a small surface of silver, hardly larger than that of the ocular ring; the apparent diameter of the rings is not changed, while that of the nebula is multiplied by the enlargement of the telescope. For photographic observations the light which has traversed the above system is collected by an objective of short focus in the focal plane of which are superposed the real image of the nebula and that of the ring. Finally, it is possible to examine the image visually with an eyepiece.

We have made a practical application of this plan to the equatorial of the Observatory of the University of Marseilles. The objective has a diameter of 26 cm, and a focal length of 3.10 m. Having removed the eye-end of the telescope, we replaced it by the apparatus represented in Fig. 1. *A* is an eyepiece with two lenses forming an optical system of 4 cm focal length, the first focus of which is in the focal plane *F* of the telescope objective. On leaving this eyepiece, the rays of the nebula give an image at infinity, enlarged eighty times. The light then traverses the interference apparatus *B*, which gives its system of interference rings. A microscope objective *C*, of 4 cm focal length, forms in its focal plane *P* a real image of the nebula, of the same size as that which would be directly given by the telescope objective; this image is crossed by the interference fringes. For photographic observations, the plate is placed at *P*. In visual observations we see the superposed image of the rings and of the nebula through a second eyepiece *D* having a focal length of 2.5 cm; the nebula is then seen with an enlargement of 120, and the rings

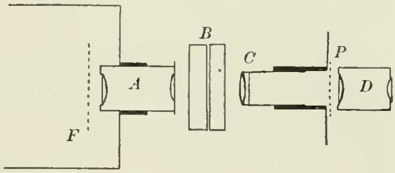


FIG. 1

with an enlargement of 1.6. The free aperture of all the lenses is such that there is no loss of light.

The interference apparatus is of the type having a fixed difference of path, which is called the *étalon interférentiel.*[1] It is difficult to apply this form of construction when the film of air has a thickness less than 1 or 2 mm; for thicknesses less than this the two silvered surfaces are simply separated by three bits of steel of suitable diameter cut from the same rod. Springs with an adjustable pressure hold the silver surfaces against the pieces of steel and permit us to obtain parallelism.

The quality of the silver surfaces is of great importance. If they are too thick, the proportion of the light transmitted is very small; if they are too thin, the reflecting power is very little, and the interferences are not sharp. We have employed films silvered by impact at the cathode. Each of the surfaces transmits for the green mercury line about 20 per cent of the incident light and reflects 50 per cent. The interference apparatus may be inclined at a slight angle to the beam of light, which permits us to displace the system of rings with respect to the image of the nebula.

The total weight of everything attached to the tube of the telescope is only 3 kilograms.

We have examined only the nebula in *Orion*. Observing visually, we established without difficulty the existence of interference rings, in successively employing differences of path of 0.6, 2, and 5 mm. The rings thus observed are due to the ray at λ 5007, which is the most intense of the visible lines in the spectrum of that nebula.

We have also made an attempt at a photographic observation. In order not to increase the time of exposure in this first experiment we worked with all the rays without interposing any absorbing medium. Since the objective of the equatorial was achromatized for the visual rays, it was impossible to obtain a good image of the stars and of the nebula: the images of stars are circles of considerable diameter, although we had made the setting for the mean of the photographic rays. That had no effect on the sharpness of the rings, the images of which are given solely by the objective *C*. With a difference of path of 0.6 mm and an

[1] Ch. Fabry and A. Perot, *Astrophysical Journal*, **15**, 81, 1902.

exposure of $1\frac{1}{4}$ hours, we obtained perfectly sharp rings on a Lumière Sigma plate. Furthermore, the images of the stars destroy part of the field. The strongest rays in the photographic spectrum of the nebula of *Orion* are the hydrogen line, γ, at λ 4341, and the line of unknown origin at λ 3727; the rings photographed result from the superposition of the rings due to these two rays.

These first attempts have no other object than to show the possibility of applying our method. We hope to be able to employ it with instruments which are more powerful and better adapted to the purpose, in particular with a reflecting telescope. All the difficulty from lack of achromatism would thus be avoided. Furthermore, the use of an objective of large diameter allows us to employ a great amount of light. It would then be possible to combine the different lenses of the apparatus in such a way that the ratio of aperture to focal length of the instrument should be large, which would permit short exposures of the photographs without making the images too small. This result may indeed be obtained whatever is the ratio of the aperture of the telescope objective. Under the conditions of our experiments, the ratio of aperture of a complete system would be only 1:12; with a large objective, it would probably not be impossible to go to 1:4.

The use of the interference method will be able to yield certain interesting results. It would be easy to measure the wave-lengths of the different rays with great precision and thus to derive the radial velocity of the object, in using the hydrogen lines. The variations of wave-length from one point to another would give us the circulatory movement of the gas. The determination of the limits of interference would give the size of the different lines; and an indication as to the temperature would be furnished by the size of the hydrogen lines, while the size of the lines of unknown origin would give us a clue to the atomic weight of the gas which forms them.

We have been able to make this experiment by the courtesy of M. H. Bourget, director of the Observatory of the University of Marseilles, who has been good enough to permit us to employ the equatorial of the observatory and has given us his personal assistance in its use. We extend to him our very sincere thanks.

Université de Marseille
March 1911

OBSERVATIONS OF *NOVA LACERTAE* AT THE YERKES OBSERVATORY

By EDWIN B. FROST

The temporary star in *Lacerta* announced by Espin on December 30, 1910, has been observed here in part as follows.

The position of the *Nova* was accurately determined with the micrometer of the 40-inch telescope by Mr. Barnard as R.A. $22^h\ 32^m\ 11^s.79$, Dec. $+52°15'19''.8$ (1911.0). He is also making a triangulation of the neighboring stars, which will be published elsewhere. Upon examining photographs of this region which he had taken in previous years, Mr. Barnard found[1] a star of about the fourteenth magnitude in exactly the position of the *Nova* on four dates, namely:

1907, August 7,	Exposure[2]	$0^h\ 47^m$	Bruce Telescope, 10-inch and 6-inch lenses
1909, August 22,		5 55	Bruce Telescope, 10-inch and 6-inch lenses
1909, August 24,		3 55	Bruce Telescope, 10-inch and 6-inch lenses
and			
1893, October 11,		6 21	Willard Lens, Lick Observatory

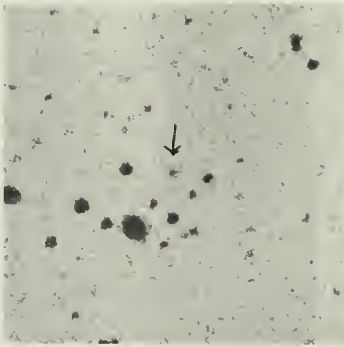
He has also taken several new photographs of the field of the *Nova* with the 10-inch and 6-inch lenses. In Plate XII, Fig. 1, is reproduced the photograph of August 7, 1907, with the 10-inch lens, enlarged twelve times. The arrow points at the *Nova*. Fig. 2 shows the same field taken with the two-foot reflector by Mr. Slocum on January 30, 1911, with an exposure of 25 minutes. It is enlarged sixfold. The lettering of the stars is that which will be used in Mr. Barnard's triangulation. The *Nova* is designated by *N*.

[1] *Astronomische Nachrichten*, 187, 63, 1911.

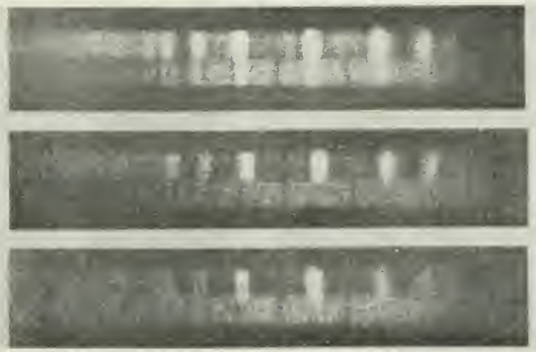
[2] Mr. Barnard advises me that the two plates in 1909 were exposed by Mr. Morehouse.

PLATE XII

3. Spectrum taken with Bruce spectrograph, January 3, 1911 (Frost)

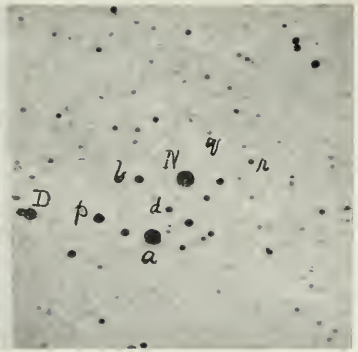


1. August 7, 1907 (Barnard)
10-inch Bruce doublet



Three spectra of *Nova* (above) and nearby star (below) taken with prismatic camera (Parkhurst)

4. January 23, 1911
5. January 30
6. February 22



2. January 30, 1911 (Slocum)
Two-foot reflector

NOVA LACERTAE

The first photographs of the *Nova* with the reflector, by Messrs. Parkhurst and Slocum, suggested the possibility of some nebulosity around the star; but the telescope had been put out of adjustment on account of some recent alterations, and subsequent exposures by Mr. Slocum, after readjustment of the instrument, did not confirm the presence of any nebulous surroundings of the star.

Photographs of the *Nova* have been obtained on two nights by Mr. Slocum with the 40-inch telescope, for the subsequent determination of its parallax, if the star remains sufficiently bright.

On the nights of January 28 and February 4, 1911, Mr. Parkhurst made three sets of measures of the brightness of a sequence of stars near the *Nova*, using the equalizing-wedge photometer attached to the 40-inch telescope. Assuming for the present that the magnitude of star $a = +51^\circ.3420$ is 8.80, as given in the *B.D.*, his values are as follows:

$a = 8.80$	. . .	$d = 12.20$
$D = 10.09$	. . .	$r = 12.93$
$p = 11.57$	. . .	$q = 14.46$
$b = 12.07$		

Mr. Parkhurst calls attention to the strongly actinic quality of the light of the *Nova*, in spite of its decidedly red color. The photographs with the two-foot reflector indicate a color-index like that of an A star. As an explanation of the anomaly that a star apparently so red should photograph as quickly as a white star, Mr. Parkhurst suggests that much of the light comes from the unusual extension of the continuous spectrum in the ultraviolet. This is clearly seen on the uppermost of the three objective-prism plates (for January 23) shown in Plate XII, and it would appear to be an entirely adequate explanation.

Mr. Barnard has referred (*loc. cit.*) to the focal peculiarity of the star when observed with the forty-inch telescope. As was the case with *Nova Geminorum* in 1903, a sharp crimson image is formed 9 mm farther from the objective than the usual image, which latter agrees in position with that for ordinary stars, and is of a whitish color surrounded by a crimson glow. The strong concentration of the light at $H\alpha$ would sufficiently account for this appearance.

Visual observations of the spectrum were made by the writer at the forty-inch telescope on December 31 and subsequently, with a small ocular spectroscope. Measurements were not possible, but the appearance was that typical of a nova, recalling early views of *Nova Aurigae* in 1892. $H\alpha$ was brilliant, and strong bright lines could be seen in the yellow and green, but could not be identified with certainty. The dark components could not be distinguished in that region of the spectrum, but could be seen faintly in the violet.

With the exception of the spectrogram secured with the Bruce spectrograph on January 3, the photographs of the spectrum have been obtained by Mr. Parkhurst with our efficient Zeiss doublet of "ultra-violet" glass (of aperture 14.5 cm and focal length 81.4 cm) with 15° objective-prism of U.V. flint. The scale is very small, 3.0 mm from $H\beta$ to $H\theta$, but much useful information may be gained from the plates of stars quite beyond the reach of our other spectrographs. The list of negatives thus far obtained is as follows:

No.	Date	Exposure	No.	Date	Exposure
414	1910 Dec. 31	12^m	420	1911 Jan. 22	$..^m$
415	31	60	421	23	60±
416	31	40	422	29	30
417	1911 Jan. 16	..	423	30	22±
418	17	42	424	Feb. 22	37
419	21	60	427	March 19	69
			428	April 6	60

The plates of January 23, January 30, and February 22 are reproduced, with 13-fold enlargement, in Plate XII. Of course this is an excessive enlargement for such negatives, on coarse-grained emulsions of high speed, but it is necessary if the engraving is to show any detail. The star *B.D.* 51°3420 (No. 7788 A.G., Cambridge, U.S.) is so close to the nova that it serves in a manner for a comparison spectrum, although shifted some 150 Å.U. toward the violet by the difference in declination of the two stars. Its spectrum falls just below that of the nova on the three plates, but on the upper one it is overlapped by the stronger lines of the nova, due to an excess of broadening in right ascension.

Only three of these objective-prism plates, namely, those taken on December 31, precede in time the Bruce spectrogram, which may now be described. Its horizontal enlargement on Plate XII is about $4\frac{1}{2}$. It will be understood that the fine lines shown are

	Violet Edge	Red Edge	Mean or Center	Displacement from Normal Position
Bright $H\epsilon$	3960.6	3984.6	(3972.6)	+2.4 Å.U
1st dark $H\delta$	4084.2	4086.5	(4085.4)	−16.5
"Split"	......	......	4086.9	−15.0
2d dark $H\delta$	4087.3	4090.5	(4088.9)	−13.0
Bright $H\delta$	4090.5	4116.7	(4103.6)	+ 1.7
Dark in bright δ	......	......	4110.3	+ 8.4
Adjacent bright max. real ?	......	......	4112.2	+10.3
1st dark $H\gamma$	4322.0	......	......	−18.6
Possible "split"	......	......	4325.8	−14.8
2d dark $H\gamma$	......	4329.6	......	−11.0
Bright $H\gamma$	4329.6	4356.2	(4342.9)	+ 2.3
Dark in bright γ	......	......	4349.5	+ 8.9
Narrow adjacent bright	......	......	4351.3	+10.7
Faint bright region	4507.5±	4525.7±	......	......
Bright region	4581.7±	4589.9±	......	......
Max. within this	......	......	4583.7	......
Broad 4640 band	4600	4708	(4654)	......
Possible dark therein	......	......	4613.5	......
Apparent bright max. in band	......	......	4640.0	......
Apparent dark	4838.4	......	......	−23.1
Apparent "split"	......	......	4842.6	−18.9
2d dark	......	4847.3	......	−14.2
Bright $H\beta$	4847.3	4879.3	(4863.3)	+ 1.8
1st bright max.	......	......	4860.8	− 0.7
2d bright max. dupl. ?	......	......	4875.2	+13.7
Dark in bright β	......	......	4890.4	......
Bright patch (helium)	......	......	4922	......
Faint bright marking (helium)	......	......	5016	......

unavoidably introduced in the process of vertical enlargement. The essential features are the broad bright lines of hydrogen and that at λ 4640; the bright maxima and possibly dark shadings within these bright lines, and the dark lines adjacent to their violet edges, which are best seen in case of $H\gamma$ and $H\delta$. The negative was obtained with an exposure of 2 hours on January 3, mid-exposure

at 12^h 46^m G.M.T., with mean hour angle 3^h 9^m West. The slit-width was 0.10 mm, and the comparison spectra were *Ti* and *Fe*. At $H\gamma$ the scale of the negative is 1 mm = 26.0 Å.U.

I made settings upon the edges and maxima of all the lines, as accurately as their very diffuse nature would permit, from which the following necessarily rough wave-lengths were deduced. The word "split" denotes a narrow separation between two components of a line, without affirming whether it is a reversal, a narrow portion of continuous spectrum, or an actual narrow line.

The value in the fourth column, entitled Mean or Center, is inclosed in parentheses when it represents the mean of the wave-length for the two edges of the band; otherwise the wave-length is that of the center of the line, upon which the setting was made.

The constitution of the hydrogen lines will be better indicated if the data of the fifth column are summarized, thus giving the displacements in Å.U. of the particular feature from its normal position.

Hydrogen Line	First Dark	Second Dark	"Split"	Center of Broad Bright	Dark within Bright	Narrow Bright
ϵ	.....	.....	.....	+2.4	.....	......
δ	−16.5	−13.0	−15.0	+1.7	+8.4	+10.3
γ	−18.6	−11.0	−14.8	+2.3	+8.9	+10.7
β	−23.1	−14.2	−18.9	+1.8	.....	+13.7
Mean.......	−19.4 Å.U.	−12.7 Å.U.	−16.2 Å.U.	+2.0 Å.U.	+8.7 Å.U.	+11.6 Å.U.

Too much weight should not be attached to results depending upon a single spectrogram, and the settings on the edges of such diffuse lines may be grossly in error; but the tolerable accordance of the displacements for the different lines gives some confidence as to the order of their magnitude. If the Doppler effect were thought to be involved (which seems highly improbable), the velocities corresponding to the displacements would for the mean of the various columns be about −1300, −880, −1100, +120, +580, and +760 km per second.

These displacements and velocities are quite parallel to those observed in case of *Nova Aurigae*, as well as *Nova Persei*. The

doubt still expressed in some quarters as to whether *Nova Lacertae* is a variable or a temporary should be wholly dispelled by the similarity of the spectrum to that of other novae.

It should be noted that such displacements may be to a considerable extent spurious, both for the dark and the bright components. This is particularly true for the dark components, which may be much narrowed on the less refrangible side by the overlapping bright radiations. However, if the "split" seen between the first and second dark components (which, by the way, was also noted in *Nova Aurigae*) is a genuine reversal, it may represent the center of the broad dark line.

The spectrogram under discussion unfortunately does not include the neighborhood of K with sufficient strength to testify whether or not K is sharp and narrow, as it was in *Nova Persei.* The plates obtained with the prismatic camera do not show it, but a very fine line could hardly be detected owing to the exceedingly small scale and great breadth of the neighboring lines.

It is much regretted that no other evening was available for the Bruce spectrograph on which the weather conditions were such as to make it worth while to attempt an exposure, until the star's brightness had declined too far.

OBJECTIVE-PRISM PLATES

I have measured the five best plates obtained with the prismatic camera, deriving the relative wave-lengths from a curve drawn from measures on other stars on the plates. The wave-length of the center of bright $H\gamma$ was assumed to be normal. The subsequent alterations in bright $H\gamma$, however, are too great for it to be a safe standard of reference. It must therefore be understood that the measures are only relative. The uncertainty of settings on the edges of such bright lines is very great, and a unit of ten Ångströms would be more suitable.

The widths of the lines are of the same order as for the spectrogram of January 3: on the later plates the duplicity of the bright lines of hydrogen is more obvious. The plate of February 22 shows bright $H\gamma$ very distinctly triple, the central component the strongest in intensity. $H\delta$, however, is clearly double, not

triple; but on April 6 $H\gamma$ has only two, approximately equal components.

WAVE-LENGTHS DERIVED FROM CURVE, THE SETTING FOR BRIGHT $H\gamma$ BEING ASSUMED

	No. 418 Jan. 17	No. 421 Jan. 23	No. 423 Jan. 30	No. 424 Feb. 22	No. 428 April 6
$H\eta$ λ 3835	....	{ 3819 { 3845	....	....	....
V.E. of dark, adjacent to	....	3850	....	....	....
$H\zeta$ bright	3880 3898	{ 3872 { 3901	3881 3895	3887	3850 3875
$H\epsilon$ bright	3963 3980	{ 3958 { 3990	3963 3982	3960 3984	3955 3978
V.E. of dark, adjacent to	....	4069	....	....	4060
$H\delta$ bright	4090 4106	{ 4082 { 4111	{ 4085 { 4115	4090 4110	4082 4102
V.E. of dark, adjacent to	....	4288	4298	....	4283
$H\gamma$ bright	4332 4348	{ 4317 { 4364	4320 4298	4328 4348 4375	4340 4364
Very faint bright	4479	4464	....	....	....
V.E. dark, adjacent to	....	4550	....	....	....
λ 4640 bright	4648	{ 4610 { 4672	{ 4619 { 4670	{ 4616 { 4675	4628
$H\beta$ bright	4882	{ 4835 { 4900	4860	4865	4863
Faint bright	....	4987	....	....	....
Faint bright	....	5045	....	5016	5013

Measures on edges are connected by braces; otherwise the settings were on centers of bands or of their separate components. V.E. denotes edge toward violet.

On the earlier spectrograms the intensities of the bright lines $H\gamma$ and $H\delta$ do not differ more than would result from the fact that the camera is focused for them; but subsequently $H\gamma$ plays a predominant part in the emission of light, and the relative waning of the helium lines λ 4922 and 5016 and of $H\beta$ is marked.

It is not possible to say from the evidence of plate 428 (April 6) whether the transformation to the nebular spectrum has begun; the scale of the plates is too small to allow us to decide whether the line previously at λ 5016 (helium) has been replaced by λ 5007,

the chief nebular line. The change in appearance of $H\delta$ since February 22 is not appreciable; at $H\gamma$ the change is the return from a triplet to the double line previously present.

The position of the star is such that these small-scale spectrograms will presumably continue to be secured until the star becomes too faint to be followed further.

Yerkes Observatory
May 2, 1911

ADDED TO PROOF SHEETS
MAY 15, 1911

Spectrograms obtained by Mr. Parkhurst on May 4 and 6 show a marked increase in intensity of the band near λ 5000. Settings on the center of this band, reduced on the assumption that the structure of $H\gamma$ is the same as on April 6, yield a wavelength of 5000, with edges roughly at λ 4962 and λ 5048. This may therefore be regarded as the first nebular line, λ 5007, although the second nebular line, λ 4959, cannot be seen separately, so that we cannot assert that it is present. The band is not sufficiently resolved so that we can declare whether the two helium lines, λ 4923 and λ 5016, have retired from the spectrum or are included within the broad band. The complex character of all the hydrogen lines is maintained as on April 6, and their appearance is not obviously different from that on the plates taken in January and February, excepting the period when $H\gamma$ was triple.

PHOTOGRAPHIC DETERMINATIONS OF STELLAR PARALLAX MADE WITH THE YERKES REFRACTOR. VI

By FRANK SCHLESINGER

Lalande 39866 ($20^h 35^m, +4° 37'$)

This 8th-magnitude star has a proper motion of a little less than 1″ a year. The fourteen plates secured were measured by Miss Ware.

TABLE I

Plates of *Lalande 39866*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
377....	1904 July 10	$-0^h.5$	S, Su, S	Fair	
431....	Aug. 25	+0.8	S, Su, S	Fair	
441....	Aug. 27	−0.6	S, Su, S	Fair	
447....	Sept. 4	+0.2	S, Su, S	Fair	
457....	Sept. 11	−0.3	S, Su, S	Fair	
509....	Oct. 30	+0.2	S. S, S	Good	
698....	1905 June 24	+1.2	Su, F	Poor	
704....	July 15	+0.1	F, Su, F	Good	
752....	Sept. 2	+0.8	Su, Su	Poor	
758....	Sept. 10	+1.5	Su, Su, Su	Good	
768....	Sept. 12	+0.8	Su, Su, Su	Poor	
939....	1906 June 5	−0.4	Su, J, Su	Good	Star (15) not measurable on second exposure
940....	June 8	−1.4	Su, Su	Poor	
941....	June 8	−1.1	Su, J, Su	Poor	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
4..........	0.72	−362	−168	+.234	+.20
15..........	0.64	−167	+112	+.093	+.10
30..........	0.63	+167	− 98	+.298	+.30
31..........	0.73	+118	+217	+.084	+.10
35..........	0.84	+244	− 63	+.291	+.30
Parallax star.	1.41	+ 31	− 58		

TABLE 2

REDUCTIONS FOR *Lalande 39866*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
377........	0.138	0.8	+0.413	−328	+.009	+″02
431........	0.176	0.7	−0.346	−282	+ 27	+.06
441........	0.153	0.7	−0.378	−280	+ 2	.00
447........	0.092	0.7	−0.499	−272	− 62	−.14
457........	0.150	0.7	−0.599	−265	− 7	−.02
509........	0.206	0.9	−0.989	−216	+ 17	+.04
698........	0.449	0.3	+0.644	+ 21	+ 18	+.03
704........	0.445	0.9	+0.337	+ 42	+ 4	+.01
752........	0.465	0.3	−0.467	+ 91	+ 3	.00
758........	0.467	0.9	−0.583	+ 99	+ 1	.00
768........	0.483	0.4	−0.610	+101	+ 15	+.03
939........	0.727	0.9	+0.858	+367	− 2	−.01
940........	0.725	0.3	+0.830	+370	− 7	−.01
941........	0.725	0.5	+0.830	+370	− 7	−.01

The normal equations are:

$$
\begin{aligned}
+3.584\pi + 8.888\mu - 0.812c &= +0.521 \\
+58.740 \quad -3.989 &= +3.620 \\
+9.000 &= +3.217
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.397 \\
\mu &= +0.0848 = +0.''226 \\
\pi &= +0.0251 = +0.''067 \pm 0.''022
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0125 = \pm 0.''033$

The residual for Plate 447 is unusually large, but a solution without this plate yields almost precisely the same parallax ($+0.''066$) with, however, a much smaller probable error ($\pm 0.''010$). Not to over-estimate the accuracy of our result, the parallax from the original solution is regarded as definitive.

The only other determination of this parallax that has been published is by Chase, $+0.''05 \pm 0.''047$.

Groombridge 3689 ($22^h 3^m, +52° 39'$)

This 8th-magnitude star has a proper motion of 0."6 per annum. The sixteen plates secured were measured in right ascension and in declination and the parallax was independently determined from the shifts in these two directions. All the measurements were made by Miss Ware.

TABLE 1

Plates of *Groombridge 3689*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
50....	1903 July 19	$-0^h.7$	S, Su, S	Good	
170A ..	Nov. 26	+0.7	S, S, S	Fair	Telescope East
181....	Dec. 6	+1.3	S, S, S	Fair	Telescope East
361....	1904 June 12	−2.0	S, Su, S	Good	
372....	June 19	−2.0	S, Su, S	Good	
379....	July 10	−0.3	S, Su, S	Good	Second exposure fair
381....	July 16	−1.4	S, Su, S	Fair	
390....	July 17	−1.8	S, Su, S	Fair	
397....	July 24	−1.7	S, Su, S	Fair	
458....	Sept. 11	−1.2	S, Su, S	Fair	
711....	1905 July 22	−1.3	Su, F	Fair	
716....	July 23	−1.2	F, Su, F	Fair	
721....	July 25	−0.5	F, Su, F	Poor	Images slightly triangular
794....	Oct. 1	−2.0	Su, Su, Su	Good	
796....	Oct. 3	−1.1	Su, J, Su	Good	
806....	Oct. 7	−0.4	Su, J, Su	Good	

COMPARISON STARS

No.	Diameter	X (right ascension)	Y (declination)	Dependence	
				Computed	Adopted
6..........	0.50	−262	+140	+.213	+.21
9..........	0.82	−206	− 94	+.072	+.07
15..........	0.52	− 57	+174	+.269	+.27
16..........	0.52	− 36	− 87	+.105	+.11
21..........	0.45	+164	+ 42	+.221	+.22
25..........	0.64	+397	−175	+.120	+.12
Parallax star.	1.21	− 5.7	+ 49.1		

TABLE 2

REDUCTIONS IN RIGHT ASCENSION FOR *Groombridge 3689*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
50........	0.493	0.8	+0.509	−385	−.033	−″.08
170A.......	0.456	0.7	−0.923	−255	+ 13	+.03
181........	0.438	0.7	−0.892	−245	+ 1	.00
361........	0.326	0.9	+0.876	− 56	− 16	−.04
372........	0.348	0.9	+0.825	− 49	+ 10	+.03
379........	0.345	0.8	+0.613	− 28	+ 21	+.05
381........	0.324	0.7	+0.537	− 22	+ 3	+.01
390........	0.336	0.7	+0.525	− 21	+ 16	+.04
397........	0.321	0.6	+0.428	− 14	+ 6	+.01
458........	0.293	0.7	−0.328	+ 35	+ 11	+.02
711........	0.101	0.5	+0.460	+349	− 8	−.02
716........	0.125	0.7	+0.446	+350	+ 17	+.04
721........	0.081	0.4	+0.418	+352	− 26	−.04
794........	0.085	0.8	−0.598	+420	+ 24	+.06
796........	0.031	0.9	−0.622	+422	− 29	−.07
806........	0.045	0.8	−0.668	+426	− 12	−.03

The normal equations are:

$$
\begin{aligned}
+4.848\pi - 3.772\mu + 1.065c &= +0.572\\
+85.615 + 8.280 &= -2.378\\
+11.600 &= +3.064
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.304\\
\mu &= -0.0569 = -0''.151\\
\pi &= +0.0069 = +0''.018 \pm 0''.014
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0115 = \pm 0''.031$

TABLE 3

REDUCTIONS IN DECLINATION FOR *Groombridge 3689*

Plate	Solution (l)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
50........	0.364	0.8	+0.858	−385	−.013	−″03
170A.......	0.304	0.7	−0.224	−255	+ 5	+.01
181........	0.292	0.7	−0.374	−245	+ 1	.00
361........	0.245	0.9	+0.501	− 56	− 6	−.02
372........	0.248	0.9	+0.590	− 49	− 4	−.01
379........	0.264	0.8	+0.802	− 28	+ 13	+.03
381........	0.254	0.7	+0.843	− 22	+ 4	+.01
390........	0.231	0.7	+0.849	− 21	− 18	−.04
397........	0.263	0.6	+0.888	− 14	+ 14	+.03
458........	0.251	0.7	+0.800	+ 35	+ 22	+.05
711........	0.129	0.5	+0.877	+349	+ 7	+.01
716........	0.127	0.7	+0.882	+350	+ 6	+.01
721........	0.105	0.4	+0.891	+352	− 17	−.03
794........	0.094	0.8	+0.603	+420	+ 5	+.01
796........	0.094	0.9	+0.577	+422	+ 7	+.02
806........	0.060	0.8	+0.526	+426	− 25	−.06

The normal equations are:

$$\begin{aligned} +5.752\pi + 8.524\mu + 7.000c &= +1.395 \\ +85.615 + 8.280 &= -0.927 \\ +11.600 &= +2.440 \end{aligned}$$

These yield

$$\begin{aligned} c &= +0.217 \\ \mu &= -0.0348 = -0.''093 \\ \pi &= +0.0299 = +0.''080 \pm 0.''018 \end{aligned}$$

Probable error corresponding to unit weight, $\pm 0.0078 = \pm 0.''021$

The mean by weight of the two determinations is

$$+0.''041 \pm 0.''020$$

I have increased the probable error of this mean over its theoretical value in view of the discordance of the two results.

Plates 170A and 181 were taken with the telescope east of the pier and at hour angles that differ considerably from those for

the other plates. A solution in right ascension without them gives the following:

$$c = +0.298$$
$$\mu = -0.0548 = -0''.146$$
$$\pi = +0.0172 = +0''.046 \pm 0''.025$$

Probable error corresponding to unit weight, $\pm 0''.032$

This value differs by only a few thousandths of a second from that obtained by uniting the two determinations from all the plates, and the latter was allowed to stand as the definitive parallax. We see from Table 3 that if we omit Plates 170A and 181, the parallax factors in declination for the other plates differ little from each other, and that it is useless to attempt a least-squares solution in this co-ordinate without these two plates.

With the Yale heliometer Dr. Chase has obtained for this parallax $+0''.03 \pm 0''.047$. Professor Flint kindly informs me, in advance of publication, that his second series with the transit circle at Madison yields $0''.00 \pm 0''.028$.

Krüger 60 ($22^{h}\ 24^{m}$, $+57^{\circ}\ 12'$)

This star is one of a wide double discovered in 1873 during the progress of the work for the *Astronomische Gesellschaft Catalog*. In 1890 Burnham found that the brighter component is itself a double; and Doolittle and Barnard have since shown that the latter is a binary with comparatively rapid orbital motion. The star was put upon my observing list at the suggestion of Professor Barnard, who was at the same time making a series of measurements for the same purpose with the micrometer of the 40-inch, and who surmised from general considerations that the parallax of this system must be large. In the *Astrophysical Journal*, **20**, 128, 1904, I published a preliminary value[1] of this parallax from eight plates; this showed that Professor Barnard's inference was correct and that the parallax is about one-quarter of a second. Professor Barnard's work confirmed this large value, and the parallax has also been investi-

[1] This preliminary work is entirely superseded by the present paper, where the same plates are definitively discussed in connection with others secured later.

gated by Professor Russell. The three results are very accordant, so that this system is not only included among the ten that are at present known to be nearest us, but is also one of an even shorter list whose distances are known with the least percentage of error.

TABLE 1

PLATES OF *Krüger 60*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
45....	1903 July 12	$-1^{h}.4$	S, Su, S	Poor	Star (21) lacking on second and third exposures
49....	July 19	−1.7	S, Su, S	Fair	Second exposure poor
64....	Aug. 3	−0.5	S, Su, S	Fair	
171A...	Nov. 26	+1.2	S, Su, S	Good	Telescope East
182....	Dec. 6	+1.8	S, S. S	Fair	Telescope East
362....	1904 June 12	−1.9	S, Su	Fair	.
373....	June 19	−1.7	S, Su, S	Fair	
378....	July 10	−1.5	S, Su, S	Fair	
382....	July 16	−1.2	S, Su, S	Good	
389....	July 17	−1.5	S, Su, S	Good	
448....	Sept. 4	−0.6	S, Su, S	Good	
459....	Sept. 11	−1.0	S, Su, S	Good	
511....	Oct. 30	−0.3	S, Su, S	Good	
712....	1905 July 22	−1.0	F, Su, F	Good	
717....	July 23	−1.0	F, Su, F	Good	
722....	July 25	−0.3	F, F	Good	
797....	Oct. 3	−0.9	Su, J, Su	Fair	
807....	Oct. 7	−0.2	Su, J, Su	Good	
850....	Nov. 12	−0.2	Su, J, Su	Fair	

COMPARISON STARS

No.	Diameter	X (longitude)	Y (latitude)	Dependence	
				Computed	Adopted
13..........	1.06	−188	− 97	+.289	+.285
15..........	1.18	−122	+136	+.076	+.08
21..........	0.71	+ 53	− 23	+.289	+.285
30..........	1.53	+257	− 16	+.345	+.35
Parallax star.	1.02	+ 40.4	− 29.9		

There is another good comparison star, numbered 22, at $X=+39$, $Y=+213$, but the dependence comes out small (+0.017) and it was therefore not used.

TABLE 2

REDUCTIONS IN RIGHT ASCENSION FOR *Krüger 60*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
45........	0.930	0.4	+0.662	−392	−.005	−″.01
49........	0.912	0.8	+0.579	−385	− 9	−.02
64........	0.884	0.8	+0.331	−367	+ 3	+.01
171A.......	0.665	1.0	−0.920	−255	+ 3	+.01
182........	0.648	0.6	−0.904	−245	− 7	−.01
362........	0.680	0.6	+0.901	− 56	+ 17	+.03
373........	0.656	0.5	+0.862	− 49	+ 3	+.01
378........	0.598	0.8	+0.676	− 28	− 19	−.05
382........	0.619	0.9	+0.606	− 22	+ 14	+.04
389........	0.606	0.9	+0.594	− 21	+ 4	+.01
448........	0.494	1.0	−0.132	+ 28	+ 5	+.01
459........	0.474	1.0	−0.240	+ 35	+ 1	.00
511........	0.368	0.9	−0.824	+ 84	− 5	−.01
712........	0.261	0.9	+0.533	+349	− 11	−.03
717........	0.271	0.9	+0.521	+350	+ 1	.00
722........	0.268	0.6	+0.494	+352	+ 2	.00
797........	0.091	0.7	−0.549	+422	− 12	−.03
807........	0.082	0.9	−0.599	+426	− 13	−.03
850........	0.059	0.7	−0.894	+462	+ 24	+.05

The normal equations are:

$$
\begin{aligned}
+6.166\pi - 4.212\mu + 0.706c &= +1.338 \\
+113.334 + 7.015 &= -6.665 \\
+14.900 &= +7.297
\end{aligned}
$$

These yield

$$
\begin{aligned}
c &= +0.527 \\
\mu &= -0.0878 = -0''.234 \\
\pi &= +0.0966 = +0''.257 \pm 0''.007
\end{aligned}
$$

Probable error corresponding to unit weight, $\pm 0.0067 = \pm 0''.018$

TABLE 3

REDUCTIONS IN DECLINATION FOR *Krüger 60*

Plate	Solution (l)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
45	0.526	0.2	+0.770	−392	+.011	″00
49	0.510	0.8	+0.827	−385	− 9	−.02
64	0.523	0.8	+0.926	−367	+ 4	+.01
171A	0.368	1.0	−0.132	−255	0	.00
182	0.360	0.6	−0.290	−245	+ 11	+.02
362	0.318	0.6	+0.422	− 56	− 2	.00
373	0.319	0.5	+0.518	− 49	− 5	−.01
378	0.347	0.8	+0.758	− 28	+ 11	+.03
382	0.340	0.9	+0.811	− 22	+ 3	+.01
389	0.347	0.9	+0.819	− 21	+ 10	+.03
448	0.318	1.0	+0.900	+ 28	− 2	−.01
459	0.301	1.0	+0.861	+ 35	− 13	−.03
511	0.227	0.9	+0.289	+ 84	− 11	−.03
712	0.161	0.9	+0.853	+349	+ 4	+.01
717	0.153	0.9	+0.861	+350	− 5	−.01
722	0.151	0.6	+0.874	+352	− 7	−.01
797	0.123	0.7	+0.662	+422	+ 19	+.04
807	0.102	0.9	+0.614	+426	+ 4	+.01
850	0.024	0.7	+0.087	+462	− 9	−.02

The normal equations are:

$$\begin{aligned} +7.373\pi + 6.735\mu + 9.001c &= +2.612 \\ +110.261 + 7.799 &= -2.871 \\ +14.700 &= +4.154 \end{aligned}$$

These yield

$$\begin{aligned} c &= +0.254 \\ \mu &= -0.0495 = -0.''132 \\ \pi &= +0.0893 = +0.''238 \pm 0.''011 \end{aligned}$$

Probable error corresponding to unit weight, $\pm 0.0054 = \pm 0.''014$

Combining these two values of the parallax in accordance with their probable errors we have as the definitive parallax of the system

$$+0.''252 \pm 0.''006$$

Plates 171A and 182 were secured with the telescope on the unusual side of the pier. A least-squares solution in right ascension without these plates yields:

$$c = +0.526$$
$$\mu = -0.0878 = -0''.233$$
$$\pi = +0.0968 = +0''.257 \pm 0''.010$$

Probable error corresponding to unit weight, $\pm 0''.019$

A similar solution in declination gives

$$c = +0.246$$
$$\mu = -0.0490 = -0''.130$$
$$\pi = +0.1003 = +0''.267 \pm 0''.018$$

Probable error corresponding to unit weight, $\pm 0''.014$

The mean by weight of these two values of the parallax is $+0''.259 \pm 0''.009$, which differs by $0''.007$ from that which results from the use of all the plates. The original mean is allowed to stand as the definitive value.

Professor Barnard has obtained $+0''.249 \pm 0''.010$ for this parallax from an extensive series of measures with the 40-inch micrometer. Professor Russell has recently derived $+0''.258 \pm 0''.013$ from measures of plates secured at Cambridge, England.

Lalande 46650 ($23^h 44^m$, $+1° 52'$)

This 9th-magnitude star has a proper motion of $1''.4$ per annum. Thirteen plates were secured as described in Table 1. There is considerable displacement in declination and the plates were accordingly measured in this direction (as well as in right ascension) but the parallax from this co-ordinate proved to have little weight.

TABLE 1

PLATES OF *Lalande 46650*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
51....	1903 July 19	$-1^h.8$	S, Su, S	Poor	
60....	Aug. 2	−1.3	S, Su, S	Fair	Second exposure poor
70....	Aug. 11	−0.5	S, Su, S	Fair	Second exposure poor
399....	1904 July 24	−0.9	S, Su, S	Good	
432....	Aug. 25	−0.6	S, Su, S	Good	
498....	Oct. 16	0.0	S, (Su, S)	Fair	Second and third exposures coincide
513....	Oct. 30	−0.4	S, Su, S	Good	
715....	1905 July 22	−0.2	F, Su, F	Fair	First exposure poor
720....	July 23	−0.6	Su, F	Fair	
724....	July 25	−0.2	F, F		First exposure fair, second good
812....	Oct. 7	+1.0	Su, J, Su	Good	
821....	Oct. 8	+1.0	Su, J, Su	Fair	Second exposure good
852....	Nov. 12	−0.5	Su, J, Su	Poor	

COMPARISON STARS

No.	Diameter	X (right ascension)	Y (declination)	Dependence	
				Computed	Adopted
3............	0.70	−363	+102	+.647	+.645=+1.00÷1.55
9............	0.63	+ 97	+158	−.038	−.032=−0.05÷1.55
11............	0.74	+266	−260	+.390	+.389=+0.60÷1.55
Parallax star....	1.25	−134	− 42		

Plates 432, 498, and 513 were measured by both Miss Ware and the writer, the others by Miss Ware alone. The accordance between the two sets of measures for these three plates is such, that if only Miss Ware's measures had been used for them (as for the other ten plates) the effect upon the parallaxes deduced from either right ascensions or declinations would have been practically nil.

TABLE 2

REDUCTIONS IN RIGHT ASCENSION FOR *Lalande 46650*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
51........	0.087	0.4	+0.801	−385	−.018	−″03
60........	0.099	0.6	+0.668	−371	− 11	−.02
70........	0.141	0.6	+0.562	−362	+ 28	+.06
399........	0.470	0.9	+0.751	− 14	− 7	−.02
432........	0.494	1.0	+0.364	+ 18	+ 9	+.02
498........	0.481	0.6	−0.426	+ 70	− 9	−.02
513........	0.490	1.0	−0.606	+ 84	− 3	−.01
715........	0.846	0.6	+0.771	+349	+ 2	.00
720........	0.849	0.5	+0.763	+350	+ 4	+.01
724........	0.845	0.5	+0.744	+352	− 1	.00
812........	0.841	0.9	−0.293	+426	− 16	−.04
821........	0.875	0.8	−0.308	+427	+ 18	+.04
852........	0.876	0.4	−0.739	+462	+ 9	+.02

The normal equations are:

$$\begin{aligned} +3.126\pi - 3.915\mu + 1.647c &= +0.526 \\ +82.185 \quad +10.078 &= +12.561 \\ +8.800 &= +5.036 \end{aligned}$$

These yield

$$c = +0.445$$
$$\mu = +0.1010 = +0.''269$$
$$\pi = +0.0606 = +0.''161 \pm 0.''014$$

Probable error corresponding to unit weight, $\pm 0.0079 = \pm 0.''021$

TABLE 3

REDUCTIONS IN DECLINATION FOR *Lalande 46650*

Plate	Solution (l)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
51........	0.924	0.4	+0.379	−485	−.023	−″04
60........	0.923	0.6	+0.333	−471	− 8	−.02
399........	0.590	0.9	+0.363	−114	+ 13	+.03
432........	0.542	1.0	+0.215	− 82	+ 2	+.01
498........	0.481	0.6	−0.129	− 30	+ 5	+.01
513........	0.468	1.0	−0.215	− 16	+ 9	+.02
715........	0.221	0.6	+0.369	+249	+ 4	+.01
720........	0.239	0.5	+0.366	+250	+ 23	+.04
724........	0.184	0.5	+0.360	+252	− 30	−.06
812........	0.119	0.9	−0.067	+326	− 6	−.02
821........	0.119	0.8	−0.074	+327	− 5	−.01
852........	0.073	0.4	−0.283	+362	− 9	−.02

The normal equations are:

$$\begin{aligned} +0.600\pi - 1.507\mu + 0.954c &= +0.601 \\ +58.022 \quad +4.050 \quad &= -3.988 \\ +8.200 \quad &= +3.329 \end{aligned}$$

These yield

$$c = +0.451$$
$$\mu = -0.0993 = -0.''264$$
$$\pi = +0.0358 = +0.''095 \pm 0.''033$$

Probable error corresponding to unit weight, $\pm 0.0081 = \pm 0.''022$

The two values of the parallax differ somewhat more than we should expect from their probable errors. Combining them in accordance with these errors we obtain for our definitive result:

$$+0.''151 \pm 0.''013$$

Other determinations of this parallax are:

Flint (transit circle)...............	+0″23	±0″092
Elkin and Smith (heliometer).......	+ .185	16
Russell (photography).............	+ .211	15

ALLEGHENY OBSERVATORY
April 1911

[*To be concluded*]

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SUBJECTS

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